TEMPERATURE AND FREQUENCY DEPENDENCE OF PRECISION CURRENT TRANSFORMER BASED ON ROGOWSKI COILS

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Abstract − This paper covers the analysis of electrical parameters which affect the accuracy of current-to-voltage transducer based on Rogowski coil. The actual transducer is designed for high-accuracy measurement of AC current (up to 20 A at power supply frequency of 50 Hz, with the aiming uncertainty of 100 parts per million). The primary source of uncertainty of ac current measured by this type of transducer is non-ideal geometry (i.e. mutual position between primary conductor and secondary coils). Except these influencing parameters, the temperature dependence of coil geometry affect the accuracy as additional source of uncertainty, as well as the self resonance frequency, especially on higher frequencies. The analysis of influence of these parameters and temperature compensation of transformer transimpedance makes the topic of this work.

Keywords: Rogowski coil, temperature compensation, frequency compensation

1. GEOMETRIC CONFIGURATION OF THE TRANSFORMERS AND ITS EQUIVALENT ELECTRICAL CIRCUIT

The description of geometry and construction details of air-core transformer based on Rogowski coils, which was designed for precision measurement of ac current, was given in [1]. Electromotive force, induced in each pair of Rogowski coils with arbitrary, most often flexible toroidal shape which enclose the primary conductor, is proportional to the derivation of total magnetic flux in coil, i.e. time derivation of measured current in primary conductor and mutual inductance M of that geometrical system:

$$e(t) = -\frac{\mathrm{d}\Psi(t)}{\mathrm{d}t} = -M \cdot \frac{\mathrm{d}i(t)}{\mathrm{d}t} \qquad (1)$$

Secondary windings of transformer are based on toroidal Rogowski coil with square cross section (Fig. 1). Mutual inductance of each toroidal coil with such geometry is expressed with the following equation:

$$M = \frac{\Psi}{I} = \mu_0 \cdot \frac{N \cdot (h+d)}{2\pi} \cdot \ln\left(\frac{2r_V + d}{2r_U - d}\right), \qquad (2)$$

where h represents height of toroidal body, d represents the external diameter of wire, while r_U and r_V indicate inner and outer radius, respectively. In our case, these parameters are $h = 19{,}24$ mm, $d = 0{,}322$ mm, $r_U = 15$ mm and $r_V = 37$ mm.
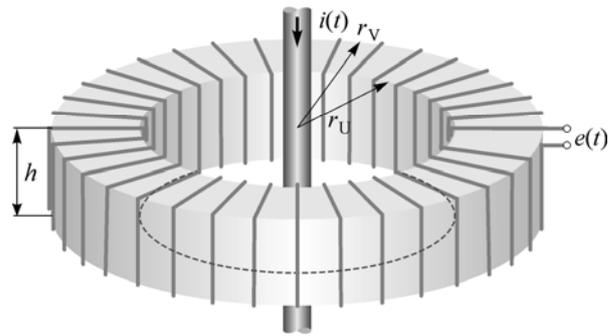
Fig. 1. Geometry of primary conductor and Rogowski coil

The configuration of secondary coils is astatic [1], with two pairs of identical toroidal coils with square cross section (Fig. 2).
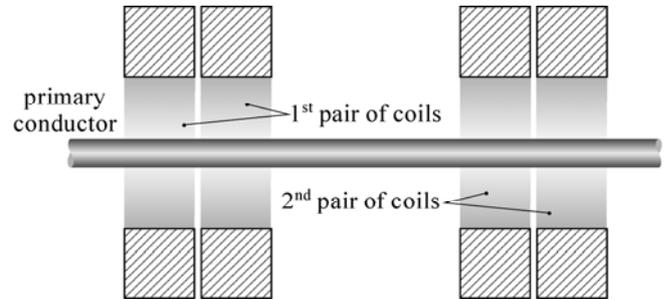
Fig. 2. Astatic configuration of secondary part

