XIX IMEKO World Congress
Fundamental and Applied Metrology
September 6−11, 2009, Lisbon, Portugal

# UNCERTAINTY EVALUATION OF MULTI-SENSOR FLOW MEASUREMENT IN A SEWER SYSTEM USING MONTE CARLO METHOD

*A. Silva Ribeiro, M. Céu Almeida and J. Palma*

Laboratório Nacional de Engenharia Civil, 1700-066 Lisboa, Portugal
asribeiro@lnec.pt; mcalmeida@lnec.pt; jpalma@lnec.pt

**Abstract** − Technical requirements and economical impacts of flow measurements in sewer systems are a issue concern in today's system's management. Thus, the quality of the measurements is considered to be a critical issue. Considering the complex nature of the measurand and the metrological requirements of local installations, the best available level of accuracy in measurement results should be sought. Therefore, both the knowledge of the measurand estimates and measurement of uncertainties are required for achieving robust results.

Within this context, the quality of measurement results depends on the knowledge of the uncertainty contributions and on the selection of an appropriate method to evaluate the measurement uncertainty. The study of these aspects can be of major importance in providing information to management of the system, namely in the selection of appropriate technology, upgrading and maintenance activities.

The Monte Carlo method is used in this paper to carry out the evaluation of the measurement uncertainty, considering its inherent capacity to deal with non-linear and multi-stage mathematical models. Influence of geometric conditions and other relevant parameters in the quality of measurements is discussed. The study was developed within the context of a specific sewer system, using a particular measurement system, from which measurement data was gathered.

**Keywords**: sewer systems, flow measurement, measurement uncertainty, Monte Carlo Method.

## 1. INTRODUCTION

Measurement of flow in sewer systems is a complex task considering the dynamic behaviour of the measurand and the effects resulting from non-ideal conditions of operation [1]. When flow measurements are regularly used for managing sewer systems, performance of the measurement system and the quality of measurement results becomes critical both to daily operation and to decision making processes within the utility.

Different solutions can be adopted in order to measure flow in free surface flow conditions in sewers [2]. One of the most common methods is the velocity-area, usually using multi-sensing flow meters composed by a combination of sensors for level and velocity measurement, often mounted in stainless steel rings or bands, to be fitted in the inner surface of sewer pipes. The flow can be calculated from measurement of different quantities, namely, level and velocity, by applying the continuity equation. The slope-area methods, using the Manning-Strickler formula or similar formulae, are sometimes used in conjunction with the velocity-area method to ensure redundancy. In both cases, calculation of the flow involves the use of non-linear mathematical models in a multi-stage system. Additionally, in general, these methods assume uniform flow conditions often difficult to ensure in actual measurement sites. For the purpose of this paper, only the continuity equation is considered.

The actual probabilistic approach of Metrology defines the measurement result has a combination of the measurand estimate and its measurement uncertainty [3]. Given the nature of the mathematical models used the Monte Carlo method was pointed out as a suitable approach to perform the measurement uncertainty evaluation [4].

The development of the uncertainty budget requires the evaluation of contributions due to different uncertainty sources, which can be grouped in eight major factors: the measurand; the instrumentation metrological performance; the calibration; the sampling; the interface; the user; the environmental conditions; and the data processing.

In the specific case under study, considering the technological development of instrumentation and data processing software, the non-ideal conditions of the measurand realization (i.e. non-uniform flow) appears to be an important contribution.

The analysis of the instrumentation assembly and its installation *in situ* shows the relevance of a number of geometric requirements: the placement of probes, measuring angles and cross-sectional geometry. In addition, hydraulic conditions associated with the inner pipe characteristics (symmetry conditions, wall roughness, hydraulic jump, drops, curves and infrastructure irregularities) can generate different types of waves, energy losses and other disturbances contributing to non-uniform flow.

In order to study the sources of measurement uncertainties and their effects, a second aim of this paper is to obtain an assessment of the conditions that make the contributions due to geometric quantities dominant in the context of the uncertainty budget. An example of a field application is used in order to illustrate the proposed discussions and conclusions.

## 2. APPROACH

Flow is a quantity measured indirectly, usually obtained by the measurement of other quantities and applying mathematical models, the continuity equation being one of the most common.

