

UNCERTAINTY OF REFERENCE FRAMES APPLIED TO COMPUTER AIDED ORTHOPEDIC SURGERY

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Abstract – In this paper, mathematical tools, like Monte Carlo simulations and direct propagation methods based on the GUM, are experienced to calculate the uncertainty of a datum reference in orthopedic surgery. In the last approach, the Jacobian matrices of the measurement process are determined either by analytical calculus or through numerical computations. The methods are applied to the determination of reference frames in total knee arthroplasty. The different approaches are employed to calculate the uncertainties and the results are compared.

Keywords Uncertainty, Mathematical tools, Computer Assisted Orthopedic Surgery (CAOS).

1. INTRODUCTION

Measurement in health applications greatly increased these last ten years. In the field of arthroplasty, several questions are however asked by surgeons on the localisation uncertainty of the prosthesis after implantation. Nevertheless, the quality of implantation of the prosthesis largely impacts its service life (currently 10 years). At this time, Computer Aided Orthopaedic Surgery (CAOS) does not permit the surgeon to know the relation between the implantation uncertainty and the precision at which typical landmarks are probed on the patient. Yet, uncertainty propagation calculation methods are already intensely used in masse, dynamic, dimensional, chemistry ... measurements.

Total knee arthroplasty requires the CAOS system defining reference frames attached to the tibia and the femur. These coordinate systems are derived from the acquisition of a set of three center points of balls fixed to each bone. Since the patient may move during the implementation process, this has to be done, at real time.

In this paper, uncertainty calculation of such reference frames will be treated by several methods (Analytical

propagation, Monte Carlo simulation). These methods will be presented and their results compared and discussed.

2. STATE OF THE ART IN UNCERTAINTY CALCULATION

Monte Carlo (MC) simulation is a common tool in uncertainty evaluation of complex measurement processes. It is used because of the lack or the difficulty to express analytical solutions. The convergence rate of Monte Carlo methods is $O(1/\sqrt{N})$, where N is the number of simulated experiments. Instead of using pseudo-random generators, it can be accelerated by employing deterministic uniformly distributed sequences known as presenting low-discrepancy. Methods based on such sequences are named Quasi Monte Carlo. Asymptotically, Quasi Monte Carlo can provide a rate of convergence of about $O(1/N)$.

Uncertainty can also be determined through computation of sensitivity coefficients. Such factors may be derived from local Taylor expansion approximations of the measurement process functions. A propagation method, based on an analytical calculus of the first order partial derivatives is thus detailed in the Guide to the Expression of Uncertainty in Measurement GUM [1]. It has already been applied successfully to classical dimensional metrology [2], [3]. In our paper, this technique will be extended to complex measurement processes, by using a numerical computation of the partial derivatives.

The sensitivity coefficients may also be defined by Sobol's approach. Sobol's methods [4], [5], [6] are variance based global sensitivity analysis techniques based upon "Total Sensitivity Indices" that account for interaction effects of the variables. The Total Sensitivity Indices of an input is defined as the sum of all the sensitivity indices involving that input. This method includes both main effect of each input as well as the interactions with the other variables [4]. Sobol's method can cope with both nonlinear and non-monotonic models, and provide a truly quantitative ranking of inputs and not just a relative qualitative measure

[5]. Effort has been done to reduce the computational complexity associated with the calculation of Sobol's indices. However, even with its most recent developments, Sobol's method remains computer time consuming.

In this paper, three methods will thus be used to calculate the uncertainty of reference frames constructed from a set of three measured points. These methods are: classical analytical propagation, hybrid propagation (numerical calculation of sensitivity coefficients) and Monte Carlo simulation method.

2. MATHEMATICAL TOOLS TO CALCULATE THE REFERENCE FRAME UNCERTAINTY

In numerous measurement procedures, it is necessary to derive an orthonormal coordinate system from a set of three measured points (Fig. 1.). This paragraph details the calculation of the components of its unit vectors and their uncertainty. The uncertainties can be estimated by several methods. In this paragraph, analytical propagation and Monte Carlo simulation are introduced.