Two pairs of secondary coils assure differential output signal and, because of derivate transfer function (1) of transformer, both secondary parts are connected to the analogue integrators, the outputs of which are connected to the differential inputs of instrumentation amplifier (Fig. 3). Significant feature of this configuration is the rejection of disturbances generated by the strange magnetic fields. On the equivalent electrical circuit (Fig. 3) of transformer, the parasitic parameters of secondary windings are shown. These parameters are self inductances, self capacitances and self resistances of each pair of coils. The calculated value of mutual inductance of each toroidal coil is $M = 1{,}0308$ μH, hence for each secondary part (i.e. each pair of the coils) these parameters are: $M_S = 2{,}0616$ μH, $L_S = 591{,}679$ μH,

$R_S = 12{,}06\ \Omega$ and $C_S \approx 57$ pF, while the calculated value of mutual inductance of entire transformer is $M_T = 4{,}1232\ \mu$H.
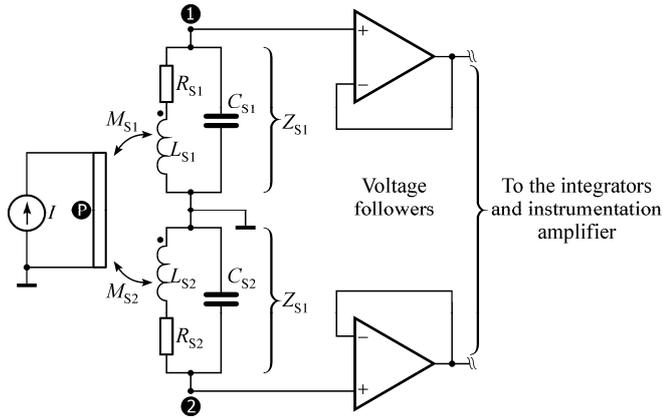
Fig. 3. The principle of ac current measurement by transducer based on Rogowski coil

2. TEMPERATURE DEPENDENCE OF MUTUAL INDUCTANCE

The influence of configuration of secondary part and its geometrical position in relation to the primary conductor had been discussed in detail in [2], and here we will concentrate on the temperature influence on the mutual inductance. This influence has double impact: the changes of temperature affect the resistance of coils, besides the stretch of wire material of which coils are made. Wire stretching results in a change of cross section of each turn, which leads to the changes of total magnetic flux, and thus change of mutual inductance. Each coil made with N turns of wire with diameter d (Fig. 4) has sides with effective length of $(r_V - r_U + d)$ and $(h + d)$, and on reference temperature T_0 the total length of wire is:

$$l_0 = 2N(r_V - r_U + h + 2d) \tag{3}$$

Assuming that the wire material has equal temperature coefficient of expansion in all three axes, the total length of wire at temperature T is then:

$$l_1 = l_0(1 + \beta \cdot \Delta T), \tag{4}$$

where β represents the linear coefficient of expansion, and $\Delta T = (T - T_0)$. Small changes in relative length allow us to assume the increase of cross-section of each turn as a consequence of relative expansion of each side for $(1 + \beta \Delta T)$. Introducing the function of temperature dependence in the expression (2), the mutual inductance of each coil at the temperature T is:

$$M = \mu_0 \cdot \frac{N \cdot (h+d) \cdot (1 + \beta \cdot \Delta T)}{2\pi} \cdot \ln\left\{ \frac{r_V + \frac{d}{2} + \left(\frac{r_V - r_U + d}{2}\right) \cdot \beta \cdot \Delta T}{r_U - \frac{d}{2} - \left(\frac{r_V - r_U + d}{2}\right) \cdot \beta \cdot \Delta T} \right\} \tag{5}$$

In case of bigger toroidal coils, the wire diameter d would be negligible in comparison to the dimensions of coil, and hence the relative deviation of mutual inductance due to the

increase of surrounding temperature from T_0 to T can be reduced to the form:

