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MULTI-RANGE TRANSFORMER BRIDGE FOR CALIBRATION OF INDUCTANCE STANDARDS

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Abstract – A high-precision multi-range unbalanced transformer bridge designed for inductance standard calibration is described. Basic theory of the bridge is discussed which enables direct measurement of differences of both impedance components (ΔL and ΔR). The instrument was designed to compare inductors from 100 μH to10 H in the frequency range from 100 Hz to10 kHz. The uncertainty of comparison in the impedance range from 1 Ω to 10 kΩ is assessed to be smaller than 10 ppm.

Keywords: inductance measurements, standard inductor, transformer bridge

1. INTRODUCTION

Systems designed for very accurate inductance measurements at audio frequencies are usually based on Maxwell-Wien (M-W) or similar four-arm bridges [1], [2], [3]. The M-W bridge has the advantage that the first order bridge equation is independent of frequency which means that the bridge has the potential to be operated in a large frequency range. But the M-W bridge has some disadvantages, particularly it is not suitable for automatic operation. Therefore transformer bridges were designed to refer inductance directly to a fixed capacitance [4] and to perform extremely accurate comparisons between inductance standards [5]. Generally, two types of measuring instruments are destined for a high-precision impedance measurements: impedance transformer bridges which measure the ratio of two impedances and impedance comparator bridges which measure the difference of two impedances. The comparator bridges are more suitable when the compared standards have similar values of inductances. A self-balancing comparator bridge for maintenance of inductance standard (named KWL4) was constructed by research team from Silesian University of Technology, Gliwice, Poland exclusively for the comparison of 10 mH inductance standards at frequencies 1 kHz and 1592 Hz [5], [6]. Nowadays, the bridge is used at Central Office of Measures (GUM) in Warsaw and at Physikalisch-Technische Bundesanstalt (PTB) in Braunschweig to compare the standards of the group with the standard defined by means of the M-W bridge, or for intercomparisons. A digitally controlled switch was designed and built to perform the intercomparison process automatically [7].

The experience gathered during earlier work and the need to extend the measurement range to another values of inductance prompted the constructors to design a new multi-range bridge which will be suitable for comparing inductance standards from 100 μH to 10 H in a wide frequency range.

2. OPERATION PRINCIPLE

The simplified diagram of the unbalanced bridge designed for comparing inductances is shown in Fig. 1.

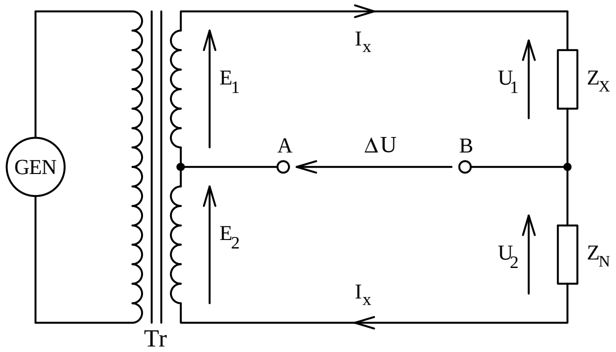

Fig. 1. Principal circuit diagram of the KWL4 bridge.

Impedances $Z_x = R_x + j\omega L_x$ and $Z_N = R_N + j\omega L_N$ representing the compared inductors are connected in series and form a part of the bridge. Two voltages with very small relative ratio-error (less than 10^{-8}) are provided by the ratio transformer. The voltage difference ΔU between the central tap of the secondary windings (point A) and the connection point of the compared inductors (point B) is the bridge unbalance voltage and is equal to

$$\Delta U = \left(Z_x - Z_N\right)\frac{I_x}{2}. \quad (1)$$

The differences of resistances and reactances can be expressed by following equations

$$R_x - R_N = \mathrm{Re}\left\{\frac{2\Delta U}{I_x}\right\}, \tag{2}$$

$$X_x - X_N = \mathrm{Im}\left\{\frac{2\Delta U}{I_x}\right\}. \tag{3}$$

The unbalance voltage ΔU is compensated by two orthogonal voltages (in-phase and quadrature in relation to the current vector I_x). Then the ratios $\mathrm{Re}\{\Delta U/I_x\}$and $\mathrm{Im}\{\Delta U/I_x\}$ are given by real numbers, which are proportional to the ΔR and ΔX differences, respectively [5]. The compensation is controlled by microcontroller and performed automatically. The comparison result is displayed on the LED indicator.

