XIX IMEKO World Congress
Fundamental and Applied Metrology
September 6–11, 2009, Lisbon, Portugal

MINIMIZATION OF THE UNEVEN SAMPLING EFFECTS ON EVALUATING ROUNDNESS WITH COORDINATE MEASURING MACHINES

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Abstract – This paper deals with the treatment of the uneven sampled data characteristic of high sampling rate roundness measurements on scanning coordinate measuring machines. Simulations using the sampling pattern presented by these machines and structured profiles resembling a multi-wave standard were performed for testing the response of several interpolation methods. The Lomb-Scargle transform for unevenly sampled data was tested for the same situations. A comparison between the techniques is carried out based on space and frequency domain characteristics of the simulated profile. Real measurements on a multi-wave standard were also performed on different machines to evaluate the behaviour of the measurement softwares with respect to the uneven sampling and to verify applicability of the simulated methods. Results show the improvements obtained on evaluated parameters with the use of the discussed techniques.

Keywords: Scanning technology, roundness profile, uneven sampling.

1. INTRODUCTION

In the industrial practice, the evaluation of geometric product specifications is one of the most critical tasks to be accomplished for product quality assurance. Coordinate measuring machines (CMM) play an important role on geometric specifications evaluation, due to the great versatility and flexibility to perform different measurement tasks. Since the scanning technology was introduced, it has been possible to evaluate form features with CMMs. Nowadays, scanning technology is the standard for point acquisition with CMMs [1], combining high-speed, accurate measurements with highly detailed information of the surface under evaluation.

In spite of the continual technology improvements, there are some inherent issues of the scanning technology that should not be overlooked. When measuring roundness in a CMM, the circular path is obtained by the combined displacement of two or more orthogonal axis (i.e. the machine guideways and the probing head). This complex kinematic gives rise to some effects that are not present in conventional form testers, which have a precision spindle that is responsible for defining the reference circle.

Among these effects, it has been shown that the points sampled by a CMM in scanning mode are seldom evenly spaced [2]. In particular, points acquired at high sampling rates tend to be spaced more irregularly. This situation affects the behaviour of the Discrete Fourier Transform (DFT), used for spectral analysis. If the points are not evenly spaced, the amplitudes become attenuated at high frequencies, providing a distorted description of the real profile.

This paper focuses on the use of several interpolation algorithms for signal reconstruction and the application of the Lomb-Scargle transform. These techniques are applied to simulated data and also to real measurement data. Conclusions are obtained on the effectiveness of the different techniques to minimize the effect of uneven spacing on the results of roundness measurements.

2. UNEVEN SAMPLING ON CMM'S

The main parameters provided by the CMM operator for point acquisition on a circular scanning are the number of points (n), the diameter of the profile (d, in mm) and the scanning speed (S, in mm/s).

The scanning pitch (p, in mm) is the relation between the perimeter and the number of points of the profile. Due to the complexity of pitch controlling over non-straight scanning paths, the acquisition is often made on a predefined sampling time (Δt, in s), as given by equation (1).

$$\Delta t = \frac{p}{S} = \frac{\pi . d}{n . S} \tag{1}$$

The sampling rate is obtained by the inverse of the sampling time.

It can be shown that the points acquired by a CMM are not evenly spaced. In some machines, this behaviour becomes more critical when the acquisition is made at high sampling rates, so to generate a strong multimodal sampling distribution (Fig. 1).

As the diameter of circular features on measured parts is determined and the number of points to be sampled is directly associated with the cut-off frequency of the filter to be applied [3], the parameter that defines the sampling rate for a given profile is the scanning speed. Higher scanning speeds will induce a stronger uneven sampling pattern, besides higher centripetal acceleration.