$$\left(\frac{\Delta M}{M}\right)_T \approx \frac{(1 + \beta \Delta T) \cdot \ln\left\{ \frac{r_V \cdot (2 + \beta \Delta T) - r_U \cdot \beta \Delta T}{r_U \cdot (2 + \beta \Delta T) - r_V \cdot \beta \Delta T} \right\} - \ln\left(\frac{r_V}{r_U}\right)}{\ln\left(\frac{r_V}{r_U}\right)} \tag{6}$$
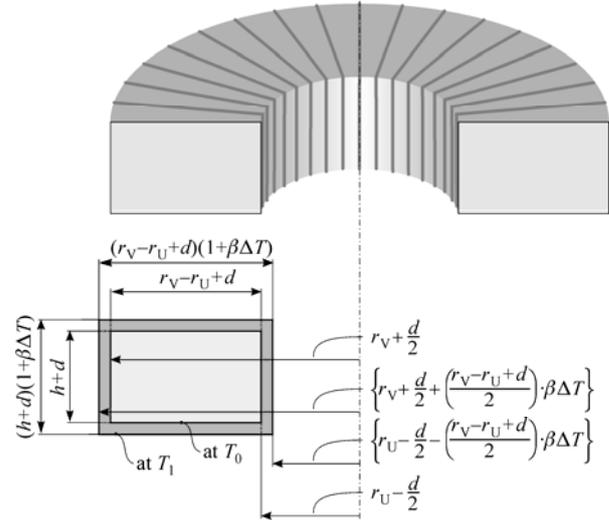
Fig. 4. Expansion in cross-section of single turn

Such calculation can predict the temperature dependence of mutual inductance for quoted toroidal geometry made by cooper wire ($\beta_{Cu} = 17 \cdot 10^{-6}/°C$ on $t = 20°C$). Diagram in Fig. 5 shows the temperature dependence of mutual inductance obtained by theoretical analysis. At a given temperature range the obtained dependence can be approximated by line and its slope corresponds to the temperature coefficient of mutual inductance; at toroidal coils made of copper wire it will be around $35 \cdot 10^{-6}/°C$, and usually does not depend on the ratio $Q = r_V/r_U$.
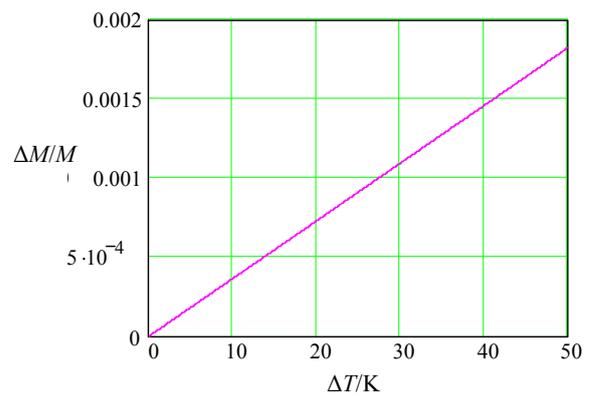
Fig. 5. Temperature dependence of mutual inductance in case of toroidal coil made of copper wire

Since the technology of coil production does not assures the tension of wire, influence of body expansion ($\beta_{Plexy} \approx 70 \cdot 10^{-6}/°C$) emphasizes only at considerably higher temperatures than those which are expected. In practical applications (laboratory conditions), the stretch of insulation body at temperature rise (i.e. temperature coefficient β_i) can be ignored and assume the coil performed without body.

The transformer coils are performed on the bodies of toroidal shape, made of polymethyl methacrylate (plexiglas), which was proven as good choice during the construction of test probes, and used in verification of analytical methods for estimation of mutual inductance of real Rogowski transducers. With temperature rise in case when $\beta_i < \beta_{Cu}$, wire can became loosely. Therefore, in the working conditions, where the larger temperature changes are expected, the temperature coefficient β_i of body material should be as close as possible to β_{Cu}.

3. THE INFLUENCE OF THE PARASITIC PARAMETERS OF COILS

On the coil equivalent circuit (Fig. 6), the self resistance R of coil appears because of the finite conductivity of the wire, and can be displayed in series with its self inductance L. The partial capacitances between turns can be replaced by the concentrated equivalent capacitance C. The same equivalent circuit is valid for the pair of coils having self resistance of R_S, self inductance of L_S and self capacitance of C_S (as it is pointed out in Fig. 3).
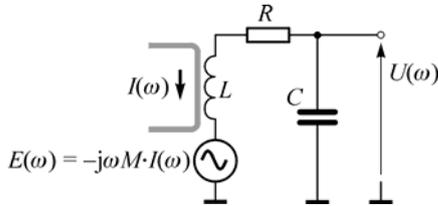
Fig. 6. Equivalent circuit of real Rogowski coil