3. MULTI-RANGE TRANSFORMER BRIDGE KWL5

Due to needed wide range of nominal inductance values (from 100 μH to10 H) and wide frequency range (100 Hz – 10 kHz), the impedances being compared cover the range between 62 mΩ (for 100 μH and 100 Hz) and 628 kΩ (for 10H and 10 kHz). It is obvious that when small impedances are compared, inevitable stray inductances should be taken into consideration. These include inductances of leads and connections. To reduce these influences the comparison should be performed using 4-wire technique. Therefore, the circuit diagram was modified to extend the impedance range (Fig. 2).

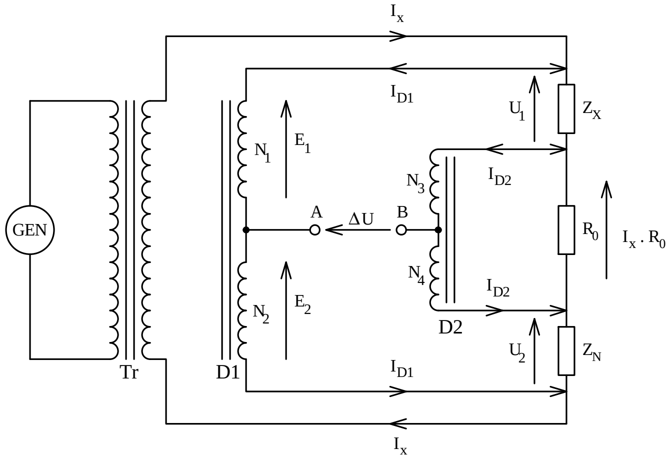

Fig. 2. Principal circuit diagram of the KWL5 bridge.

When through the impedances Z_x and Z_N flows the same current I_x, and turn ratios of the autotransformers D1 and D2 are denoted by

$$n = \frac{N_1}{N_2}, \quad m = \frac{N_3}{N_4}, \tag{4}$$

than the following equation describing the circuit can be written

$$2\Delta U = \frac{I_x R_0 (n-1)}{n+1} - \frac{I_x R_0 (m-1)}{m+1} + \frac{2(U_1 - nU_2)}{n+1}. \tag{5}$$

If the turn ratios of the D1 and D2 are equal ($m = n$) the equation (5) simplifies to

$$2\Delta U = \frac{2(U_1 - nU_2)}{n+1}. \tag{6}$$

Since

$$U_1 = Z_x I_x, \quad U_2 = Z_N I_x, \tag{7}$$

and the number of turns $N_1 = N_2 = N_3 = N_4$ ($n = m = 1$), then the equation (7) has the same form as equation (1) given for the bridge KWL4

$$Z_X - Z_N = \frac{2\Delta U}{I_X}. \tag{8}$$

Therefore, the KWL5 circuit has the same advantage as the KWL4: direct measurement of the inductance and series resistance differences of compared inductors, and additionally enables comparing small impedances. In the modified circuit impedances of leads and connections have no influence into the unbalance voltage ΔU. The voltage depends only on the difference of compared impedances and the measurement current I_x.

3.1. Shielding

When high impedance values are compared the measurement result can be incorrect because of stray capacitances between cables used and because of stray capacitances of autotransformers D1 and D2. In Fig. 3 the KWL5 bridge circuit with system of equipotential shields is shown. The technique is highly effective for minimizing the errors caused by the capacitive currents occur in the circuit.

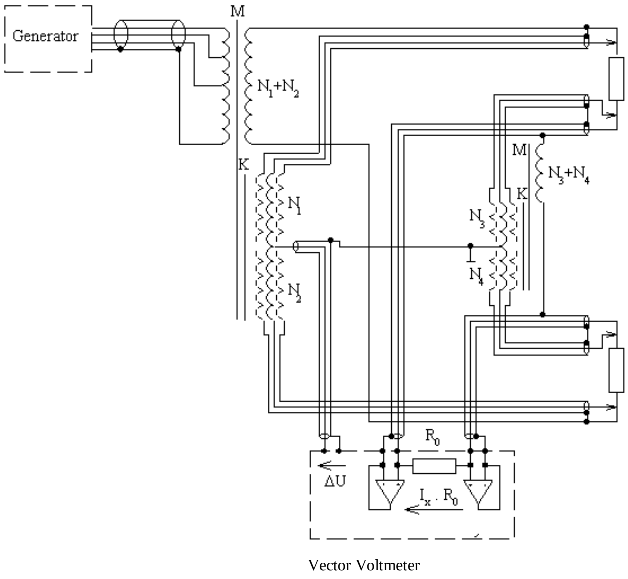

Fig. 3. KWL5 bridge circuit with shielding.

