XIX IMEKO World Congress
Fundamental and Applied Metrology
September 6–11, 2009, Lisbon, Portugal

ANALYSIS ON THE EFFECTS OF STIFFNESS IN MASS MEASUREMENT USING RELAY FEEDBACK OF DISPLACEMENT

Takeshi Mizuno [1], Yuji Ishino [2] and Masaya Takasaki [3]

[1] Saitama University (Department of Mechanical Engineering), Saitama, Japan, mizar@mech.saitama-u.ac.jp
[2] Saitama University (Department of Mechanical Engineering), Saitama, Japan, yishino@mech.saitama-u.ac.jp
[3] Saitama University (Department of Mechanical Engineering), Saitama, Japan, masaya@mech.saitama-u.ac.jp

Abstract – This paper treats a relay-feedback mass measurement system that uses a relay with dead zone and feeds back the displacement of the object. The efficacy of the proposed method has been already confirmed experimentally. In actual measurement, however, the object to be measured was guided by a spring for avoiding rotation. The effect of stiffness produced by this spring on mass estimation has not been investigated analytically. In this paper, an estimating equation including this effect is derived. The derived equation indicates that the estimated mass is smaller than the actual one when the conventional estimating equation is used. The experiments carried out in the fabricated apparatus support well the validity of the derived estimating equation.

Keywords : mass measurement, relay feedback

1. INTRODUCTION

The completion of international space station (ISS) will increase opportunities of experiment in space. It will raise the necessity of mass measurement under weightless conditions accordingly.

Various methods of measuring mass under weightless conditions have been proposed [1-10]. The authors have proposed to apply relay feedback to mass measurement [9, 10]. There are two types. One of them uses a relay with hysteresis and feeds back the velocity of the object [9]. The other uses an on-off relay with dead zone and feeds back the displacement of an object to be measured [10]. The advantage of the latter over the former is the cost of generating switching signals and their quality. Switching positions can be detected with a compact and low-cost sensor such as photo interrupter. This paper, therefore, focuses on the latter.

According to the principle of measurement, the object to be measured makes a linear reciprocating motion. In actual measurement systems, the other motions such as rotation must be prevented with some mechanism. A plate spring was used in the previous work [10] because it is simple and virtually free from friction. The effect of the stiffness on mass estimation, however, has not been investigated analytically. In this paper, the estimating equation considering this effect is derived.

2. PRINCIPLE OF MEASUREMENT

Figure 1 shows a physical model and a block diagram of the proposed mass measurement system. It is made up of four elements:

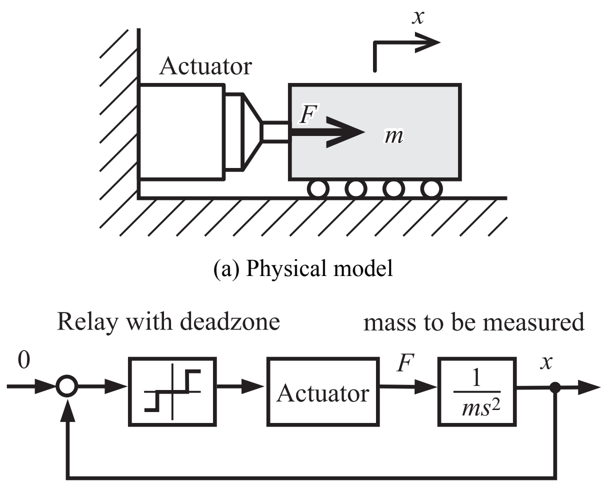
- actuator for moving an object to be measured,
- sensor for detecting the displacement of the object,
- controller for producing switching signals,
- amplifier for driving the actuator.

The operation of the measurement system is shown in Fig.2. The force $F(t)$ produced by the actuator is switched in relation to the displacement x of the object to satisfy

$$\left.\begin{aligned} F(t) &= +F_0 \quad \text{when } x \le -X_0, \\ F(t) &= 0 \quad \text{when } -X_0 < x < X_0, \\ F(t) &= -F_0 \quad \text{when } X_0 \le x. \end{aligned}\right\} \tag{1}$$

When the generator is controlled in this way, a periodic oscillation is excited as shown in Fig.2.