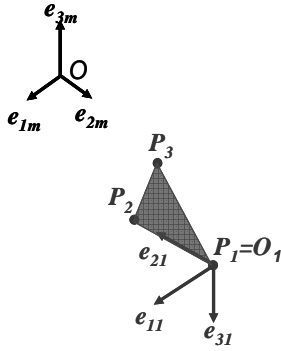


Fig. 1. Reference frame $(\vec{e}_{11}, \vec{e}_{12}, \vec{e}_{13}, O_1)$

3.1. Analytical reference frames derived from three measured points

An example of reference frame constructed from a set of three points P_1, P_2, P_3 , digitalized by a coordinate measuring machine (CMM) is represented in Fig. 1.

This coordinate system is composed of orthogonal unit vectors e_{11}, e_{21} and e_{31} . Equations (1) to (3) detail the calculation of these vectors.

$$\vec{e}_1 = \frac{\overrightarrow{P_1 P_2} \times \overrightarrow{P_1 P_3}}{\|\overrightarrow{P_1 P_2} \times \overrightarrow{P_1 P_3}\|} = \left(\frac{\vec{B}}{\|\vec{B}\|} \right) \cdot \vec{B} \quad (1)$$

$$\text{With } \vec{B} = \overrightarrow{P_1 P_2} \times \overrightarrow{P_1 P_3}$$

$$\vec{e}_2 = \frac{\overrightarrow{P_1 P_2}}{\|\overrightarrow{P_1 P_2}\|} = \left(\frac{\vec{C}}{\|\vec{C}\|} \right) \cdot \vec{C} \quad (2)$$

$$\text{With } \vec{C} = \overrightarrow{P_1 P_2}$$

$$\vec{e}_3 = \vec{e}_1 \times \vec{e}_2 = \vec{D} \quad (3)$$

3.1. Analytical propagation

The classical method of the GUM is used to propagate the uncertainties of the measured points P_1, P_2, P_3 , thus leading to following equation:

$$\text{Cov}(\vec{e}_1, \vec{e}_2, \vec{e}_3, O_1) = J \cdot \text{Cov}(P_i) \cdot J^T \quad (4)$$

$$(12 \times 12) = (12 \times 9) \cdot (9 \times 9) \cdot (9 \times 12)$$

The numbers under equation (4) define the dimensions of the matrices. Jacobian matrix J is defined by:

$$J = \begin{pmatrix} J_{e_1/a_i} \\ J_{e_2/a_i} \\ J_{e_3/a_i} \\ J_{O_1/a_i} \end{pmatrix} \quad (5)$$

Where a_i represents one of the nine point coordinates by which the functions are derived.

The components of the previous global Jacobian are formulated in equations (6) to (9):

$$J_{e_1/a_i} = (\vec{B} \cdot \vec{B})^{-\frac{3}{2}} \cdot [I \cdot (\vec{B} \cdot \vec{B}) - (\vec{B} \otimes \vec{B})] \cdot J_{\vec{B}/a_i} \quad (6)$$

$$J_{e_2/a_i} = (\vec{C} \cdot \vec{C})^{-\frac{3}{2}} \cdot [I \cdot (\vec{C} \cdot \vec{C}) - (\vec{C} \otimes \vec{C})] \cdot J_{\vec{C}/a_i} \quad (7)$$

$$J_{e_3/a_i} = (\vec{D} \cdot \vec{D})^{-\frac{3}{2}} \cdot [I \cdot (\vec{D} \cdot \vec{D}) - (\vec{D} \otimes \vec{D})] \cdot J_{\vec{D}/a_i} \quad (8)$$

$$J_{O_1/a_i} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (9)$$

Where: I is the unit matrix.