ISBN 978-963-88410-0-1

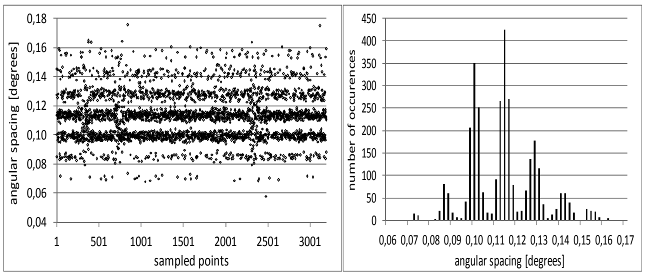

Fig. 1. Uneven angular spacing of a real CMM roundness measurement (left) and its multimodal distribution (right).

A direct consequence of this uneven distribution is the frequency spreading and amplitude attenuation on the spectral analysis via DFT (Fig. 2), so that frequency domain based filters may be affected by this behaviour. Comparison of profiles by their harmonic content may also be impaired.

Another consequence of this behaviour arises when a space domain, point-by-point comparison of profiles using resampling techniques is performed (for example, given two measured profiles, obtained in the same CMM and under the same conditions, but with different uneven spacing distribution). In this case, the difference between the profiles may be severely overestimated. Convolution on the space domain for filtering with evenly distributed weighing kernels may be affected as well.

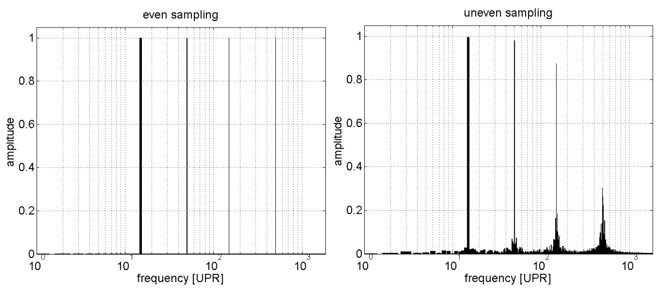

Fig. 2. Frequency spectra of a simulated structured profile containing four frequency components (15, 50 150 & 500 UPR, unitary amplitudes), evaluated using even sampling (left) and a CMM-based uneven sampling pattern (right).

There are two major alternatives discussed in the literature to deal with the spectral analysis of unevenly spaced samples: interpolation methods to obtain a regularly spaced sample [4], and transform methods that do not require evenly spaced samples [5]. The first group is often quoted for signal reconstruction, or when is desirable to perform mathematical operations on the frequency domain, while the second is usually applied for periodicity analysis on signals.

The next sections describe the results of a comparative analysis of these methods with respect to measurements of roundness with CMMs.

3. MATERIALS AND METHODS

The evaluations carried out for this paper are presented in two stages: a computational one based on numerical simulations; and an experimental one based on real measurements on a multi-wave standard [6].

3.1. Simulations

The goal of the simulations was to verify the influence of the CMMs uneven sampling pattern over roundness profiles evaluations in absence of other errors and to compare methods to minimize the uneven sampling effects.

For the simulations, a high point density profile (360.000 points per revolution) was generated. This profile has a structured pattern with multiple frequencies (15, 50, 150 & 500 UPR, with unitary amplitudes), resembling a multi-wave standard. Subsequently, white noise has been added at two levels ($\sigma = 3$ and $\sigma = 10$) to the high point density profile, and for each iteration of the simulation process, the noise was regenerated. To avoid aliasing, an anti-aliasing filter with a step transfer characteristic at the Nyquist frequency have been implemented and applied before the sampling routines.

For the next step, sampling of the high density profile, two different sampling routines were implemented. The first one was an even sampling that provides the reference profiles for all subsequent comparisons. The second one was the uneven sampling routine, which consisted in the generation of a multimodal sampling pattern emulating the behaviour of some CMMs at high sampling rate (Fig. 3). Both sampling methods were executed with same quantity of points (around 3667) on each iteration.

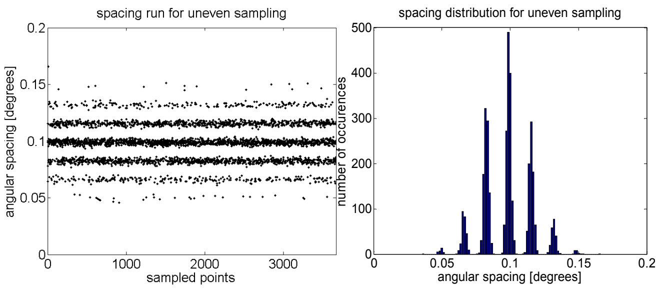

Fig. 3. Simulated sampling pattern with 3667 points (left) and its multimodal distribution (right).