The continuity equation, as given by (1), is a functional relation that yields the volumetric flow rate, $Q$, as a function of the mean velocity, $U$, and the cross sectional area of flow, $A$, according to the principle of conservation of mass.

$$Q = U \cdot A \qquad (1)$$

In practice, the input quantities of this mathematical model, obtained by indirect measurement of other measurands, create a multi-stage metrological problem with several input and output quantities, and functional relations between them, to reach the final output measurand, $Q$.

The flow through a given surface $S$ is defined as the result of an integration of a velocity field over that target surface. Thus, $U$ is the average of the field velocities over $S$. The pattern of the velocity field spatial distribution may vary significantly according to the type of flow (e.g. in completely filled pipes or free surface flow) and local conditions.

The best approximation to the average velocity $U$ in a given flow should be obtained by measuring velocities in a large number of points distributed over the target surface, $S$.

The measurement of $U$ is often carried out by transducers that capture the effect of the velocities along a straight line or, more realistically, along the conical dispersion of the beam [5,6], by assuming that certain flow distribution and symmetry conditions are well known and that yield feasible solutions. Then, the average velocity $U$ is obtained from a measured value (which can be either a beam average value or its maximum value) multiplied by an appropriate calibration factor.

The complexity of the relations established between quantities is presented in Fig. 1, showing several stages where some quantities are simultaneously output of some stage and input to the next stage. The complete set of quantities used is described in Table 1, being based on the formulation presented in Fig. 1.

The experimental performance of flow measurement in sewers implies that some influence quantities related to the method deviations, $\delta Q_i$, should additionally be taken into account in the mathematical model, as included in ($f_8$). This modification of the mathematical model (1) is required in order to evaluate the measurement uncertainty. Both relations are in agreement if the average values of these quantities are null (as usually expected).

Table 1. Set of quantities applied

| Symbol | Description |
|---|---|
| $c_{\mathrm{us,w}}$ | Ultrasound velocity in water (at reference conditions) |
| $f_S$ | Emitter frequency |
| $\beta$ | Angle of sound propagation |
| $\Delta f$ | Doppler frequency shift |
| $u_{\max}$ | Peak flow velocity |
| $C_u$ | Peak to average flow velocity factor |
| $\overline{U}$ | Average flow velocity |
| $c_{\mathrm{us,air}}$ | Ultrasound velocity in air (at reference conditions) |
| $t_{\mathrm{tr,air},i}$ | Wave time of transit |
| $d_i$, $\hat{d}$, $\overline{d}$ | Displacement, estimate and average values |
| $D$ | Diameter of pipe (at flow depth section measurement) |
| $d_o$ | Displacement offset of the acoustic emitter |
| $\delta h_{\mathrm{us}}$ | Flow depth variation in the measurement surface |
| $h_{\mathrm{us}}$ | Flow depth (measured with acoustic us instrument) |
| $p_{\mathrm{w}}$, $p_{\mathrm{atm}}$ | Pressure of fluid (water) and atmospheric pressure |
| $g$ | Gravity |
| $\rho_{\mathrm{w}}$ | Density of water (at reference conditions) |
| $h_p$ | Flow depth (measured with pressure depth instrument) |
| $r_{\mathrm{c}}$ | Radius of conduit (at the cross-section area) |
| $A$ | Cross-section "wet" area |
| $\delta Q_i$ | Flow influence quantities related with the method and with computational processing |
| $Q$ | Volumetric flow rate |

The random variable flow depth, $h_{\mathrm{us}}$, can be estimated from two different measurement approaches (Doppler effect or fluid column pressure), allowing to have redundant information about system performance.