The expressions of the intermediate Jacobian matrixes are derived in equations (10) to (12):

$$J_{\vec{B}/a_i} = \begin{bmatrix} 0 & (Z_2 - Z_3) & (Y_3 - Y_2) & 0 \\ (Z_3 - Z_2) & 0 & (X_2 - X_3) & -(Z_3 - Z_1) \\ (Y_2 - Y_3) & (X_3 - X_2) & 0 & (Y_3 - Y_1) \\ (Z_3 - Z_1) & -(Y_3 - Y_1) & 0 & -(Z_2 - Z_1) & (Y_2 - Y_1) \\ 0 & (X_3 - X_1) & (Z_2 - Z_1) & 0 & -(X_2 - X_1) \\ -(X_3 - X_1) & 0 & -(Y_2 - Y_1) & (X_2 - X_1) & 0 \end{bmatrix} \quad (10)$$

$$J_{\vec{C}/a_i} = \begin{pmatrix} -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \quad (11)$$

$$J_{\vec{D}/a_i} = \begin{pmatrix} J_{11} & \dots & J_{19} \\ \vdots & & \vdots \\ J_{31} & \dots & J_{39} \end{pmatrix} \quad (12)$$

The components of this last matrix are easily calculated:

$$\begin{aligned}
J_{11} &= (Z_3 - Z_2) \cdot (Z_2 - Z_1) - (Y_2 - Y_3) \cdot (Y_2 - Y_1) \\
J_{12} &= (X_2 - X_3) \cdot (Y_3 - Y_1) - (X_3 - X_1) \cdot (Y_2 - Y_1) \\
&\quad - (X_3 - X_2) \cdot (Y_2 - Y_1) \quad (13)
\end{aligned}$$

$$\begin{aligned}
J_{38} &= (Z_1 - Z_2) \cdot (Y_2 - Y_1) \\
J_{39} &= (Y_2 - Y_1)^2 + (X_2 - X_1)^2
\end{aligned}$$

The three points P_1, P_2, P_3 , are assumed to be measured independently. The global covariance matrix of the whole nine acquired coordinates is thus composed of three independent sub-matrices:

$$\text{Cov}(P_1, P_2, P_3) = \begin{pmatrix} \text{Cov}(P_1) & 0 & 0 \\ 0 & \text{Cov}(P_2) & 0 \\ 0 & 0 & \text{Cov}(P_3) \end{pmatrix} \quad (14)$$

For all three points P_i , the measurements are assumed to be uniformly scattered in a spherical localisation zone of diameter W (Fig. 2).

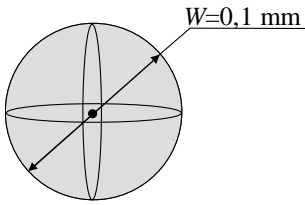


Fig. 2. Localisation zone of measured points

The accuracy of the CMM is thus considered to be the same in any direction and, consequently, each basic covariance matrix takes a spherical form. The standard deviation of the nine independent coordinates is:

$$u_{\text{measure}} = \frac{W}{2\sqrt{3}} \quad (15)$$

The covariant matrix of the datum reference is finally deduced from all these equations.

3.2. Hybrid propagation

In complex processes it is very hard to calculate the Jacobian matrix of the measurement function as it has been done in previous method. We propose therefore now to calculate the components of the Jacobian matrices numerically. Reference frame (e_1, e_2, e_3, O_1) is characterized by the nine cosines of its unit vectors and the three coordinates of its origin. Let us name f one of these twelve components. Such function depends on the nine mean coordinates $(P_{11}, P_{12}, P_{13}), (P_{21}, P_{22}, P_{23}), (P_{31}, P_{32}, P_{33})$, of the measured points (P_1, P_2, P_3) and their random perturbations. A given component of the Jacobian matrix is just the partial derivative $\partial f / \partial P_{ij}$ of function f . It can be evaluated numerically through a third degree polynomial Taylor expansion. As already pointed out, each point P_i is

assumed to be scattered in a spherical localisation zone of diameter W . For that reason, function f is calculated for the four values $P_{ij} \pm W/4$ and $P_{ij} \pm W/2$ of parameter P_{ij} , keeping all other coordinates fixed to their mean value. The partial derivative of the function f can finally be evaluated through following equation:

$$\frac{\partial f}{\partial P_{ij}} = \frac{\delta f_{(P_{ij}+W/4)} - \delta f_{(P_{ij}-W/4)} + f_{(P_{ij}+W/2)} - f_{(P_{ij}-W/2)}}{6 \cdot W} \quad (16)$$

Where index i references the point and j one of its three coordinates.