Ideally, when the self resistance, self capacitance and self inductance can be neglected, each pair of coils induces electromotive force $E_S(\omega) = -j\omega M_S \cdot I(\omega)$, and their transimpedance is therefore:

$$Z_{Si}(\omega) = -j\omega M_S \qquad (7)$$

In real case, which is shown in Fig. 6, the voltage $U_S(\omega)$ induced on secondary terminals of coils is a consequence of the total magnetic flux, and it is dependent of mutual inductance M_S, but it also depends of the parameters L_S, R_S and C_S. Therefore, unloaded real pair of secondary coils has the transimpedance of:

$$Z_S(\omega) = \frac{U_S(\omega)}{I(\omega)} = -j\omega M_S \cdot \frac{-j\dfrac{1}{\omega C_S}}{j\omega L_S + R_S - j\dfrac{1}{\omega C_S}} \qquad (8)$$

In frequency domain, the modulus of relative deviation of real transimpedance $Z_S(\omega)$ from ideal transimpedance $Z_{Si}(\omega)$ is:

$$\frac{|Z_S(\omega)| - |Z_{Si}(\omega)|}{|Z_{Si}(\omega)|} = \frac{1}{\sqrt{(\omega R_S C_S)^2 + (\omega^2 L_S C_S - 1)^2}} - 1 \qquad (9)$$

For the interpretation of the expression (9), Fig. 7 shows the frequency dependence of quoted deviation for a concrete example. Due to the existence of equivalent capacitance, the

parallel self resonance [3, 4] will occur and therefore, by increasing the frequency of primary current, the deviation of induced voltage U_S from the electromotive force E_S will rise.
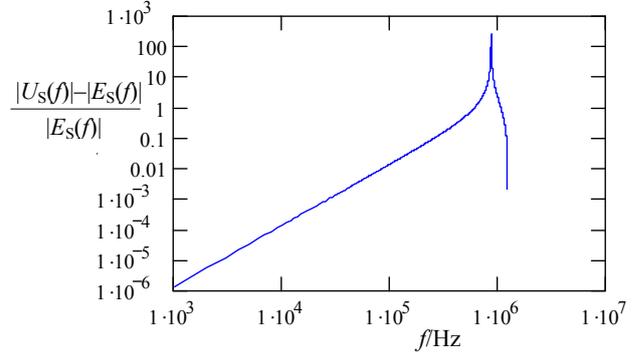
Fig. 7. The deviation of induced voltage due to the self resonance for each pair of secondary coils ($M_S = 2,062$ μH, $L_S = 591,67$ μH, $C_S = 57$ pF and $R_S = 12,06$ Ω)

In some cases, the conditions for the oscillations of transformer can be established, supported by energy taken from the primary circuit, and then the secondary circuit need some fixed burden. The resistance divider created by burden resistor adds additional systematic deviations of induced voltage, which should be taken into account.

4. THE COMPENSATION OF TEMPERATURE INFLUENCE ON MUTUAL INDUCTANCE

Besides the influence on the geometry of secondary circuit, the temperature also changes the resistance R_S of each pair of coils. Since the copper has a positive temperature coefficient of resistance α_{Cu}, this change of resistance may be used for the compensation of temperature dependence of induced voltage. With a certain (temperature-independent) loading resistance R_T, the secondary voltage can be made almost independent of temperature, thus the temperature dependence of total flux in coils does not change, but only apparently compensates.

The burden resistance R_T with resistance R_S forms a voltage divider, so the voltage on the input terminals of integrator will be somewhat smaller than expected. Under constant product ωI of angular frequency and current, the electromotive force E_S can be expressed as kM_S, so the voltage induced on R_T is:

$$U_S = \frac{E_S R_T}{R_S(1 + \alpha_{Cu}\Delta T) + R_T} = \frac{kM_S(1 + TC_M\Delta T)R_T}{R_S(1 + \alpha_{Cu}\Delta T) + R_T}, \qquad (10)$$

where TC_M represents the temperature coefficient of mutual inductance. If we equalize the derivation $\partial U_S/\partial(\Delta T)$ with zero, we can express the compensation resistance R_T in the following form:

