

FEM ANALYSIS OF ROGOWSKI COILS COUPLED WITH BAR CONDUCTORS

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Abstract – We study the behavior of Rogowski coils coupled with bar conductors under quasi-static conditions. Through a finite element (FEM) analysis, we estimate the current distribution across the bar and the flux linked by the transducer for various positions of the primary conductor and for various operating frequencies. Simulation and experimental results are reported in the text.

Keywords: Rogowski coil, FEM analysis, quasi-static conditions.

1. INTRODUCTION

Rogowski coils are often employed for measuring high alternating currents or high current pulses [1], [2]. The basic principle of operation is based on the mutual coupling between the current transducer and the primary conductor. Apart from the ideal case, the mutual inductance M depends on the current distribution across the primary conductor and on the relevant positions between the active edges of the winding and the primary conductor itself. In many actual configurations, the accurate estimation of M should require a full electromagnetic analysis.

Once we know the current distribution across the primary conductor, the estimation of M can be obtained from the geometry of the system by means of analytical or quasi-analytical methods. Biot-Savart's law, partial inductance technique, or, more in general, magnetic energy method can be adopted to solve the problem [3], [4]. On the other hand, the current distribution across a massive conductor obeys to a diffusion equation and, except for particular geometries and paths of the primary conductor a numerical solution must be performed.

Through a frequency analysis carried out by a commercial FEM code we estimate the variation of the flux linked by circular Rogowski coils with rectangular cross sections coupled with an infinitely long bar conductor. Our research objective consists in improving the design technique and possibly the calibration of such current transducers. In particular, we focus on Rogowski coils with small number of turns being the number of turns very critical in terms of variation of M [3].

2. BASIC CONFIGURATIONS

In Fig. 1, we show a schematic of a circular Rogowski coil with 8 turns crossed by a primary bar conductor. The internal and external radii of the torus are $a = 3$ cm and $b = 4,5$ cm, respectively, while its height is $w = 3,2$ cm. Dimensions of the bar are width $c = 3$ cm and thickness $d = 0,5$ cm. The primary conductor is assumed to be infinite in the axial direction.

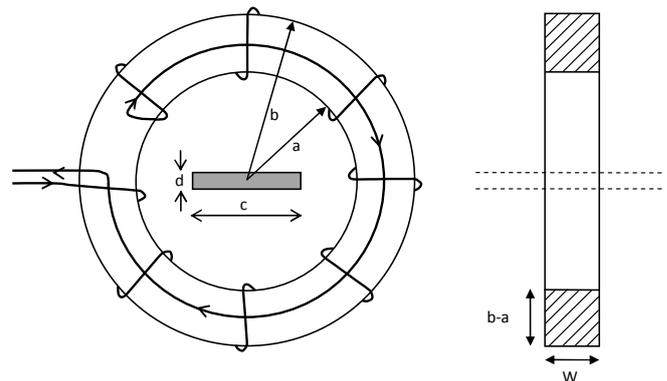


Fig. 1. Schematic representation of the reference configuration – position 1.

As to be expected the variation of M reduces significantly with the number of turns N . Indeed, as known from the theory, a Rogowski coil with a very small pitch, i.e., a very high number of turns, can be used to show experimentally the Ampère's law.

In Fig. 2, we show the primary conductor shifted of 2,25 cm along the y -direction, while in Fig. 3 the bar is shifted of 1,45 cm along the x -axis.

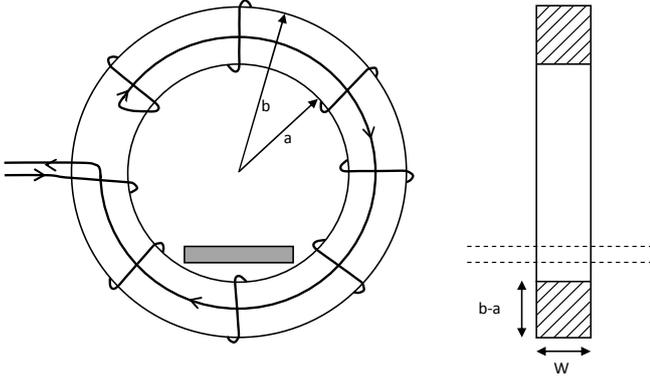


Fig. 2. Second configuration with the primary bar conductor shifted along the y -direction of 2,25 cm. Position 2.