The uneven sampled profile was then interpolated with four distinct methods. For noise added profiles, filtering was used on the unevenly sampled profile and on the interpolated profiles. The data were then fitted to a reference least-squares circle for the roundness evaluation. The methods of interpolation, domain transformations, data fitting and filtering will be presented after.

A comparison was carried out between the non-interpolated unevenly sampled profiles and the interpolated profiles, using always the evenly sampled profile as reference. The parameters used on the comparison were the following:

- roundness deviation bias (parameter related with the functional characteristic of profiles);
- standard deviation of the point-by-point bias (parameter related with space domain comparison of profiles);
- amplitude bias of the 500 UPR component (parameter related with analysis of profiles by their harmonic content).

The situations included noiseless and noise-added high point density profiles and unfiltered and filtered (500 & 150

UPR cut-off frequencies) sampled profiles. A total of 10.000 iterations were performed for data generation.

3.2. Real CMM measurements

To evaluate the applicability of the results obtained by simulation to the real world, an experiment was carried out, involving three industrial portal-type scanning-probe CMM's and a multi-wave standard (5, 50, 150 & 500 UPR frequencies, 2 µm of amplitude each, machined on a 80 mm external diameter). Two measurement strategies were used (with high and low sampling rates, see Table 1). Three measurement cycles were performed for each strategy.

The evaluated parameters were the roundness deviation with a 500 UPR cut-off frequency filter and the amplitudes of the 500 & 150 UPR frequency components. The parameters' values were obtained as provided by the CMM software and by external processing (with and without interpolation).

Table 1. Measurement strategies defined for each CMM.

	STRATEGY 1 - LOW SAMPLING RATE		
CMM #	1	2	3
Max. sampling rate (points/s)	450	450	67
Employed sampling rate (points/s)	75	75	15
Scanning speed (mm/s)	5	5	1
Scanning pitch (mm)	0.067	0.067	0.067

	STRATEGY 2 - HIGH SAMPLING RATE		
CMM #	1	2	3
Max. sampling rate (points/s)	450	450	67
Employed sampling rate (points/s)	400	400	60
Scanning speed (mm/s)	27	27	4
Scanning pitch (mm)	0.068	0.068	0.067

3.3. Processing algorithms

Four interpolation methods have been compared:

1. Nearest neighbour – This is the simplest method, on which the amplitude of the uneven sampled point nearest to the nominal grid is adopted.
2. Linear interpolation – The value of the amplitude attributed to the point on the nominal grid lies over the line defined by its two neighbours.
3. Cubic spline interpolation – Between each two points there is a piecewise cubic curve. The second and first derivatives at the endpoints of each adjacent cubic curve are set equal, providing a continuous second derivative [7].
4. Hermit cubic spline interpolation – Similar to the former, but with a discontinuous second derivative. It becomes less susceptible to outliers, having no overshoots and less oscillation than the continuous second derivative cubic spline, preserving monotonicity of the data [8].

For domain transformations the MATLAB FFT function was used. The Lomb-Scargle transform for uneven data, as described by [9], was implemented and tested for frequency spectrum evaluations. Although the oversampling factor[1] (*ofac*) was implemented, it has not been used, because the purpose was to compare the amplitudes at the evaluated frequencies, not only to verify if these frequencies are present on the profile. Thus, a modification on the definition of the lowest independent frequency parameter[1] (*T*) was implemented, where n is the number of points of the profile, and θ are the angular coordinates of these points. The modified definition of the lowest independent frequency parameter is presented on equation (2).

$$T \equiv \frac{n}{(n-1)}\left[\max(\theta)-\min(\theta)\right] \tag{2}$$

One peculiarity of the Lomb-Scargle algorithm is that it evaluates absolute values (amplitudes) of the transform, but not the phase, thus it is not possible to neither perform operations on the frequency domain nor calculate the inverse transform.