The periods during which $F(t) = \pm F_0$ and $F(t) = 0$ are designated as T_1 and T_2, respectively. The mass is assumed to pass the threshold positions $\pm X_0$ at velocities of $\pm V_0$.



(b) Block diagram of the closed-loop system

Fig.1 Measurement system

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When $F(t) = F_0$, the equation of motion is given by

$$m\ddot{x} = F_0 . \quad (2)$$

Solving (2) with $x(0) = -X_0$ and $\dot{x}(0) = -V_0$ leads to

$$2V_0 = \frac{F_0}{m} \cdot \frac{T_1}{2} . \quad (3)$$

When $F(t) = 0$, the equation of motion is given by

$$m\ddot{x} = 0 . \quad (4)$$

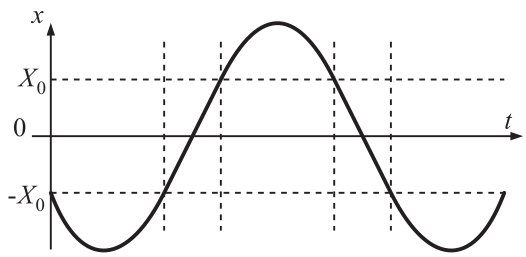
Solving (4) with $x(T_1/2) = -X_0$ and $\dot{x}(T_1/2) = V_0$ leads to

$$2X_0 = V_0 \cdot \frac{T_2}{2} . \quad (5)$$

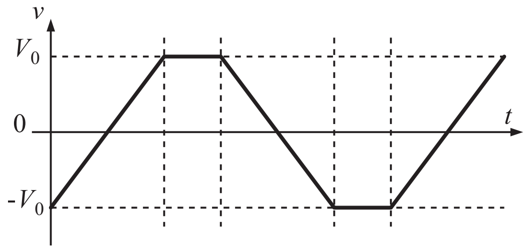
Combining (3) with (5) gives

$$m = \frac{F_0}{16X_0} T_1 T_2 . \quad (6)$$

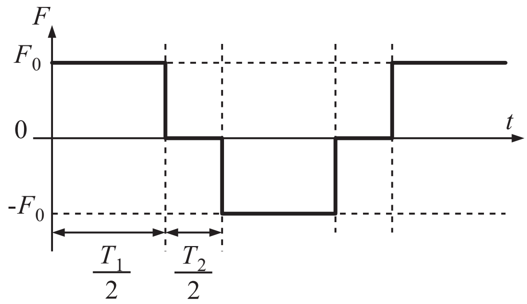
Therefore, the mass of measurement object is determined from the time interval measurement of T_1 and T_2. It is to be noted that mass is determined independently of the velocity V_0.



(a) Displacement



(b) Velocity



(c) Force

Fig.2 Operation of the measurement system

3. ESTIMATION CONSIDERING STIFFNESS

In the original configuration shown in Fig.1, the object makes a linear reciprocating motion and only the force of the actuator acts on the object. In actual measurement systems, however, it is necessary to prevent the other motions such as rotation. One method of prevention is to guide the object with linear-motion bearings. Rolling-element and slide bearings accompany friction, which increases uncertainty in measurement. Air bearing is virtually free from friction but needs an air source that should be avoided in space applications. Another method is to suspend the object with plate springs. This method has several advantages over the other methods. It is virtually free from friction, simple in structure, light in weight, inexpensive and in addition, easily replaceable even in space so that this paper focuses on this method.

The equations of motion including the effect of spring are given by

$$m\ddot{x} = +F_0 - kx \quad \text{when } x \le -X_0 , \quad (7)$$

$$m\ddot{x} = -kx \quad \text{when } -X_0 < x < X_0 , \quad (8)$$

$$m\ddot{x} = -F_0 - kx \quad \text{when } X_0 \le x , \quad (9)$$

where k is the stiffness of spring. It is assumed that the steady-state response makes a closed trajectory in the phase plane as shown in Fig.3. Solving (7) with $x(0) = -X_0$ and $\dot{x}(0) = -V_0$ leads to

$$x(t) = \frac{\alpha}{\omega^2} - (\frac{\alpha}{\omega^2} + X_0)\cos\omega t - \frac{V_0}{\omega}\sin\omega t , \quad (10)$$

$$v(t) = \omega(\frac{\alpha}{\omega^2} + X_0)\sin\omega t - V_0 \cos\omega t , \quad (11)$$