This procedure is applied to all nine coordinates of the three measured points P_i . The 12x9 components of the Jacobian matrix J are thus obtained. The spherical covariance matrix of the measured points is finally propagated to the reference frame components using equation (4).

3.3. Monte Carlo simulation

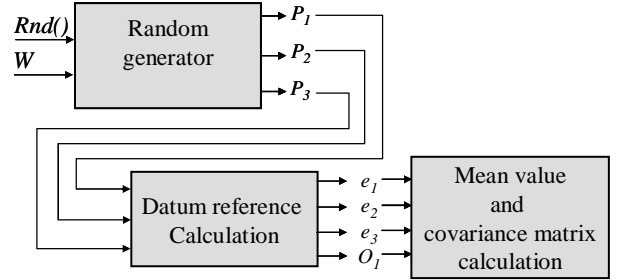


Fig. 3. MC simulation method

The procedure used to calculate the mean value and the covariance matrix of the reference datum is shown in Fig.3. First, the three coordinates of each measured point is generated randomly using the specific procedure which will be presented below. Second, the components of the unit vector characterizing the reference datum are deduced from equations (1) to (3). After numerous simulations, their mean values and covariances are finally computed through classical statistical analysis.

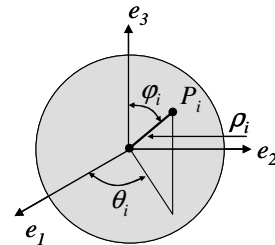


Fig. 4. Random parameters of spherical coordinate system

Monte Carlo simulation method has been applied in major scientific problems. It is very easy to use but requires all input variables to be independent. A classical Cartesian representation of the deviations to the mean coordinates

cannot thus be used to simulate a uniform random repartition of measured points in a spherical localisation zone. A spherical coordinate system parameterization $(\theta_i, \varphi_i, \rho_i)$ of the deviations, as presented in Fig. 4, is thus useful to solve the problem.

An accurate simulation of measurement data needs however accounting for the non uniform distribution of the independent variables $(\theta_i, \varphi_i, \rho_i)$. This leads to following random generation equations:

$$\begin{aligned}\rho_i &= \frac{W}{2} \sqrt[3]{Rnd()} \\ \varphi_i &= \text{Arccos}(1 - 2 \cdot Rnd()) \\ \theta_i &= 2 \cdot \Pi \cdot Rnd()\end{aligned} \quad (17)$$

In these equations, $Rnd()$ is the random generation of a uniformly distributed real variable in the interval $[0,1]$. Mersenne-Twister pseudo-random generation algorithm has been used to avoid sequential correlations between these variables.

4. CASE STUDY

Figure 5 shows an example of anatomic landmark measurement carried out during total knee arthroplasty. An external optical measuring machine acquires the three points P_1 , P_2 and P_3 of a ball tip pointer. These points permit calculating the coordinates, in the measurement reference datum, of the landmark M_i pointed by the surgeon. The knowledge of calibrated vector P_1M_i permits obtaining its coordinates in the global reference frame:

$$\overrightarrow{O_m M_i} = \overrightarrow{O_m P_1} + \overrightarrow{P_1 M_i} = \overrightarrow{O_m P_1} + a \cdot \vec{e}_1 + b \cdot \vec{e}_2 + c \cdot \vec{e}_3 \quad (18)$$

Where: a, b, c , are the calibrated components of vector P_1M_i .

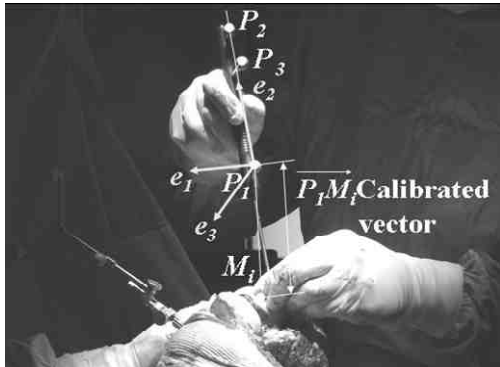


Fig. 5. Point measurement technique in total knee arthroplasty.