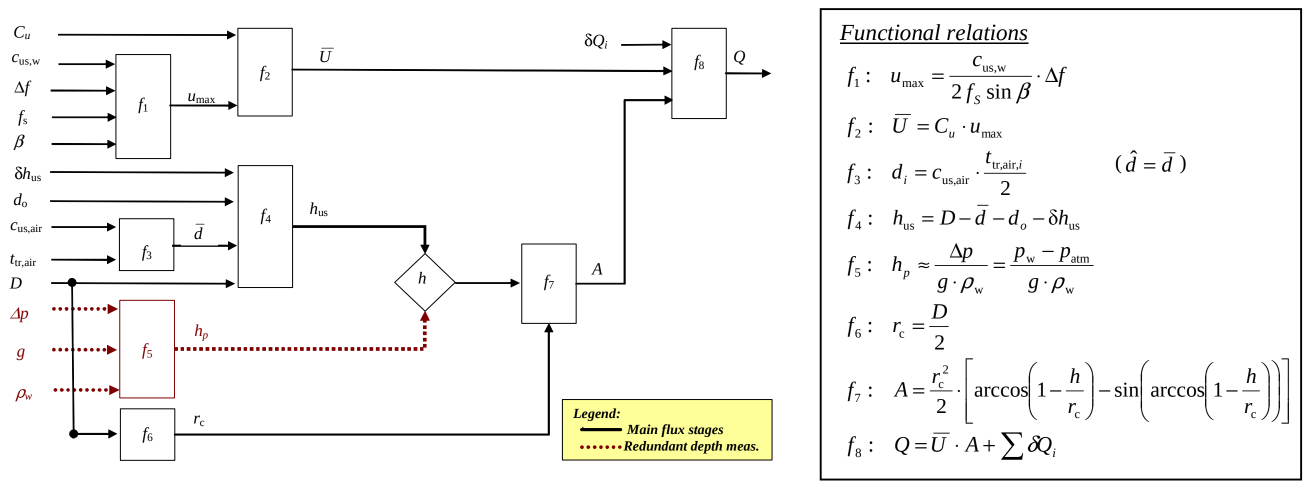


Figure 1. Input quantities and functional relations to obtain volumetric flow rate

In order to test the proposed approach as a means for evaluating the flow measurement uncertainty, measurement data from a large sewer system were used. This regional sewer system has circa 60 flow measurement locations, and measurement accuracy constitutes an important issue since data is used for billing purposes. The measurement approach used in most locations is based on the velocity-area method and the cross-section shape on locations selected is circular (Figures 2 and 3).

The information obtained allows to calculate estimates of the measurement uncertainty contributions and to discuss the model sensitivity to different parameters such as those related with the geometric conditions.

Figure 2. Flow measurement using the velocity-area method

Figure 3. Flow measurement device: detail of ultrasound device (four pairs) for flow depth measurement

In most of the measurement locations, mounting the instrumentation is made under adverse conditions, usually in places where flow performance can be strongly affected by the geometry of pipes and by irregularities in joints. Furthermore, in conditions where flow imposes strong impulses on the instrumentation, dislocations of the instrumentation supporting ring causes permanent changes in the setup, dragged objects and debris might damage the instrumentation, and sediment grease and oil accumulation can obstruct the sensors. These unpredictable events eventually identified during maintenance operations or data processing, can lead to significant measurement errors. However, incorporation of these effects as contributions to measurement uncertainty proves to be difficult.

Thus, it is expected that the error sources are strongly dependent of local conditions at each measurement location.

The evaluation of the measurement uncertainty contributions is based on the analysis of the variability of average values obtained from several locations. The probability distributions were derived from the observation of some variables during the measurement process, together with estimated values provided by the manufacturers or by referenced bibliography.

## 3. EVALUATION OF MEASUREMENT UNCERTAINTY USING MONTE CARLO METHOD

The modern approach to measurement science has a framework where the measurement result is composed by the estimate of measurement and by the measurement uncertainty.

A general procedure to evaluate the measurement uncertainty was introduced by the GUM. However, exact solutions are obtained only when applied to linear or slightly nonlinear mathematical models.

These restrictions were a strong motivation for the study of other methods suitable to the evaluation of measurement uncertainties related with complex, nonlinear mathematical models.

Monte Carlo method has proved to be particularly suitable to this purpose [4], providing the opportunity to obtain robust solutions in the evaluation of measurement uncertainties in a multi-stage nonlinear model as the described above.

Regarding the process used by the Monte Carlo Method (MCM) to perform the evaluation of measurement uncertainties, the relations (mathematical models) of the multi-stage system are used directly, together with the input data obtained by sampling from probability density functions (PDFs) of each input quantity. The computation of the algorithm gives the propagation of distributions in order to obtain the output quantities PDFs and their statistical parameters of interest (namely, measurands best estimates and variances).