where v is the velocity of the object and

$$\omega = \sqrt{\frac{k}{m}} , \quad (12)$$

$$\alpha = \frac{F_0}{m} . \quad (13)$$

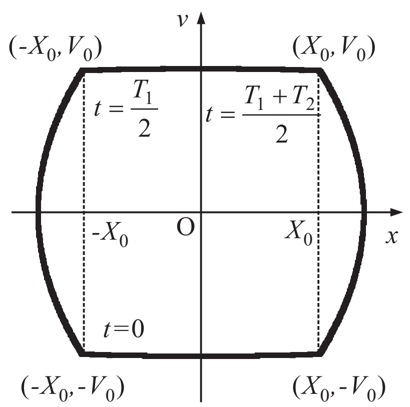


Fig.3 Trajectory of the steady-state response

Considering that $x(T_1/2) = -X_0$ and $v(T_1/2) = +V_0$ on the trajectory, we derive (14) from (11) as

$$V_0 = \omega(\frac{\alpha}{\omega^2} + x_0)\tan\frac{\omega T_1}{4}. \quad (14)$$

Solving (8) with $x(T_1/2) = -X_0$ and $\dot{x}(T_1/2) = +V_0$ leads to

$$x(t) = -X_0\cos\omega(t - \frac{T_1}{2}) + \frac{V_0}{\omega}\sin\omega(t - \frac{T_1}{2}), \quad (15)$$

$$v(t) = \omega X_0 \sin\omega(t - \frac{T_1}{2}) + V_0\cos\omega(t - \frac{T_1}{2}). \quad (16)$$

Define T by $T = T_1 + T_2$. Considering that $x(T/2) = +X_0$ and $v(T/2) = +V_0$, we derive (17) from (16) as

$$V_0 = \omega X_0 \cot\frac{\omega T_2}{4}. \quad (17)$$

From (14) and (17), we get

$$(\frac{\alpha}{\omega^2} + X_0)\tan\frac{\omega T_1}{4}\tan\frac{\omega T_2}{4} = X_0. \quad (18)$$

When the effect of stiffness is small, that is $\omega T \ll 1$, applying the approximation $\tan\theta \cong \theta$ to (18) and neglecting the second-order term of ωT lead to

$$\frac{\alpha T_1 T_2}{16} \cong X_0. \quad (19)$$

$$\therefore \quad m \cong \frac{F_0}{16 X_0} T_1 T_2 \quad (20)$$

This is same as the conventional estimation equation (6).

Substituting (12) and (13) into (18) gives

$$(\frac{F_0}{k} + X_0)\tan\frac{T_1}{4}\sqrt{\frac{k}{m}}\tan\frac{T_2}{4}\sqrt{\frac{k}{m}} = X_0. \quad (21)$$

Since it is impossible to express m in an analytical form explicitly, we derive an approximate equation by using

$$\tan\left(\frac{T}{4}\sqrt{\frac{k}{m}}\right) \cong \frac{T}{4}\sqrt{\frac{k}{m}} + \frac{1}{3}\left(\frac{T}{4}\sqrt{\frac{k}{m}}\right)^3. \quad (22)$$

From (21) and (22), we get

$$m \cong \frac{F_0}{16 X_0} T_1 T_2 \left\{1 + \frac{k X_0}{F_0}\left(1 + \frac{T_1^2 + T_2^2}{3 T_1 T_2}\right)\right\}. \quad (23)$$

Equation (23) indicates that the estimated mass is smaller than the actual mass by $k(T_1^2 + 3T_1T_2 + T_2^2)/48$ when the conventional estimating equation (6) is used.