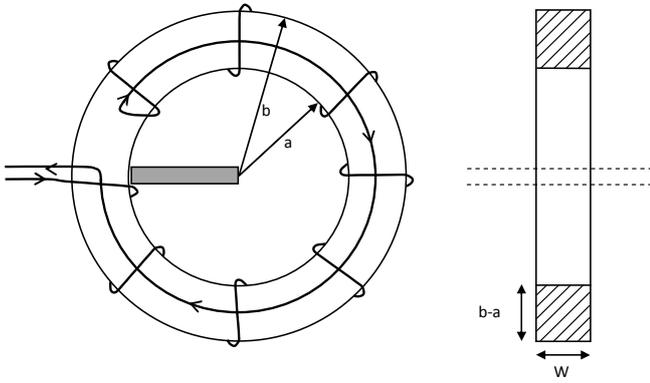


Fig. 3. Third configuration with primary bar conductor shifted along the x -direction of 1,45 cm. Position 3.

3. FEM ANALYSIS

We adopted a time-harmonic quasi-static formulation to solve the field distribution of our problems. Thus, starting from the two Maxwell's equations :

$$\vec{E} = -i\omega\vec{A} - \nabla V \quad (1)$$

$$\vec{B} = \nabla \times \vec{A} \quad (2)$$

introducing two new potentials:

$$\vec{A}' = \vec{A} + \nabla \psi \quad (3)$$

$$V' = V - i\omega\psi \quad (4)$$

and fixing the gauge

$$\psi = \frac{V}{i\omega} \quad (5)$$

we obtain the driving equation:

$$\nabla \times \mu_0^{-1} \nabla \times \vec{A}' + i\omega\vec{A}' = \vec{J}^e \quad (6)$$

We started our simulations solving a 2D problem representing an indefinite conducting cylinder. Simulation results have been compared with the analytical calculation and data were in complete agreement with respect to our purposes. Percentage differences were less than one part per ten thousand. In particular, the flux linked by the modelled Rogowski has been calculated through the circulation of the magnetic vector potential along its active edges and compared with the direct application of the Ampère's law. On the other hand, the current distribution across the round wire has been compared with the analytical solution expressed in terms of Bessel functions.

In Fig. 4, we show the colour map of the current density distribution through the bar at 200 Hz, 1000 Hz, and 2000 Hz. For all the simulations the calculation of the flux linked by the Rogowski is done via the circulation of the magnetic vector potential A . The 2D computation domain is represented by a circular region of radius $r = 0,15$ m. The number of triangular-type elements is about 153000 corresponding to a number of degrees of freedom of about 307000. Simulations were performed on a 64-bit 16 GB Ram platform and the computational time for each simulation was about 1 minute.

It is apparent how skin effects are practically negligible at 200 Hz, while they are significant at 2000 Hz. A rough estimation of the eddy current effect can be evaluated for a copper lamination by the calculation of the penetration depth δ :

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} \quad (7)$$

where σ is the electrical conductivity and μ the magnetic permeability. From (7) we obtain a penetration depth of about 4,7 mm, 2,1 mm and 1,5 mm at 200 Hz, 1000 Hz and 2000 Hz, respectively. In more detail, the hell blue color (upper image – 200 Hz and centre image – 1000 Hz) corresponds to a current density of 0,06 A/mm², while the red colour (lower plot – 2000 Hz) indicates a current density value of about 0,35 A/mm².

In Fig. 5, we show the colour map of the magnetic field distribution in the computation domain. Data are evaluated at 2000 Hz.

4. EXPERIMENTAL SETUP

In Fig. 6, it is shown a part of the adopted experimental setup. It is possible to recognize the bar, the Rogowski coil and the output signal cable. A one meter copper bar (thickness 0.5 cm, width 3 cm) is closed to form a rectangular loop of 4 m \times 3 m to avoid proximity effects between the Rogowski coil and the rest of the circuit.

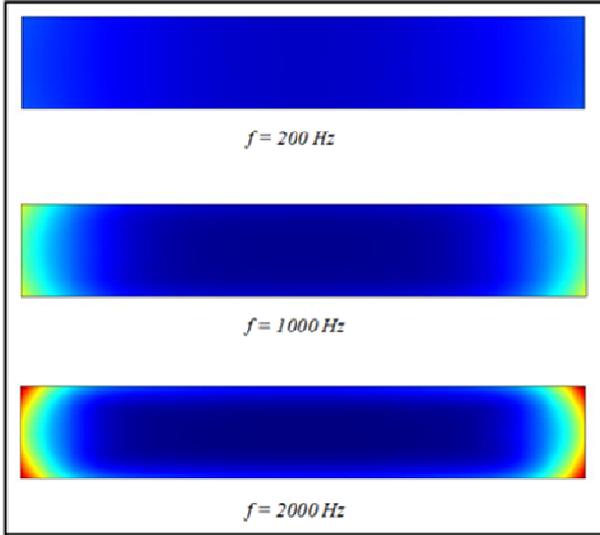


Fig. 4. Current density distribution through the bar conductor at 200 Hz, 1000 Hz and 2000 Hz.

