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UNCERTAINTIES IN THE WHOLE RANGE OF THE CALIBRATION OF A THERMOCOUPLE

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Abstract − This contribution describes the procedure of evaluating the calibration of thermocouple by means of its comparison with the thermocouple standard. In the process of thermocouple calibration by means of comparison, the resulting uncertainty specified by applying the generalized procedure for evaluating the calibration of measuring devices with continuous scale. The advantage of this method of evaluation is the determination of uncertainties in the whole range of the calibration. The conclusion of this paper states the illustrated differencies between cases when covariances are into account or are not.

Keywords: **calibration, thermocouple uncertainties**

1. INTRODUCTION

The best way for increasing the accuracy of measurement in modern metrology is often the application of modern mathematical-statistical method which until now has not been sufficiently utilized for the evaluating the calibration of instrument. This attitude is reasonable because current technical solutions are so perfect, that their development stagnates. For measuring instrument with continuous scale a generalized procedure for evaluating the calibration uncertainties and covariances has been developed by Palenčár, Wimmer [1,2] and Kubáček [6]. In this paper authors are presenting these procedures for evaluating uncertainties of the calibration of a thermocouple (hereafter TC only) type S by means of comparison.

2. CALIBRATION PROCEDURE

Calibration is carried out by comparison of the unit under test TC type S against standard TC type S calibrated in defined fixed points according to ITS-90 (Fig.2.1). Thermoelectric voltage (emf) is measured by digital voltmeter connected to PC through GPIB port for simultaneous recording of values. As a source of heat is used the horizontal pipe calibration furnace. Here the TC's measuring junction is placed and reference junction is maintained at 0 °C in Dewar flask. Calibration is carried out in the range from 0 °C to 1100 °C. In each calibration point measurement is repeated ten times. Ambient temperature is 23 °C ± 1 °C. The calibration is represented as a curve fitted to the measured values of the deviation E-E_{ref} and generally given as a function of temperature t. This curve is representing deviation function.

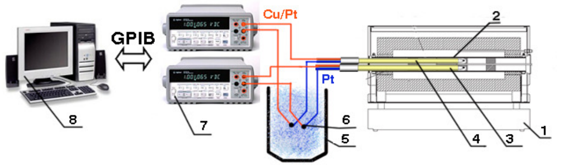


Fig. 2.1: Scheme of calibration
1- Calibration furnace, 2- Isothermal block, 3- Standard TC, 4- Unit under test, 5- Dewar flask, 6- Reference junction of TC's, 7- Voltmeters, 8- Computer with GPIB port

3. METHODOLOGY

We consider the case, when number of calibration points r is higher than number of unknown parameters p, $r>p$ the model is overdetermined. Calibration model should be established using following relations (3.1)

$$\begin{aligned} W_{100} &= a_0 + a_1 \cdot t_{100} + a_2 \cdot t^2{}_{100} + a_3 \cdot t^3{}_{100} + a_4 \cdot t^4{}_{100} \\ W_{200} &= a_0 + a_1 \cdot t_{200} + a_2 \cdot t^2{}_{200} + a_3 \cdot t^3{}_{200} + a_4 \cdot t^4{}_{200} \\ &\vdots \\ W_{1100} &= a_0 + a_1 \cdot t_{1100} + a_2 \cdot t^2{}_{1100} + a_3 \cdot t^3{}_{1100} + a_4 \cdot t^4{}_{1100} \end{aligned} \tag{3.1}$$

in matrix notation

$$\boldsymbol{W} = \boldsymbol{Ta} \tag{3.2}$$

where $\boldsymbol{T}$ is a matrix, which contains values, arithmetical means of series of measurements in each calibration points (3.3) measured by standard TC.

$$T = \begin{pmatrix} 1 & t_{100} & t^2{}_{100} & t^3{}_{100} & t^4{}_{100} \\ 1 & t_{200} & t^2{}_{200}2 & t^3{}_{200} & t^4{}_{200} \\ 1 & t_{300} & t_{300}2 & t^3{}_{300} & t^4{}_{300} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & t_{1100} & t^2{}_{1100} & t^3{}_{1100} & t^4{}_{1100} \end{pmatrix} \tag{3.3}$$