$$R_T = \frac{R_S(\alpha_{Cu} - TC_M)}{TC_M} \qquad (11)$$

It follows that the compensation resistance for each pair of coils with self resistance of $R_S = 12,06$ Ω and temperature coefficient of resistance $\alpha_{Cu} = 3,9 \cdot 10^{-3}/°C$, should be $R_T = 1332$ Ω. Compensation resistors should be temperature

independent, and connected to the clamps of both pair of secondary coils, in parallel to the inputs of voltage followers (Fig. 3). In this case, the transimpedance of loaded transformer on low frequencies $Z_{S,lf}(\omega)$ may be expressed as:

$$Z_{S,lf}(\omega) = \frac{U(\omega)}{I(\omega)} = \underbrace{-j\omega M_S \frac{R_T}{R_S + R_T}}_{\text{for low frequencies}}, \qquad (12)$$

The connection of compensation resistors has resulted in strong attenuation or even complete absence of self resonance [3], and also reflected in the deviation of voltage U_S in the frequency domain. This effect can be analyzed through the expression for transimpedance of loaded transformer:

$$Z_S(\omega) = \frac{-j\omega M_S \cdot \left(\dfrac{-jR_T}{-j+\omega C_S R_T}\right)}{j\omega L_S + R_S + \left(\dfrac{-jR_T}{-j+\omega C_S R_T}\right)} \qquad (13)$$

Knowing all the parameters of the coils, we can determine the deviations of transimpedance in frequency domain (i.e. frequency dependence of secondary voltage induced by constant primary current). As a reference value we will take the value at low frequencies, given by (12), where the influences of self inductance L_S and parasitic capacitance C_S can be neglected.

Fig. 8. The influence of compensation resistance $R_T = 1332\ \Omega$ on relative deviation of transimpedance ($M_S = 2,062\ \mu H$, $L_S = 591,67\ \mu H$, $R_S = 12,06\ \Omega$, $C_S = 57\ pF$) in relation to the ideal case ($M_S = 2,062\ \mu H$, $R_S = 12,06\ \Omega$, $L_S = 0$, $C_S = 0$)

The diagram in Fig. 8 shows the mentioned suppress of self resonance, even to the extent of rejection of voltage induced at higher frequencies. The influence of frequency dependent part starts at frequencies higher than 1 kHz, so that the deviation from the reference value $Z_{S,lf}(\omega)$ at frequency of 10 kHz is about $2,5 \cdot 10^{-4}$.

The compensation of these deviations is possible with decreasing the self resonant frequency of transformer, with adding compensation capacitance C_K in parallel to the compensation resistance R_T. In the quoted function of relative deviation, instead of their self capacitance C_S, we shall put the sum $(C_S + C_K)$ which leads to the following expression:

$$\frac{\left|Z_S(\omega)\right| - \left|Z_{S,lf}(\omega)\right|}{\left|Z_{S,lf}(\omega)\right|} = \frac{R_S + R_T}{\sqrt{\omega^2\left[(C_S + C_K)R_S R_T + L_S\right]^2 + \left[\omega^2 L_S (C_S + C_K)R_T - R_S - R_T\right]^2}} - 1 \qquad (14)$$

The compensation is achieved when (14) is equal to zero, and the needed value of compensation capacitance to obtain this is:

$$C_K = \frac{L_S R_T \pm L_S \sqrt{R_T^2 - \left(R_S^2 + \omega^2 L_S^2\right)}}{R_T(R_S^2 + \omega^2 L_S^2)} - C_S \qquad (15)$$

With the known self capacitance of each secondary winding ($C_S = 57\ pF$), it follows that the compensation capacitance is $C_K = 110\ pF$. In Fig. 9 it is notable that the frequency response of transimpedance $Z_{S,k}(\omega)$ is linearized at higher frequencies. The relative deviation is just 10^{-5} at the frequency of 40 kHz, which means that the expression (12) for the transimpedance is valid up to that frequency. This is important result which strongly limits the usable frequency range of the whole system, having in mind the targeted level of uncertainty.