In total knee arthroplasty, this procedure is applied at lot of times to acquire all the anatomic landmarks used to characterize the mechanical axes of the tibia and the femur. This permits finally defining the cutting planes of the bones

whose variations are the main source of uncertainty of the final adjustment of the prosthesis.

In next paragraph, the uncertainty of a given measured point M_i will be determined using the three methods presented in previous section. As shown in Fig. 5. These schemes will be applied to the measurement of the lateral femoral epicondyle. The mean coordinates of the related points P_i , expressed in the coordinate system of the measuring machine, are given in Table 1.

Table 1. Coordinates of points P_i

P_1	P_2	P_3
1007,285	876,633	941,468
1746,624	1791,046	1744,374
663,364	604,570	616,577

The calibrated lengths of the ball tip pointer vector P_1M_i are specified in Table 2.

Table 2. Ball tip pointer calibrated lengths.

a	0
b	-100
c	0

The accuracy of the optical measuring machine is fixed, as defined by the manufacturer, to $W=0.1$ mm.

5. RESULTS

The three methods are employed to calculate, the covariance matrix of the reference frame defined by the three points P_i of the ball tip pointer. The covariance matrix of the coordinates of the related measured point M_i is then derived from this data. This paragraph details the obtained results.

5.1. Analytical propagation

The coordinates of Table 1 are entered in equations (4) to (14) to compute the covariance matrix of the reference frame (e_1, e_2, e_3, O_1) . Figure 6 presents the results of these calculations.

5.7E-08	-6.0E-10	3.8E-08	-3.2E-08	-3.2E-08	4.6E-08	5.5E-08	-1.9E-08	2.5E-08	2.5E-06	2.5E-06	-3.6E-06
-6.0E-10	9.3E-07	6.5E-07	1.1E-08	1.1E-08	-1.6E-08	-4.3E-07	-5.8E-07	8.3E-07	4.7E-06	4.8E-06	-6.9E-06
3.8E-08	6.5E-07	4.8E-07	-1.4E-08	-1.4E-08	2.1E-08	-2.6E-07	-4.2E-07	5.9E-07	5.0E-06	5.1E-06	-7.3E-06
-3.2E-08	1.1E-08	-1.4E-08	1.8E-08	1.9E-08	-2.5E-08	-3.6E-08	4.6E-09	-5.5E-09	-1.3E-06	-1.4E-06	1.9E-06
-3.2E-08	1.1E-08	-1.4E-08	1.9E-08	6.8E-08	8.6E-09	-4.1E-08	2.2E-08	-2.9E-08	-1.4E-06	-5.1E-06	-6.4E-07
4.6E-08	-1.6E-08	2.1E-08	-2.5E-08	8.6E-09	6.3E-08	4.8E-08	6.2E-09	-1.0E-08	1.9E-06	-6.4E-07	-4.7E-06
5.5E-08	-4.3E-07	-2.6E-07	-3.6E-08	-4.1E-08	4.8E-08	3.9E-07	2.5E-07	-3.7E-07	-3.2E-06	5.5E-07	-7.4E-06
-1.9E-08	-5.8E-07	-4.2E-07	4.6E-09	2.2E-08	6.2E-09	2.5E-07	3.8E-07	-5.4E-07	-5.8E-06	-4.8E-06	4.2E-06
2.5E-08	8.3E-07	5.9E-07	-5.5E-09	-2.9E-08	-1.0E-08	-3.7E-07	-5.4E-07	7.7E-07	8.2E-06	6.7E-06	-5.7E-06
2.5E-06	4.7E-06	5.0E-06	-1.3E-06	-1.4E-06	1.9E-06	-3.2E-06	-5.8E-06	8.2E-06	8.3E-04	0.0E+00	0.0E+00
2.5E-06	4.8E-06	5.1E-06	-1.4E-06	-5.1E-06	-6.4E-07	5.5E-07	-4.8E-06	6.7E-06	0.0E+00	8.3E-04	0.0E+00
-3.6E-06	-6.9E-06	-7.3E-06	1.9E-06	-6.4E-07	-4.7E-06	-7.4E-06	4.2E-06	-5.7E-06	0.0E+00	0.0E+00	8.3E-04
	e_1		e_2		e_3		O_1				

Fig. 6. Covariance matrix of the reference frame.