Left side of the model (3.1) or (3.2), the observation vector $\boldsymbol{W}$ is presenting the measurement model of unit under test TC

$$\boldsymbol{W} = \boldsymbol{\Delta E} + \boldsymbol{C}_{\mathrm{K}} \boldsymbol{\Lambda} \tag{3.4}$$

where $\boldsymbol{\Delta E}$ is the vector of deviations from the reference function (3.5). Reference function is given by IEC 584.2 standard (3.6)

$$E_{\mathrm{ref}\,i} = \sum_{k=1}^{8} b_k t_i^{\,k},\ i = 100, 200, \ldots 1100 \tag{3.5}$$

$$\boldsymbol{\Delta E} = \begin{pmatrix} \overline{E}_{100} - E_{\mathrm{ref}_{100}} \\ \overline{E}_{200} - E_{\mathrm{ref}_{200}} \\ \overline{E}_{300} - E_{\mathrm{ref}_{300}} \\ \vdots \\ \overline{E}_{1100} - E_{\mathrm{ref}_{1100}} \end{pmatrix} \tag{3.6}$$

in product of $\boldsymbol{C}_{\mathrm{K}} \boldsymbol{\Lambda}$ fills every influences of measurement.

Vector of correction $\boldsymbol{\Lambda}$ is given by

$$\boldsymbol{\Lambda}^{\mathrm{T}}{}_{1\times 20} = \begin{pmatrix} \delta E_{\mathrm{IH}} & \delta E_{\mathrm{RV}} & \delta E_{\mathrm{K}} & \delta E_{\mathrm{D}} & \delta E_{\mathrm{CK}} \\ & \delta E_{\mathrm{N}} & \delta t_{\mathrm{R0}} & \delta t_{\mathrm{F}} & \delta t_{\mathrm{RF}} \end{pmatrix} \tag{3.7}$$

where

δE_{IH}	- correction linked to the reading of voltmeter
δE_{RV}	- correction linked to the resolution voltmeter
δE_{K}	- correction obtained from the calibration of voltmeter
δE_{D}	- correction linked to the drift of voltmeter
δE_{CK}	- correction linked to the compensation cable
δE_{N}	- correction due to the inhomogenity of the thermocouple wires
δt_{R0}	- correction due to the deviation of the ice bath temperature
δt_{F}	- correction linked to the nonuniformity of the temperature profile
δt_{RF}	- error of reference function

and matrix $\boldsymbol{C}_{\mathrm{K}}$ is the known matrix, usually its elements are sensitivity coefficients.

Our aim is to get estimation for unknown parameters of deviation function. This aim could be reached by using least-square method. Uncertainties are taken into account as well. We apply following expression iteratively because of stochastic character of quantity t.

$$\hat{\boldsymbol{a}} = \left(\boldsymbol{T}^{\mathrm{T}} \boldsymbol{U}_{\boldsymbol{W}}{}^{-1} \boldsymbol{T}\right)^{-1} \boldsymbol{T}^{\mathrm{T}} \boldsymbol{U}_{\boldsymbol{W}}{}^{-1} \boldsymbol{W} \tag{3.8}$$

Initial values of unknown parameters $\hat{\boldsymbol{a}}$ of deviation function are determined by zero estimation. Then covariance matrix of input quantities $\boldsymbol{U}_{\boldsymbol{W}}$ is

$$\boldsymbol{U}_{\boldsymbol{W}} = \boldsymbol{U}_{\boldsymbol{\Delta E}} + \boldsymbol{C}_{\mathrm{K}} \boldsymbol{U}_{\boldsymbol{\Lambda}} \boldsymbol{C}_{\mathrm{K}}{}^{\mathrm{T}} \tag{3.9}$$

where

$\boldsymbol{U}_{\boldsymbol{\Delta E}}$ - covariance matrix of the vector $\boldsymbol{\Delta E}$ is diagonal matrix, principal-diagonal elements present square of uncertainties estimated by type A method

$\boldsymbol{C}_{\mathrm{K}} \boldsymbol{U}_{\boldsymbol{\Lambda}} \boldsymbol{C}_{\mathrm{K}}{}^{\mathrm{T}}$ - product of these matrix gives diagonal covariance matrix, principal-diagonal elements present square of uncertainties estimated by type B method

$\boldsymbol{U}_{\boldsymbol{\Lambda}}$ - uncertainties of correction measurement by unit under test TC are included in this covariance matrix

Covariance matrix $\boldsymbol{U}_{\hat{\boldsymbol{a}}}$ is represented by matrix of the uncertainties of the estimates

$$\boldsymbol{U}_{\hat{\boldsymbol{a}}} = \left(\boldsymbol{T}^{\mathrm{T}} \boldsymbol{U}_{\boldsymbol{W}}{}^{-1} \boldsymbol{T}\right)^{-1} \tag{3.10}$$