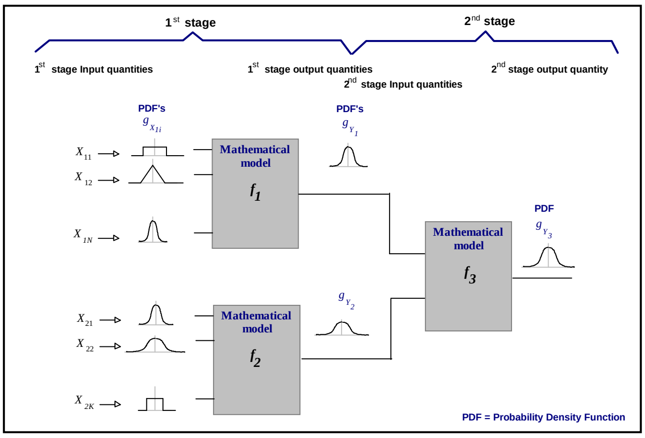


Figure 4. Propagation of density probability functions in a two stage measurement system

The propagation of PDFs from one stage to the following is illustrated in Fig. 4, assuming that the output numerical sequence of one stage (with its own PDF) is taken as the input numerical sequence of the next stage, while keeping the statistical properties (such as correlations) characteristic of the each specific random variable.

Since the MCM can be applied in the absence of mainstream GUM requirements, such as symmetry of the input probability functions (or others), the method proves to be especially suitable to be applied to non-linear mathematical models.

Development of MCM numerical simulations is carried out by generating sequences of up to $10^6$ values for each quantity, depending on the required computational accuracy. The draws were based on the Mersenne Twister uniform random number generator [7] and the PDFs were obtained using validated methods like the Box-Muller transformation and the inverse cumulative distribution function (CDF) method [8]. Tests to verify the computational accuracy of the output PDFs were also made according with [9] allowing to conclude that the numerical simulations provide robust and accurate solutions for the metrological problem proposed. In Table 2, the experimental input data and PDF parameters adopted are presented. Some observations to this table are:

a) The quantity $u_{max}$ includes the contributions from input quantities presented in Fig. 1 (mathematical model $f_1$) combined with the contributions due to the resolution, linearity and drift of the indication device.
b) The quantity $c_{us,air}$ incorporates the temperature influence of ± 0,17 % /ºC and the influence of pressure of ± 0,1 % of the readings.
c) The quantity related with the pipe diameter estimate includes the resolution effect of the measurement instrument and the roundness error effect.
d) The quantity $\delta h_{us}$ includes surface wave effects.
e) The quantity $\delta Q_{geom}$ includes effects due to pipe slopes and other geometry constrains (based on the instrumentation manufacturer information).
f) The quantity $\delta Q_{overfalls}$ includes the geometric influence due to proximity to drops at entrance to downstream manhole.
g) The quantity $\delta Q_{inst\_ring}$ includes effects due to instrumentation ring setup and installation geometry.
h) The quantity $\delta Q_{comp}$ includes effects due to computational process performed with modified off-the-shelf software.

Table 2. Experimental input data and PDFs adopted

| Random variables | PDF parameters | Units |
|---|---|---|
| $u_{max}$ a) | $\mathcal{N}$(0,79; 0,01) | m·s$^{-1}$ |
| $C_u$ | $\mathcal{R}$ (0,85; 0,95) | adim. |
| $c_{us,air}$ b) | $\mathcal{R}$ (340,1; 346,9) | m·s |
| $t_{tr,air,i}$ | $\mathcal{R}$ (8,43; 8,93) | ms |
| $D$ c) | $\mathcal{R}$ (1793; 1803) | mm |
| $d_o$ | $\mathcal{R}$ (5; 15) | mm |
| $\delta h_{us}$ d) | $\mathcal{R}$ (10; 20) | mm |
| $\delta Q_{geom}$ e) | $\mathcal{N}$(0; 0,0025·$q$) | L·s$^{-1}$ |
| $\delta Q_{overfalls}$ f) | $\mathcal{N}$(0; 0,005·$q$) | L·s$^{-1}$ |
| $\delta Q_{inst\_ring}$ g) | $\mathcal{N}$(0; 0,005·$q$) | L·s$^{-1}$ |
| $\delta Q_{comp}$ h) | $\mathcal{N}$(0; 0,001·$q$) | L·s$^{-1}$ |