The variances of unit vectors e_1 and e_3 , and origin O_1 are of the same order of magnitude, whereas those of e_2 are much smaller. As established by equation (2) this is linked to the direct calculus of the last vector from the coordinates of points P_1 and P_2 .

In the analytical method, the uncertainty of the measured point M_i is obtained by propagation of the reference frame covariance matrix:

$$Cov_{(O_m M_i)} = J_1 \cdot Cov_{(e_1, e_2, e_3, O)} \cdot J_1^T \quad (19)$$

With:

$$J_1 = \begin{pmatrix} a & a & a & b & b & b & c & c & c & 1 & 0 & 0 \\ a & a & a & b & b & b & c & c & c & 0 & 1 & 0 \\ a & a & a & b & b & b & c & c & c & 0 & 0 & 1 \end{pmatrix}$$

The mean coordinates and the covariance matrix of the point probed by the ball tip pointer are thus derived. The results are presented in Table 3.

Table 3. Results of the analytical propagation method.

Covariance matrix			Mean value
2,54E-03	2,33E-03	1,96E-03	1094,386
2,33E-03	3,79E-03	2,59E-03	1717,010
1,96E-03	2,59E-03	3,05E-03	702,560

Such calculation permits optimizing the design of the ball tip pointer. In fact, to avoid excess uncertainty propagation to the probed point M_i , the largest length of the calibrated vector $P_1 M_i$ must be set in direction of unit vector e_2 . The best configuration is thus to put the end point of the ball tip pointer in the prolongation of line $P_1 P_2$.

The standard deviations of the measured coordinates can finally be estimated. The enlarged uncertainties of the measured point, for a confidence ratio of $k=2$, are presented in Table 6.

5.2. Hybrid propagation

The evaluation of the covariance matrix of the probed point M_i does not require computing the Jacobian matrix of the reference frame (e_1, e_2, e_3, O_1). In fact, numerical calculation of partial derivatives, as explained in paragraph 3.2, can be extended directly to the three coordinates of the probed point M_i . This allows straight evaluation of the Jacobian matrix J_2 between the nine parameters characterizing the set of measured points (P_i) and these coordinates. The classical propagation formula of the GUM is finally used to evaluate the covariances of the probed point M_i :

$$Cov_{(O_m M_i)} = J_2 \cdot Cov_{(P_1, P_2, P_3)} \cdot J_2^T \quad (20)$$

$$(3 \times 13) = (3 \times 9) \cdot (9 \times 9) \cdot (9 \times 3)$$

The numbers under equation (20) define the dimensions of the matrices. The results of this hybrid propagation method are presented in Table 4.

Table 4. Results of the hybrid propagation method.

Covariance matrix			Mean value
1,28E-03	4,78E-04	-6,32E-04	1094,386
4,78E-04	2,52E-03	2,15E-04	1717,010
-6,32E-04	2,15E-04	2,40E-03	702,560

The enlarged uncertainties of the measured point, for a confidence ratio of $k=2$, are presented in Table 6.

5.3. Monte Carlo simulation

The Monte Carlo Simulation method presented in paragraph 3.3 has been applied to the acquisition procedure of point M_i , in order to evaluate the uncertainties of its coordinates. 30000 sets of points (P_i) were therefore built randomly. The mean values and the covariance matrix of the simulated coordinates were finally computed through classical statistical analysis:

$$\bar{X} = \frac{1}{N} \cdot \sum_{i=1}^N X_i \quad (21)$$

$$Cov(X, Y) = \frac{1}{N-2} \cdot \sum_{i=1}^N (X_i - \bar{X})(Y_i - \bar{Y})$$

Where: X_i and Y_i are simulated coordinates.