As described above, the compensation of temperature dependence of mutual inductance without significant degradation of other features was carried out. Finally, the overall transimpedance Z_T of transformer, including the temperature compensation with resistance R_T and frequency compensation with capacitance C_K is:

Fig. 9. Relative deviation of transimpedance $Z_{S,k}(f)$ for each pair of coils compensated with $C_K = 110\ pF$ ($M_S = 2,062\ \mu H$, $L_S = 591,67\ \mu H$, $R_S = 12,06\ \Omega$, $C_S = 57\ pF$) in relation to the ideal case ($M_S = 2,062\ \mu H$, $R_S = 12,06\ \Omega$, $L_S = 0$, $C_S = 0$) with loading resistance of $R_T = 1332\ \Omega$

$$Z_T(\omega) = -2j\omega M_S \cdot \frac{\left(\dfrac{-jR_T}{-j+\omega(C_S + C_K)R_T}\right)}{j\omega L_S + R_S + \left(\dfrac{-jR_T}{-j+\omega(C_S + C_K)R_T}\right)}, \qquad (16)$$

which can be expressed using "effective" mutual inductance as:

$$Z_{\mathrm{T}}(\omega) \cong -\mathrm{j}\omega \cdot \underbrace{\left(2M_{\mathrm{S}} \cdot \frac{R_{\mathrm{T}}}{R_{\mathrm{S}} + R_{\mathrm{T}}} \right)}_{\substack{\text{effective mutual inductance} \\ \text{for } f \leq 40 \text{ kHz}}} \qquad (17)$$

From the obtained function it follows that the connection of compensating resistance R_{T} has a consequence that the "effective" mutual inductance is decreased by factor $R_{\mathrm{T}}/(R_{\mathrm{S}} + R_{\mathrm{T}})$ in comparison to the real mutual inductance M_{S}.

5. MEASURING RESULTS

The principle of mutual inductance measurement is presented in schematic diagram in Fig. 10. The frequency of primary current is measured by HP 5316B Universal Counter, the induced voltage is measured by the HP 3458A Digital Multimeter marked as V_1, driven by *Swerlein's* algorithm [5], while the primary current is determined as voltage on Fluke A40 shunt measured on second HP 3458A Digital Multimeter, marked as V_2.
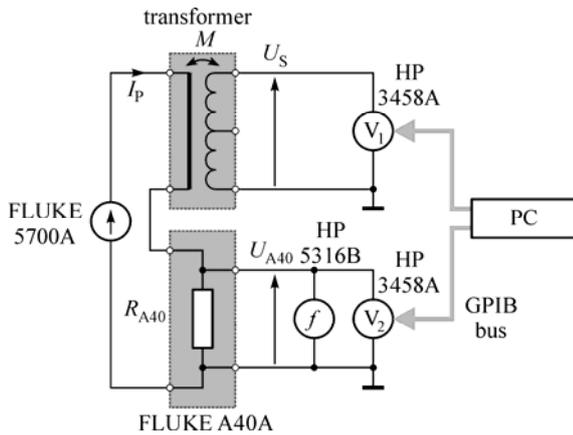
Fig. 10. Principle of measurement of the transformer mutual inductance

The measurements was carried out with nominal primary current I_{P} of 1 A, and its frequency $f_{\mathrm{P}} = 100$ Hz. The uncertainty of this measurement depends mostly of uncertainties of measured voltages. In quoted algorithm, for frequency of 100 Hz it is around 20 ppm. Mutual inductance of the entire transformer is estimated from the following expression:

$$M_{\mathrm{T}} = \frac{U_{\mathrm{S}}}{2\pi f_{\mathrm{P}} \cdot I_{\mathrm{P}}} = \frac{1}{2\pi \cdot f_{\mathrm{P}}} \cdot \frac{U_{\mathrm{S}}}{U_{\mathrm{A40}}} \cdot R_{\mathrm{A40}} \qquad (18)$$