Windings of the D1 and D2 autotransformers are wound on toroidal cores using coaxial cables that the influence of the stray capacitances is mostly reduced. The inner conductors of the coaxial cables work as voltage lines and are connected to the voltage sense terminals of the bridge. The voltages on current and sense terminals are nearly equal. It should be pointed out that connecting the shields of the coaxial cables to the current terminal is very advantageous because it eliminates potential differences between the inner conductor and the shield in each point of the circuit. Thus, it effectively eliminates capacitative currents in the circuit.

The orthogonal components of the unbalance voltage ΔU and the voltage proportional to the measurement current I_x are measured by the vector voltmeter. Components of the impedance difference ΔL and ΔR are calculated according to the formulas (2) and (3), and the comparison result is displayed on the LCD panel.

4. MEASUREMENT OPTIONS

The instrument has two inputs for connection of two external inductance standards to be compared. The inputs are marked A and B. The instrument measures differences in impedance components of inductors connected to the input A and input B

$$\Delta L = L_A - L_B, \tag{9}$$

$$\Delta R = R_A - R_B. \tag{10}$$

The KWL5 bridge is designed for 4-terminal measurements and it is equipped with two 4-wire coaxial cables ended by special double fork copper terminals. Each wire is separately screened. The amplitude and the frequency of the measuring current can be set by the operator. The current value ranges from 30mA to 10 mA. The extremely low measuring current minimises the self-heating effect, however high instrument sensitivity, repeatability, and accuracy is retained. The frequency range from 100 Hz to 10 kHz covers all frequency values at which the calibration in NMI laboratories is usually made. The current and frequency values can be set using the instrument keyboard as well as precisely adjusted using handwheel encoder. Set frequency and current values are displayed on the lower LCD before starting the measurement procedure. After starting the measurement, the subsequent voltage measurement results are displayed in-pairs on the lower display, while on the upper display the last results ΔL and ΔR are displayed. After performing all of 32 measurements[1] the final results of differences are calculated again and new values are displayed on the upper LCD.

The measuring circuit is protected against the exceeding the voltage and frequency ranges by the microprocessor program. In this case the operator is informed by the message “overload” on the lower display.

5. SELF-CALIBRATION

There are three different calibration procedures applied to the comparator bridge:

a) The comparator bridge performs automatically the internal procedure of the calibration twice during each measurement of ΔL and ΔR. It consists in checking the AD converters of the voltmeter, and it is performed after each change of the converter input. This takes place before the 1st and the 17th voltage measurement. This is normal procedure of the 24-bit ΣΔ converter used in the vector voltmeter, implemented by its manufacturer (Analog Devices).

b) The procedure “autocalibration” removes the effects of undesirable charges in the switched capacitors, and enlarges the dynamic range of the converter. It is started by the pressing the “Autocal” key. The operator is strongly encouraged to initiate the “autocalibration” procedure after each change of the measuring frequency.

c) The user can periodically perform the external calibration procedure, which is needed in case of changing (e.g. by the aging) the reference resistance value R_0. The procedure should be performed individually for each current range. It consists in writing to the internal program of the bridge corrected number depended among other things on the resistance of the reference resistor used in the circuit. This should be made by connecting to the bridge inputs two resistors of precisely known resistance values (or at least their difference), free of residual inductance, and then measuring their difference and putting the corrected number to the program. The calibration procedure affects simultaneously both component results (ΔL and ΔR), because the same resistance R_0 is the reference for the both.

In the prototype instrument only its producer has the access to perform this calibration procedure.

6. TESTING

The multi-range bridge for inductance standard calibration was preliminary tested in October 2005 in Laboratory of Inductance and Capacitance Measurements at PTB in Braunschweig. Then the coherence test (commonly named as “triangle method”) based on checking the formula (11) was performed. Coherence can be checked using following equation

$$|(L_1 - L_2) + (L_2 - L_3) + (L_3 - L_1)| \leq \lambda, \tag{11}$$

where λ is arbitrary set level of coherence, for example $\lambda = 5$ nH.