The results obtained by the Monte Carlo simulation method are presented in Table 5.

Table 5. Results of the Monte Carlo Simulation method.

Covariance matrix			Mean value
1,47E-03	5,95E-04	-5,19E-04	1094,385
5,95E-04	3,01E-03	1,75E-04	1717,009
-5,19E-04	1,75E-04	1,58E-03	702,617

The enlarged uncertainty ($k=2$) of the coordinates of probed point can be derived from the covariance matrix. These results are shown in Table 6.

6. DISCUSSION

Table 6 summarises the results of the three propagation methods. All components are expressed in the coordinate system of the measuring machine. By principle, the mean values calculated by the analytical and hybrid methods are the same, since they derive directly from the mean coordinates of the measured points (P_i). The Monte Carlo method, on the contrary, shows small bias. This may be due to the nonlinearity of the equations which leads to non symmetrical probability densities of the calculated results.

The uncertainties of the three methods slightly differ, but remain of the same order. In direction 2, the uncertainty is about 0.1mm. The largest value is obtained through

analytical propagation ($U(k=2)=0.123\text{mm}$) and the smallest estimation through hybrid propagation ($U(k=2)=0.1\text{mm}$). Monte Carlo simulation gives a median result ($U(k=2)=0.110\text{mm}$).

In the two other directions, the uncertainties evaluated by hybrid propagation or Monte Carlo simulation are close. Analytical propagation leads to slightly overestimated values.

Table 6. Uncertainty results.

Analytical Method		Monte Carlo Method	
Mean value	$U(k=2)$	Mean value	$U(k=2)$
1094,386	0,101	1094,385	0,077
1717,010	0,123	1717,009	0,110
702,560	0,111	702,617	0,080

Hybrid Method	
Mean value	$U(k=2)$
1094,386	0,072
1717,010	0,100
702,560	0,098

7. CONCLUSIONS

In this paper, three methods were used to define the uncertainties of a reference frame built from three points acquired by a measuring machine.

The first method is a classical analytical propagation derived from the Guide to the Expression of Uncertainty in Measurement (GUM). It required a tedious calculus of partial derivatives defining the Jacobian matrix which permits propagating the covariances of the measured coordinates. It allows, nevertheless, expressing clearly the sensibility of the results versus each input parameter.

The second propagation technique is a hybrid solution based on a numerical estimation of the partial derivatives. The third method is a Monte Carlo simulation process. Our study showed very close results of the two last approaches, while analytical propagation led to slightly overestimate the uncertainties.

This study will permit improving the design of the ball tip pointer used in orthopaedic surgery and find a best practice for reference frame calculations.

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The authors thank the patient for its permission to watch the complete process plan of surgery.

REFERENCES

- [1] International Organization for Standardization, *Guide to the Expression of Uncertainty in Measurement, First Edition*, 1993.
- [2] J.M. Sprael, J.M. Linares, J. Bachmann and P. Bourdet, "Uncertainties in CMM Measurements, Control of ISO Specifications", *Annals of CIRP*, vol.51, n°.2, pp.423-426, 2003
- [3] J.M. Linares, J. Mailhé and J.M. Sprael, "Uncertainties of multi sensors CMM measurements applied to high quality surfaces", *XVIII IMEKO World Congress*, CdRom, Rio de Janeiro, Brazil, Sept. 2006.
- [4] I.M. Sobol, "Sensitivity estimates for nonlinear mathematical models", *Mathematical Modeling and Computation*, vol.1, n°.4, pp.407-414, 1993.
- [5] A. Saltelli, "Sensitivity analysis for importance assessment", *Risk Analysis*, vol.22, n°.3, pp.1-12, 2002.
- [6] A. Saltelli, "Making best use of model evaluations to compute sensitivity indices", *Computer Physics Communication*, vol.145, n°.2, pp.280-297, 2002.