Critical conditions such as backwater flow, very low and off-axis flow velocity components and their relation with mean flow velocity were not considered given the difficulty to quantify the consequences on the measurement due to these extreme effects. However, special care should be taken when selecting the measurement locations to avoid large errors derived from this type of effects.

MCM simulations were carried out using Table 2 values as input parameters.

The estimate of the measurement result obtained for the output quantity, volumetric flow rate, including its standard uncertainty is

$$Q_v = (229{,}7 \pm 14{,}1)\ \mathrm{L/s} \qquad (2)$$

and the related output PDF is presented in Fig. 5.

Computation results confirm the significant advantage of using the MCM approach, since it allowed the evaluation of measurement uncertainty despite the use of a nonlinear function, $f_7$,

$$A = \frac{r_c^{\,2}}{2} \cdot \left[ \mathrm{acos}\left(1 - \frac{h}{r_c}\right) - \sin\left[\mathrm{acos}\left(1 - \frac{h}{r_c}\right)\right]\right] \qquad (3)$$

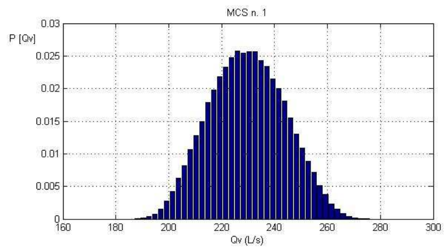


Figura 5. Output PDF of flow rate obtained for MCM n. 1

In fact, results are consistent, giving low computational accuracy values, as shown in Table 3.

## 4. SENSITIVITY ANALYSIS

The sensitivity analysis was developed in two ways aiming at comparing the influence of the several input quantities on the result, in order to find those that can be considered dominant and to study the effect of the geometric quantities uncertainties on the output flow measurement uncertainty.

Apparently, the output PDF has a gaussian shape (Figure 5), which is the usually predicted. However, detailing the statistical study, results show a deviation from normality, namely, due to the excess kurtosis coefficient value of 0,50 (typical for a logistic distribution).

In order to study the behaviour of the output PDF and the relations with the several model parameters, a sensitivity analysis focusing the measuring uncertainties was also carried out, allowing the identification of critical parameters for the measurement uncertainty magnitude and the output PDF shape.

The sensitivity analysis clearly showed that the *wave time of transit* has the higher influence in the output measurement uncertainty. To illustrate this fact, additional MCM simulations were carried out. Overall simulations were done considering the typical standard uncertainties of ± $5\cdot10^{-5}$ s (MCS n. 01), ± $8\cdot10^{-5}$ s (MCS n. 02) and ± $1\cdot10^{-4}$ s (MCS n. 03). Results summarised in Table 3 show that the increase of the standard uncertainty causes an increase of both the skewness (to the right) and of the excess kurtosis coefficient, which exhibits an increasing departure from the gaussian shape (Figures 6 and 7).

Table 3. Summary results for first set of MCM simulations

| Quantities and parameters | MCM Estimates | | |
|---|---|---|---|
| | MCS 1 | MCS 2 | MCS 3 |
| Volumetric flow estimate | 229,7 L/s | 229,4 L/s | 229,2 L/s |
| Standard uncertainty | 6,5 % | 9,2 % | 11 % |
| Computational accuracy | ± 0,15 % | ± 0,2 % | ± 0,25 % |
| Skewness | 0,04 | - 0,007 | - 0,03 |
| Excess kurtosis | - 0,50 | - 0,74 | - 0,84 |