4. EXPERIMENT

4.1 Experimental Apparatus

Figures 4 and 5 show a schematic drawing of an apparatus developed for experimental study, and the outline of control

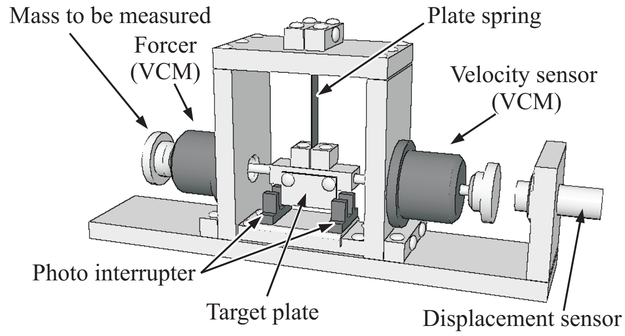


Fig.4 Experimental apparatus

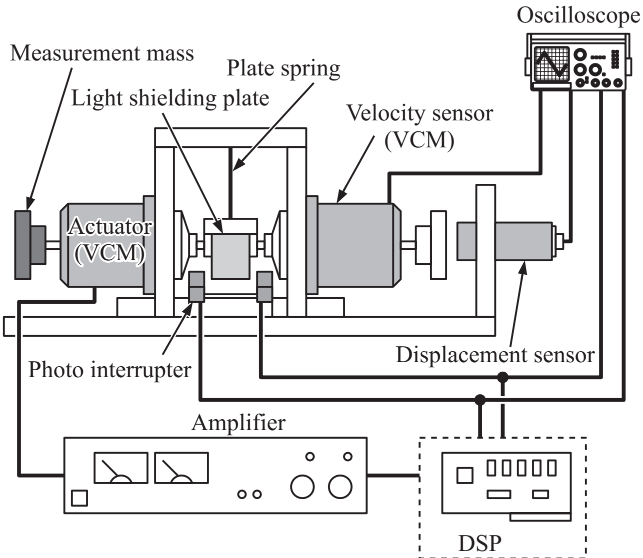


Fig.5 Control and measurement system

and measurement system [10]. It has two voice coil motors (VCM's) whose maximum output force is 9.8[N]. One of them is used as an actuator, which is driven by a current-output power amplifier. The other is used for velocity monitoring or generating disturbances such as stiffness and damping to study their effects on measurement accuracy. The movers of the VCM's are connected with a mechanical coupling to move together. A plate spring is also connected to them for preventing rotation and keeping the position at a middle point without control.

A pair of photo interrupters is used to detect the switching positions $\pm X_0$. The value of X_0 is adjusted by changing the width of a target plate that is attached to the connected movers. It is to be mentioned that the cost and weight of photo detector are much less than those of velocity sensors such as voice coil motor [9] and Doppler-type sensors [8].

Weights for changing the mass to be measured are attached to one end of the connected movers. At the other end, a sensor target for an eddy-current displacement sensor is fixed. The sensor is used for monitoring the position of the movers.

The control algorithm described by (1) is implemented with a DSP-based digital controller. The controller sends command signal (0, $\pm I_0$) to the power amplifier through a D/A converter. The electromagnetic force is switched to the

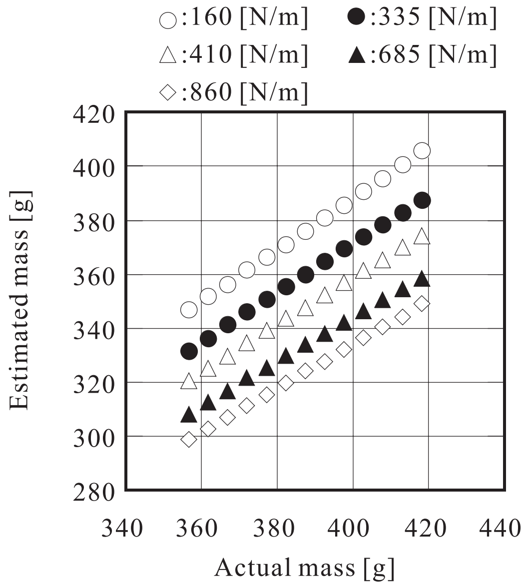


Fig.6 Estimation according to (6) (neglecting stiffness)

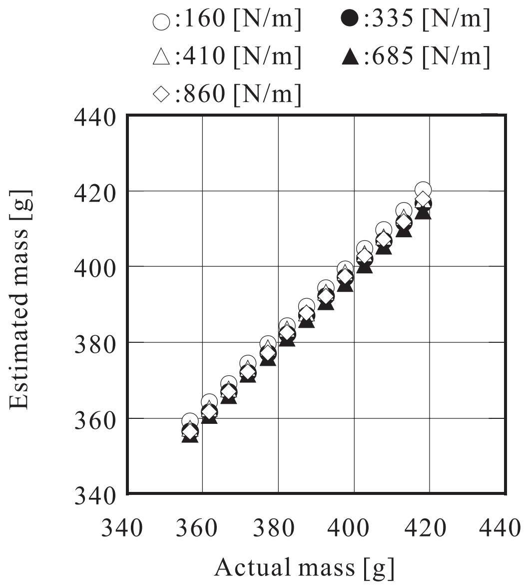


Fig.7 Estimation according to (23) (considering stiffness)

corresponding values (0, $\pm F_0$). The periods T_1 and T_2 are measured with a digital oscilloscope.