The coherence test idea is presented in details in the [6]. To eliminate systematic errors cause by impedance of cables, the standards are interchanged in position. So two differences ΔL_{ij} (ΔR_{ij}) and ΔL_{ji} (ΔR_{ji}) are measured for each pair of standards. For the calculation of coherence the mean value of both measurements $\Delta L_{ij,e}$ ($\Delta R_{ij,e}$) is taken (see formulas in Tables 1 and 3).

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[1] The vector voltmeter enables to measure the components of the unbalance voltage ΔU and the voltage drop $\Delta U_0 = I_x.R_0$:

- for all four pairs of reference phases (0^0-90^0, 90^0-180^0, 180^0-270^0 and 270^0-0^0),
- for both polarities of the input signal,
- by the two channels of vector voltmeter ,

so totally 32 measurements are performed for each cycle and the mean value of the measurements is calculated.

The first coherence tests were done for three groups of standards (100 μH group, 10 mH group and 1 H group). Then designed bridge failed the coherence test but results obtained during the test helped the constructors of the KWL5 bridge to improve the instrument.

The further coherence tests were done for two groups of standards (1 mH group and 10 mH) in July 2007 at PTB. Each group consisted of three standard inductors of different inductance, resistance and factor values. Only in such case the coherence test is a powerful tool to prove the lack of most systematic errors and large random errors in the measurement procedure realized automatically by the instrument. The comparisons was performed at 1 kHz and 10 kHz and at different values of measuring currents (from 0.5 mA to 5 mA). The standards were compared using improved KWL5 bridge connected as shown in Fig. 4. Results of differences measurements for inductances and resistances of standards were inserted in Tables 1 and 3. In Tables 2 and 4 the calculation results for $\Delta L_{12,e}$, $\Delta L_{23,e}$, $\Delta L_{31,e}$, $\Delta R_{12,e}$, $\Delta R_{23,e}$, $\Delta R_{31,e}$ were inserted. Sums of calculated differences placed in the last row of Tables 2 and 4 correspond to the level of coherence of the comparison results. When we refer these level of coherence to the nominal value of the compared standards and multiply the ratio by million than the level of coherence will be expressed in ppm.

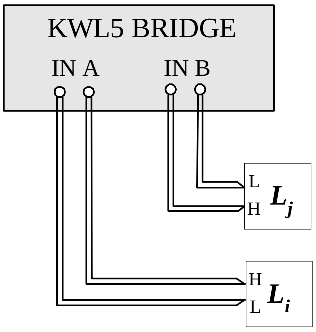

Fig. 4. Connection of standards to the bridge (i, j = 1, 2, 3 and $i \neq j$).

Table 1. Results of differences measurements for group consisted of three 1 mH standards (S_1 – impedance of standard GR1482E No. 18473, S_2 – impedance of standard GR1482E No.18460, S_3 – impedance of standard QT1482E No. 5500219).

Impedance connected to bridge input		Measured impedance difference	Comparison results ΔL_{ij} [μH]		Comparison results ΔR_{ij} [mΩ]	
Input A	Input B		f = 1 kHz I_x =5mA	f = 10kHz I_x =5mA	f = 1 kHz I_x =5mA	f = 10kHz I_x =5mA
S_1	S_2	$\Delta S_{12} = S_1 - S_2$	0.773	0.068	-2.1	1.1
S_2	S_1	$\Delta S_{21} = S_2 - S_1$	-0.783	-0.682	3.4	0.1
S_2	S_3	$\Delta S_{23} = S_2 - S_3$	-0.731	-0.215	-18.0	-21.2
S_3	S_2	$\Delta S_{32} = S_3 - S_2$	0.722	0.198	19.6	22.2
S_3	S_1	$\Delta S_{31} = S_3 - S_1$	-0.054	-0.479	22.3	21.8
S_1	S_3	$\Delta S_{13} = S_1 - S_3$	0.046	0.465	-20.6	-20.5

Table 2. Coherence calculations for results inserted in Table 1.