Deviation associated with the reference function is solved by $\boldsymbol{\Delta} \hat{\boldsymbol{E}} = \boldsymbol{T} \hat{\boldsymbol{a}}$ (3.11)

uncertainty of the deviation can be achieved by application of law of propagations of uncertainties

$$\boldsymbol{u}^2{}_{\boldsymbol{\Delta E}} = \boldsymbol{T} \cdot \boldsymbol{U}_{\hat{\boldsymbol{a}}} \cdot \boldsymbol{T}^{\mathrm{T}} \tag{3.12}$$

Zero estimation of vector $\hat{\boldsymbol{a}}$ is biased (see Fig.4.1(a)). It is caused by stochastic characters of the quantity t. Therefore the model is nonlinear and requires a solution procedure. It is linearized by application of Taylor series and higher elements of estimated values are neglected. After linearization left side of model vector $\boldsymbol{W}$ will be

$$\boldsymbol{W} = \boldsymbol{\Delta E} + \boldsymbol{C}_{\mathrm{K}} \boldsymbol{\Lambda} + \boldsymbol{D}\left(\boldsymbol{\delta t}_1 + \boldsymbol{C}_{\mathrm{S}} \boldsymbol{\delta t}_2\right) \tag{3.13}$$

$\boldsymbol{D} = \mathrm{diag}\left(d_{100} \quad d_{200} \quad \ldots \quad d_{1100}\right)$ - is the known matrix, obtained by application of expansion of Taylor series

$$\begin{aligned} d_{100} &= \hat{a}_1 + 2\hat{a}_2 t_{\mathrm{T}_{100}} + 3\hat{a}_3 t_{\mathrm{T}_{100}}^2 + 4\hat{a}_4 t_{\mathrm{T}_{100}}^3 \\ d_{200} &= \hat{a}_1 + 2\hat{a}_2 t_{\mathrm{T}_{200}} + 3\hat{a}_3 t_{\mathrm{T}_{200}}^2 + 4\hat{a}_4 t_{\mathrm{T}_{200}}^3 \\ &\vdots \\ d_{1100} &= \hat{a}_1 + 2\hat{a}_2 t_{\mathrm{T}_{1100}} + 3\hat{a}_3 t_{\mathrm{T}_{1100}}^2 + 4\hat{a}_4 t_{\mathrm{T}_{1100}}^3 \end{aligned} \tag{3.14}$$

$\left(\boldsymbol{\delta t}_1 + \boldsymbol{C}_\mathrm{S}\boldsymbol{\delta t}_2\right)$ - this part is valid for standard,
vector $\boldsymbol{\delta t}_1$ - error given by standard
product $\boldsymbol{C}_\mathrm{S}\boldsymbol{\delta t}_2$ - which contains influence quantities concerning a standard TC, same way as in experession (3.4) for unit under test.

After linearization covariance matrix $\boldsymbol{U_W}$ has the form

$$\boldsymbol{U_W} = \boldsymbol{U}_{\Delta \boldsymbol{E}} + \boldsymbol{C}_\mathrm{K}\boldsymbol{U}_\Lambda \boldsymbol{C}_\mathrm{K}{}^\mathrm{T} + \boldsymbol{D}\left(\boldsymbol{U}_{\boldsymbol{\delta t}_1} + \boldsymbol{C}_\mathrm{S}\boldsymbol{U}_{\boldsymbol{\delta t}_2}\boldsymbol{C}_\mathrm{S}{}^\mathrm{T}\right)\boldsymbol{D}^\mathrm{T} \tag{3.15}$$

where

$\boldsymbol{U}_{\boldsymbol{\delta t}_1}$ - covariance matrix of the vector $\boldsymbol{\delta t}_1$ is diagonal matrix, principal-diagonal elements present square of uncertainties estimated by type A method

$\boldsymbol{C}_\mathrm{S}\boldsymbol{U}_{\boldsymbol{\delta t}_2}\boldsymbol{C}_\mathrm{S}{}^\mathrm{T}$ - product of this matrix is given by diagonal covariance matrix, principal-diagonal elements present square of uncertainties estimated by type B method

$\boldsymbol{U}_{\boldsymbol{\delta t}_2}$ -uncertainties of correction of measurements by standard are included in this covariance matrix

Now in new iteration we consider the observation vector $\boldsymbol{W}$ (3.13) and covariance matrix $\boldsymbol{U_W}$ (3.15) and we use formula for estimation of parameters (3.8).
Numerically, in the most cases design matrix $\boldsymbol{T}$ is badly scaled and its columns are nearly linearly dependent. For this it is reasonable to transform quantities of t to interval $-1 \le t \le 1$ according to following relationship

$$t_\mathrm{T} = \frac{(t_i - t_{\max}) - (t_{\max} - t_i)}{t_{\max} - t_{\min}}; i = 100, 200, \ldots 1100 \tag{3.16}$$

Feedback transformation is carried out by multiplicating the row of vector $\boldsymbol{t}_\mathrm{T}$ determined by (3.16) and columns of vector of estimated parameters $\hat{\boldsymbol{a}}$ and columns of $\boldsymbol{U}_{\hat{\boldsymbol{a}}}$ as first multiplied from left side then right side.