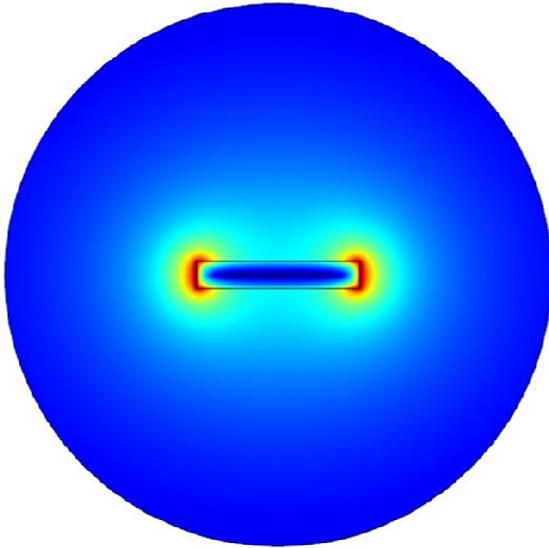


Fig. 5. Magnetic field (H) in the computation domain at 2000 Hz

An arbitrary function generator (Agilent 33220A) is connected to the loop via a transconductive power amplifier (not shown in the figure), so that it works as a current generator. A Labview[®] platform drives all the system through the IEEE 488[®] protocol. A sinusoidal current with amplitude 10 A is automatically set at each investigated frequency.

The current through the circuit is measured by means of a hall probe. Another DMM – Agilent 34401A measures the induced voltage at the remote ends of the Rogowski coil.

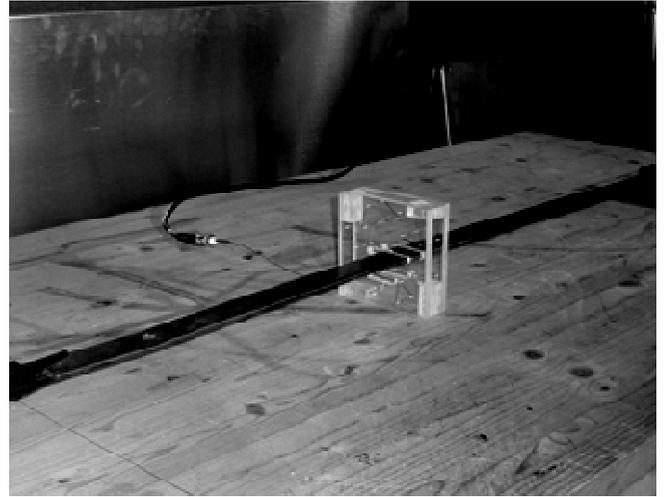


Fig. 6. Part of the adopted experimental setup.

We wound two times eight turns on two plexiglas dielectric supports of thickness 8 mm and placed at a distance of 16 mm between the internal faces, obtaining a total height of 32 mm of the transducer. The internal and external diameters are 30 mm and 45 mm, respectively. In Fig. 7, it is shown the adopted Rogowski coil and a segment of the primary bar conductor placed as shown in Fig. 1.

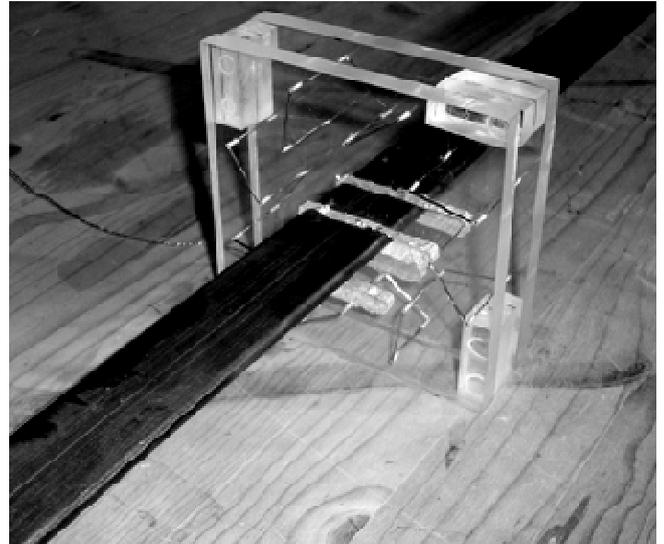


Fig. 7. Rogowski coil coupled with the bar conductor in the position 1.

5. RESULTS AND DISCUSSION

In Table I, we report the M values for the described configurations provided by the FEM simulation at 200 Hz, 1000 Hz, and 2000 Hz. For all the simulations and measurements, position 1 refers to Fig. 1, position 2 to Fig. 2 and position 3 to Fig. 3, and position 1 is the benchmark.