The measured mutual inductance of the whole transformer is $M_{\mathrm{Tm}} = 4{,}1270 \cdot 10^{-6} \cdot (1 \pm 4 \cdot 10^{-5})$ H. The temperature dependence of mutual inductance was determined by the measurement in two points; on laboratory temperature $t_0 = 23\ ^\circ$C and increased temperature $t = 28\ ^\circ$C. Measurement results given in Figs. 11 and 12 show behaviour of M in cases with and without temperature compensation.
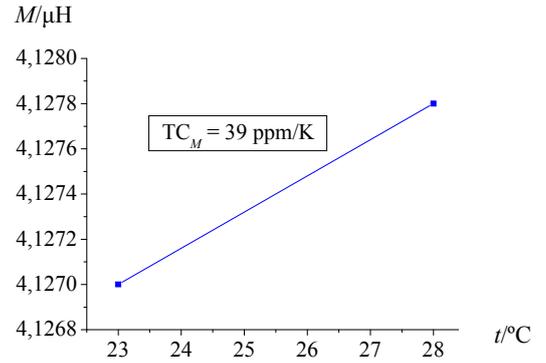
Fig. 11. Temperature coefficient of mutual inductance without temperature compensation
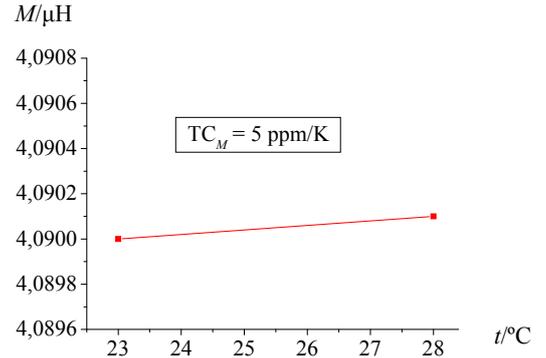
Fig. 12. Temperature coefficient of mutual inductance with temperature compensation

6. CONCLUSION

Based on the previous analysis, the calculated mutual inductance of the whole transformer is equal to $M_{\mathrm{Tc}} = 2M_{\mathrm{S}} = 4{,}1232 \cdot 10^{-6}$ H, while the real mutual inductance, measured in the laboratory conditions at temperature $t = 23\ ^\circ$C, is $M_{\mathrm{Tm}} = 4{,}1270 \cdot 10^{-6} \cdot (1 \pm 4 \cdot 10^{-5})$ H. The relative difference between M_{Tm} and M_{Tc} of $9{,}2 \cdot 10^{-4}$ is achieved, indicating the possibility of creating a generic calculable current-to-voltage converter. The quoted results were obtained without temperature compensation. On the other hand, with the introduced temperature compensation by loading the secondary coils with compensation resistors R_{T}, the measured „effective" mutual inductance of transformer is $M_{\mathrm{Tm,eff}} = 4{,}0900 \cdot 10^{-6} \cdot (1 \pm 4 \cdot 10^{-5})$ H, and the difference is taken into account as corrective term.

The incomplete temperature compensation comes due to the fact that the theoretical temperature coefficient of mutual inductance is somewhat less than the real one, but in the given temperature range the compensated transformer has TC_M of only ≈ 5 ppm/K, which is a satisfactory result.

It is important to emphasize that the temperature compensation by loading with R_{T} has a strong influence on the frequency characteristic of this transformer, in which the much lower deviation from ideal transfer function is possible, which enables the wider frequency range of its use with the lower uncertainty.

REFERENCES

[1] L. Ferković, D. Ilić, I. Leniček, "Laboratory current transformer based on Rogowski coil", *Proceedings of the*

16th IMEKO TC4 International Symposium, pp.283-288 (CD publication), Florence, Italy, September 22-24, 2008.

[2] L. Ferković, D. Ilić, R. Malarić, "Mutual Inductance of a Precise Rogowski Coil in Dependence of the Position of Primary Conductor", *IEEE Transactions on Instrumentation and Measurement*, vol. 58, no.1, pp.122-128, January 2009.

[3] D.A. Ward, J. La T. Exon, "Using Rogowski coils for transient current measurements", *Engineering Science and Education Journal*, pp. 105−113, June 1993.

[4] Ray W. F., Hewson C. R., "High Performance Rogowski Current Transducers", *IEEE Industry Applications Conference Proceedings*, Vol. 5, pp. 3083−3090, Rome, 2000.

[5] Ronald L. Swerlein, "A 10 ppm accurate digital AC measurement algorithm", Hewlett Packard Co.,1991.