From the viewpoint of the user relevant results are the temperature values and their uncertainties. Temperature value can be obtained by interpolation table which can be edited from deviation function and its uncertainty is determined by application of theorem for implicit function.

$$f(E, t, a) = E - g(t, \boldsymbol{a}) = 0 \tag{3.17}$$

$$\begin{aligned} g(t, \boldsymbol{a}) &= a_0 + (a_1 + b_1)\cdot t + (a_2 + b_2)\cdot t^2 + \\ &+ (a_3 + b_3)\cdot t^3 + (a_4 + b_4)\cdot t^4 + b_5 t^5 + b_6 t^6 + b_7 t^7 + b_8 t^8 \end{aligned} \tag{3.18}$$

we get it by adding up deviation function and reference function, where variable E is representing the current measured value of emf. Now consider function t=(h,$\boldsymbol{a}$) is defined from the implicit function. This function is continous and we has the partial derivation

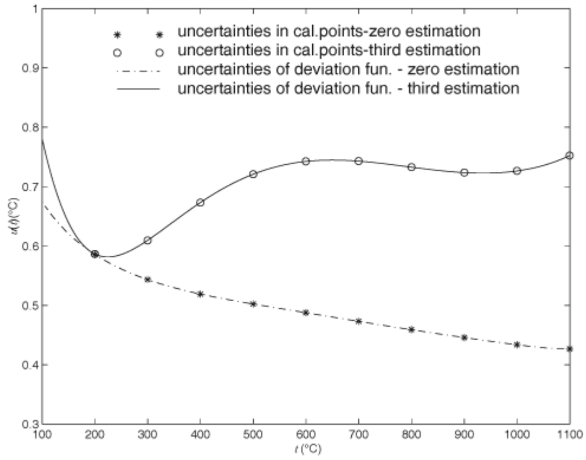
$$\frac{\partial h}{\partial a_\mathrm{k}} = -\frac{\dfrac{\partial f}{\partial a_\mathrm{k}}}{\dfrac{\partial f}{\partial t}} = -\frac{\dfrac{\partial g}{\partial a_\mathrm{k}}}{\dfrac{\partial g}{\partial t}} = -\frac{t^\mathrm{k}}{S(t)}, \mathrm{k} = 0, \ldots 4 \tag{3.19}$$

Derivation (3.19) is presenting elements of vector $\boldsymbol{h}$. Standard uncertainty is then obtained from the (3.20) relation

$$u^2(t) = \boldsymbol{h}^\mathrm{T} \cdot \boldsymbol{U}_{\hat{a}} \cdot \boldsymbol{h} \tag{3.20}$$

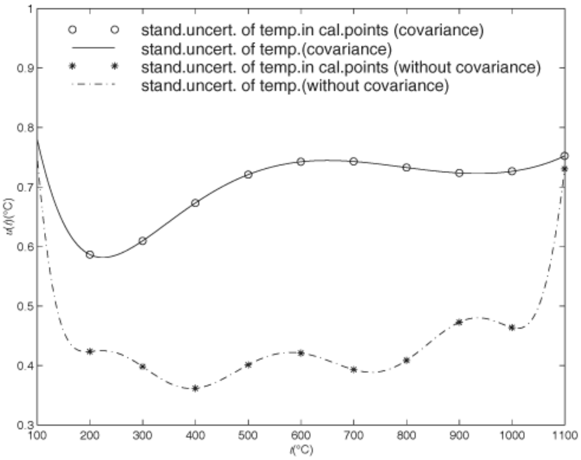
4. CONCLUSIONS

Procedure for evaluating the calibration of TC was applied to demonstrate whether considering the covariances has an impact on final result of standard uncertainty. For this reason was carried out the evaluation twice. The difference is shown in Fig. 4.1(b).



(a)

As a conclusion we can claim that covariances had significant effect on final result of a calibration.



(b)

Fig 4.1: Standard uncertainties of deviation function: (a) Difference between zero and third estimation of parameters, (b) Standard uncertainties derived from third estimation when consider covariance and not

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