4.2 Experimental Results

In the following experiments, F_0 and X_0 were set as $F_0 = 9.37$[N] and $X_0 = 1.25$[mm]. The stiffness of the plate spring was identified as $k = 160$[N/m] so that preparatory measurement results fitted well to the analytical ones given by (23).

To estimate the effects of stiffness, the stiffness was varied flexibly by operating the VCM for velocity monitoring as actuator and feeding back the displacement of the object. The total stiffness k of the plate spring and the displacement feedback was adjusted as

(a)160, (b)335, (c)410, (d)685, (e)860 [N/m],

by the displacement feedback gain. The measurement results are shown in Figs.5 and 6 where mass was estimated according to (6) and (23), respectively. When stiffness is neglected, mass is apparently underestimated and the difference increases as stiffness increases. In contrast, mass estimation is insensitive to stiffness when (23) is used. It demonstrates well the validity of the derived equation.

5. CONCLUSIONS

An estimating equation considering the effect of stiffness on mass estimation was derived for the mass measurement system using relay feedback of displacement. The analytical result indicates that mass is underestimated when the effect of stiffness is neglected. The validity of the derived equation was demonstrated experimentally.

ACKNOWLEDGMENTS

The authors wish to express their gratitude to Mr. Takeuchi Minoru for help with the experiments.

REFERENCES

[1] Sarychev, V.A., Sazonov, V.V., Zlatorunsky, A.S., Khlopina, S.F., Egorov, A.D. and Somov, V.L., Device for Mass Measurement under Zero-Gravity Conditions, Acta Astronautica, Vol.7, pp.719-730, 1980.

[2] Ono, T. and Shimaoka, H., Dynamic Mass-Measurement under Weightless Conditions (in Japanese), Trans. SICE, Vol.21, No.11, pp.1184-1190, 1985.

[3] Ono, T., Uozumi, H., Honda, O. and Nagata, K., Mass-Measurement under Weightless Conditions by the Frequency-Controlled Method, Measurement, Vol.22, pp.87-95, 1997.

[4] Maeda, C., R. Masuo, and T. Baba, Mass Measurement Using Centrifugal Force Under Weightless Condition (*in Japanese*), Proc. of the 32nd SICE Annual Conference, Domestic Session, pp.899-890, 1993.

[5] Mizuno, T. and Araki, K., Mass Measurement Using a Dynamic Vibration Absorber under Weightless Conditions (in Japanese), Trans. SICE, Vol.32, No.8, (1996), pp.1145-1151.

[6] Mizuno, T. and Negishi, T., Mass Measurement Using a Dynamic Vibration Absorber (*in Japanese*), Trans. JSME, Series C, Vol.65, No.636, pp.3122-3128, 1999.

[7] Mizuno, T., and Minowa, J., Measurement Based on the Law of Conservation of Momentum --- Mass Measurement System Using Electromagnetic Impulsive Force (*in Japanese*), Trans. SICE, Vol.36, No.12, pp.1059-1064, 2000.

[8] Fujii, Y.,: Method for Measuring Mass of Non-rigid Objects for Microgravity Conditions (*in Japanese*), Trans. SICE, Vol.38, No.4, pp.337-344, 2002.

[9] Mizuno, T., Adachi, T, Takasaki, M. and Ishino, Y., Mass Measurement System Using Relay Feedback with Hysteresis, Journal of System Design and Dynamics, Vol.2, No.1, pp.188-196, 2008.

[10] Mizuno, T., Takeuchi, M., Takasaki, M. and Ishino, Y., Mass Measurement Using a System Containing an On-Off Relay with Dead Zone, Trans. SICE, Vol.41, No.1, pp.1-6, 2005.