TABLE I. Mutual inductance values for the investigated configurations. (Values are in μH).

	f = 200 Hz	f = 1000 Hz	f = 2000 Hz
Position 1	43,49	43,49	43,49
Position 2	42,12	41,80	41,81
Position 3	45,41	46,30	46,55

In Table II, the percentage variations with respect to the centred position are reported.

TABLE II. Percentage variation of M for the investigated configurations (simulated data).

	f = 200 Hz	f = 1000 Hz	f = 2000 Hz
Position 1	0	0	0
Position 2	-3,2	-3,9	-3,9
Position 3	+4,4	+6,4	+7,0

In Table III, we report the measured percentage variations of M for the investigated configurations.

TABLE III. Percentage variation of M for the investigated configurations (measured data).

	f = 200 Hz	f = 1000 Hz	f = 2000 Hz
Position 1	0	0	0
Position 2	-3,0	-3,6	-3,8
Position 3	+3,9	+5,1	+5,6

The obtained results are summarized in Fig. 8 that reports the measured (circle symbols) and computed (cross symbols) percentage variations of the mutual inductance M for the investigated configurations. At each frequency, lower and upper plots refer to position 2 and 3, respectively. Results show how the frequency and position play an important role and how it is important an accurate knowledge of the relevant positions among the active elements and calculation of the current density distribution for a correct reconstruction of the primary current.

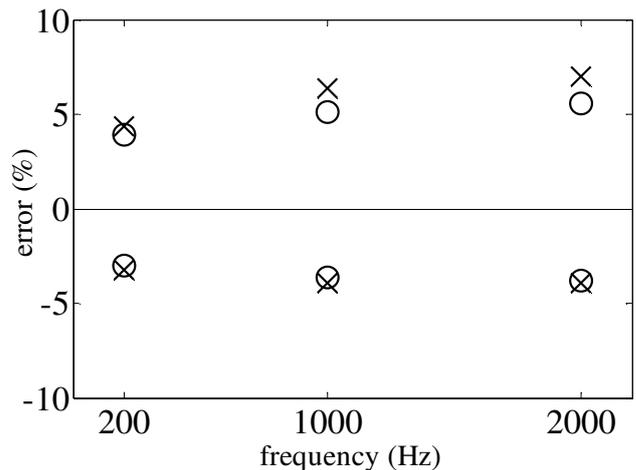


Fig. 8. Measured (circles) and computed (crosses) variations of the induced voltage with respect to the benchmark for position 2 (lower plot) and position 3 (upper plot).

In particular, for our implemented geometries the maximum percentage variation of M has been observed at 2000 Hz from $-3,9\%$ up to $+7,0\%$ with respect to the benchmark. This can make the Rogowski an inaccurate transducer if such parameters are not taken into account.

It is apparent how a numerical simulation can be a useful tool in the designing phase of air-cored current transducer. In addition, an accurate evaluation of the parameter M over the frequency range of interest can be very important for a correct reconstruction of distorted primary currents. Indeed, the intrinsic linearity of the Rogowski coil makes possible to evaluate the spectrum of the output voltage and then via the $M(f)$ relation to obtain the nonsinusoidal primary current.

Preliminary results appear to be promising for extending the FEM analysis to more complex geometries as that shown in Fig. 9. Further investigations are currently in progress to improve the design of such current transducers.

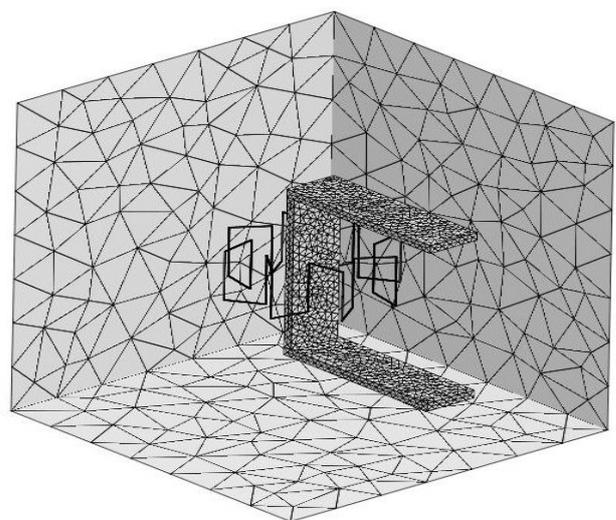


Fig. 9. 3D problem, rectangular C-shaped primary conductor.

6. CONCLUSIONS

Our main purpose has been to develop a FEM analysis for investigating how the current density distribution can affect the estimation of the mutual coupling between Rogowski coils and bar conductors. Results have shown how frequency and position are critical parameters for such transducers, especially for Rogowski coil having a small number of turns. The FEM analysis appears to be an effective tool to improve the design of such current transducers.

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