Formula	Calculation results $\Delta L_{ij,e}$ [μH]		Calculation results $\Delta R_{ij,e}$ [mΩ]	
	f = 1 kHz	f=10 kHz	f = 1 kHz	f=10 kHz
$\Delta S_{12,e} = 0.5(\Delta S_{12} - \Delta S_{21})$	0.778	0.675	-2.75	0.50
$\Delta S_{23,e} = 0.5(\Delta S_{23} - \Delta S_{32})$	-0.727	-0.207	-18.8	-21.70
$\Delta S_{31,e} = 0.5(\Delta S_{31} - \Delta S_{13})$	-0.050	-0.472	21.45	21.15
|SUM|	**0.002**	**0.003**	**0.10**	**0.05**

Table 3. Results of differences measurements for group consisted of three 10 mH standards (S_1 – impedance of standard GR1482H No.17864, S_2 – impedance of standard GR1482H No. 17862, S_3 – impedance of standard GR1482H No. 17859).

Impedance connected to bridge input		Measured impedance difference	Comparison results ΔL_{ij} [μH]		Comparison results ΔR_{ij} [mΩ]	
Input A	Input B		f = 1 kHz I_x =2.5mA	f = 10kHz I_x =0.5mA	f = 1 kHz I_x =2.5mA	f = 10kHz I_x =0.5mA
S_1	S_2	$\Delta S_{12} = S_1 - S_2$	4.548	1.71	-144.3	-132
S_2	S_1	$\Delta S_{21} = S_2 - S_1$	-4.563	-1.98	145.6	198
S_2	S_3	$\Delta S_{23} = S_2 - S_3$	-1.048	-0.14	-202.0	-193
S_3	S_2	$\Delta S_{32} = S_3 - S_2$	1.030	-0.10	203.0	253
S_3	S_1	$\Delta S_{31} = S_3 - S_1$	-3.524	-1.96	348.1	420
S_1	S_3	$\Delta S_{13} = S_1 - S_3$	3.504	1.70	-347.0	-361

Table 4. Coherence calculations for results inserted in Table 3.

Formula	Calculation results $\Delta L_{ij,e}$ [μH]		Calculation results $\Delta R_{ij,e}$ [mΩ]	
	f = 1 kHz	f=10 kHz	f = 1 kHz	f=10 kHz
$\Delta S_{12,e} = 0.5(\Delta S_{12} - \Delta S_{21})$	4,556	1.845	-144.95	-165
$\Delta S_{23,e} = 0.5(\Delta S_{23} - \Delta S_{32})$	-1.039	-0.020	-202.50	-223
$\Delta S_{31,e} = 0.5(\Delta S_{31} - \Delta S_{13})$	-3.514	-1.830	347.55	390.5
|SUM|	**0.002**	**0.005**	**0.10**	**2.5**

For 1 mH group the coherence level is 2 ppm at 1 kHz and 3 ppm at 10 kHz. For 10 mH group the result of coherence is smaller than 1 ppm (0.2 ppm at 1 kHz and 0.5 ppm at 10 kHz). The results obtained after modification of KWL5 confirmed that the bridge is very useful for comparison of inductance standards in National Measurement Institutes, where the highest accuracy is required.

4. CONCLUSIONS

The unbalance bridge described proved to be very convenient for the calibration of the unknown standard values with reference to a group of standards. For this purpose comparisons between standards, calibrated and referenced, should be performed and a value of a calibrated standard can be then estimated by statistical methods [6].

Older types of the unbalance transformer bridges (KWL4 type) cooperated with digitally controlled switches are still successfully used in National Measurement Institutes in Germany and Poland (PTB and GUM) for calibration of 10 mH inductance standards at frequencies 1 kHz and 1592 Hz. The uncertainty of inductance difference measurement for these instruments is smaller than 10 ppm. It is expected that in the nearest future the KWL4 bridges will be replaced by the KWL5 which will enable 1:1 comparison of inductors from 100 μH to 10 H in the frequency range from 100 Hz to 10 kHz with uncertainty smaller the 10 ppm.

Nowadays, the KWL5 bridge is tested at Physikalisch-Technische Bundesanstalt in Braunschweig, Germany. First investigations confirmed high accuracy and usefulness of the instrument for the inductance standard calibrations in the wide impedance and frequency ranges. An additional activities will be taken to improve properties and meet high accuracy requirements for the smaller and the highest impedance measurement.

ACKNOWLEDGMENTS

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