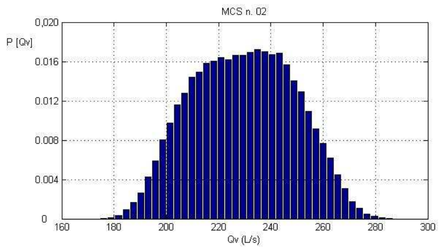


Figure 6. Flow rate output PDF for MCS n. 2

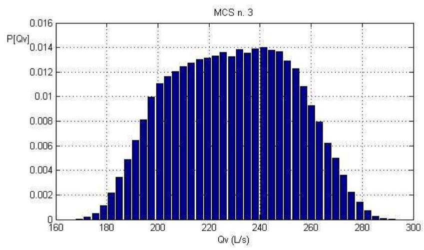


Figure 7. Flow rate output PDF for MCS n. 3

The PDFs presented in Fig. 6 and 7 support the conclusion that the output PDF is non-symmetric and non-gaussian, and that flow rate measurement uncertainty increases significantly with the wave time transit measurement uncertainty.

The shape of the flow rate output PDF (especially the one presented in Figure 7) suggests the existence of an input variable with similar shape, which has large influence in certain circumstances. The analysis of the input/output quantities of the multi-stage system presented on Fig. 1 leads to a most probable quantity, the nonlinear cross-section "wet" area, $A$, obtained using the function referred on (3).

An MCM simulation carried out in order to obtain the output PDF associated with this variable showed that this explanation was correct. In fact, the shape of this quantity (Figure 8) is similar to the shape of Fig. 6 and Fig. 7, giving the observed non-symmetry and non-normality.

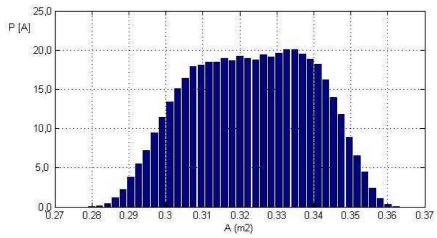


Figure 8. Output PDF for the cross-section "wet" area

It should be emphasized that this conclusion is only possible because MCM provides the PDF information essential to this analysis. In fact, most methods to assess measurement uncertainties only provide the quantity estimates and the confidence interval limits.

Another study was carried out to assess the influence of the angle of sound propagation, $\beta$, in the output results. In fact, this influence is expected considering the direct relation between the average velocity, $U$, and the flow rate (see $f_8$).

The nominal angle is usually given as 45º, which can be difficult to establish in practice due to the mounting conditions and adverse flow conditions, as mentioned at section 3.

The sensitivity analysis was performed using MCM in a two step procedure:

Step 1. Evaluation of the measurement uncertainty of the peak flow velocity considering a standard uncertainty of the angle $\beta$ of ± 1% (manufactures condition) or, at extreme conditions, of ± 5%;

Step 2. Evaluation of the measurement uncertainty of the corresponding flow rate.

A synthesis of results obtained is presented in Table 4, showing 30% increase in the output measurement uncertainty due to the angle uncertainty increase from ± 0,45 º to ± 2,25 º (respectively 1% and 5 % of the nominal angle of 45º). This increase is shown in Figures 9 and 10 which have the same scales.

Table 4. Summary results for the second set of MCM simulations

| Quantity | Relative Standard uncertainty | |
|---|---|---|
| Angle ($\beta$) | ± 1 % | ± 5 % |
| Peak velocity ($u_{max}$) | ± 0,8 % | ± 3,9 % |
| Flow rate (Q) | ± 6,4 % | ± 8,3 % |

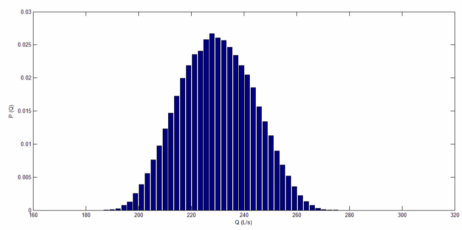

Figure 9. Flow rate output PDF for u(β) =± 1 %

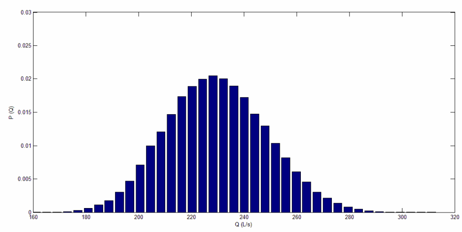

Figure 10. Flow rate output PDF for u(β) =± 5 %

Again, the use of MCM provides quantitative information regarding the relation between the input quantity (angle) and the output quantity (flow rate) based on a probabilistic approach, essential for the conclusions obtained.

## 5. DISCUSSION AND CONCLUSIONS

The success of the evaluation of measurement uncertainties depends on the nature of the metrological problem considered, being particularly relevant the nature of the mathematical models used.

The development of metrological studies showed that the conventional GUM approach cannot lead to exact solutions when there are strongly nonlinear models, and alternative approaches such as the Monte Carlo Method have to be used (as suggested on GUM Supplement 1).

Flow measurement in sewer systems is a typical non-linear, multi-stage metrological problem. Using the velocity area method a nonlinear relation exists on the definition of the cross-section "wet" area, therefore requiring the use of an alternative approach for determining measurement uncertainty.

In fact, the studies carried out showed that, for this type of problems, MCM is suitable to overcome the difficulties due to the nonlinear problem of the model, thus providing robust estimates of the measurement uncertainties.

Sensitivity analysis on the model parameters was carried out to define the uncertainty contributions, allowing a comparison of the sources of uncertainty effects into the output quantity (flow) uncertainty, as well as giving information on the best way to increase system accuracy. Furthermore, the analysis allowed to quantify the relation between the measurement uncertainty of the angle of sound propagation and the flow rate and to confirm the need to assure that this quantity is obtained with the best accuracy possible.

The MCM approach is also known for allowing a deeper analysis of the stochastic problems, namely, because it provides the output PDFs. This fact became especially relevant, since results showed that the output PDF can change from a nearly gaussian shape to a non-symmetric and non-gaussian shape depending on the individual contributions of some input quantities. This fact is significant as it increases the measurement expanded uncertainty interval.

The analysis of the data allowed concluding that the nonlinear function that provides the cross-section "wet" area generates a non-symmetric and non-gaussian PDF whose shape is quite similar to some of the output PDFs obtained. This conclusion can only be achieved by using a MCM approach.

The studies carried out are considered to be relevant to improve knowledge on this type of measurement systems, identifying critical points to its accuracy, to the identification of improvement opportunities, and providing useful information to support management decisions within the context of quality management.

## ACKNOWLEDGMENTS

The authors wish to acknowledge the interest and cooperation of SANEST, S.A. (Sistema Multimunicipal de Saneamento da Costa do Estoril), especially for providing historical information essential to carry out this study.

## REFERENCES

[1] Larrarte, F., "Velocity fields within sewers: An experimental study", *Flow Measurement and Instrumentation*, n. 17, pp. 282-290, 2006.

[2] Bertrand-Krajewski, J-L, Laplace, D., Joannis, C., Chebbo, G., *Mesures en hydrologie urbaine et assainissement*. Techniques & Documentation, 2000.

[3] *Guide to the Expression of Uncertainty in Measurement*, BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, OIML. Geneva, Switzerland, 1995.

[4] *Evaluation of measurement data − Supplement 1 to the "Guide to the expression of uncertainty in measurement"− Propagation of distributions using a Monte Carlo method*, BIPM, Sèvres, France, 2008.

[5] Edelhauser, M., "A Comparison of Continuous Wave Doppler vs. Pulsed Doppler Profiling Technology", *MGD Technologies*, Inc., February, 1999.

[6] Jaafar, W., Fischer, S., Bekkour, K., "Velocity and turbulence measurements by ultrasound pulse Doppler velocimetry", article in press - corrected proof, *Measurement*, 2008.

[7] M. Matsumoto and T. Nishimura, "Mersenne Twister: a 623-dimensionally equidistributed uniform pseudorandom number generator", *ACM Transactions on Modelling and Computer Simulations*, vol. 8, nº1, pp. 3-30, January 1998.

[8] Gentle, J. E., *Random Number Generation and Monte Carlo Methods*, 2nd Ed. Springer-Verlag. New York, 2003.

[9] M. G. Cox, M. P. Dainton and P. M. Harris, *Software specifications for uncertainty calculation and associated statistical analysis*, National Physical Laboratory Report CMSC 10/01, NPL, Teddington, Mar. 2001.