Filtering operations were performed with convolution on both space and frequency domain using a Gaussian filter kernel with 50% of transmission on the cut-off frequency. For data fitting, the Newton-Gauss least-squares circle algorithm was used, because of its fast convergence. Fitting and filtering algorithms were implemented as described by [10].

All externally processed and simulated measurements used the same algorithms for interpolation, space-frequency domain transformations, and data fitting and filtering.

4. RESULTS

4.1. Simulations

The comparison for the roundness deviation bias obtained from the simulations is presented in Fig. 4. It can be observed that for unfiltered noiseless profiles, roundness values of the unevenly sampled profiles are nearly the same as for the evenly sampled ones. However, for noise-added filtered profiles, the unevenly sampled profiles showed positive bias.

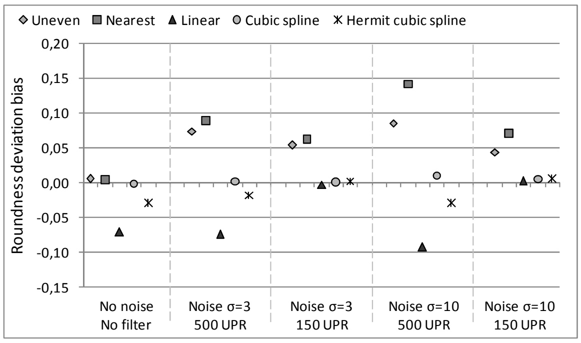

Fig. 4. Comparison of the mean roundness deviation bias resulting from the uneven sampling and the four interpolation methods after 10.000 iterations.

Regarding the interpolation methods, it can be seen that, in general, the cubic spline interpolation performs better, being closest to the evenly sampled profiles roundness values.

The results of the comparison for the point-to-point bias are shown in Fig. 5. As can be noted, there is a substantial difference between the evenly and the non-interpolated unevenly sampled profiles. This results leads to the conclusion that a quantitative comparison of unevenly sampled profiles on space domain makes no sense unless an

[1]For details on these parameters, refer to [9].

interpolation of the profiles is performed. For this purpose, the cubic spline also presented the best results, thought linear and hermit cubic spline methods performed reasonably well too.

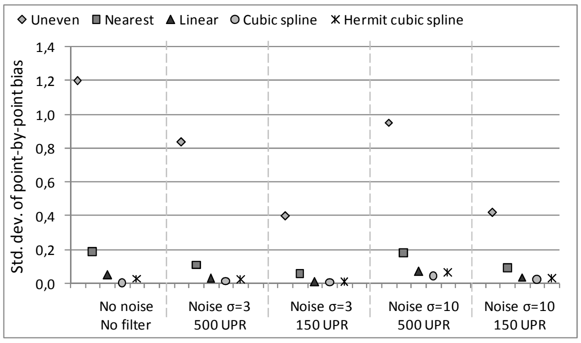

Fig. 5. Comparison of the mean standard deviation of point-by-point bias resulting from the uneven sampling and the four interpolation methods after 10.000 iterations.

The results shown on Fig. 4 and on Fig. 5 were obtained by frequency domain filtering. The space domain filtering showed nearly the same results, with relative differences of individual values smaller than 0,1%. This shows that convolution on both domains is affected the same way by the uneven sampling of points.

The last parameter evaluated was the 500 UPR component amplitude bias, for which comparative results are shown in Fig. 6. It can be seen that the cubic spline interpolation showed near-zero amplitude bias even for the noise-added profiles, proving to be an effective method to minimize the effects of a CMM multimodal sampling pattern over frequency spectra calculation.

Although not showed here, it has been noted that the performance of all interpolation methods decays as the evaluated frequency rises. However, even for the 1500 UPR frequency, which would be the highest frequency of interest for most of the practical cases, the mean relative amplitude bias after the cubic spline interpolation was less then 0,1%.

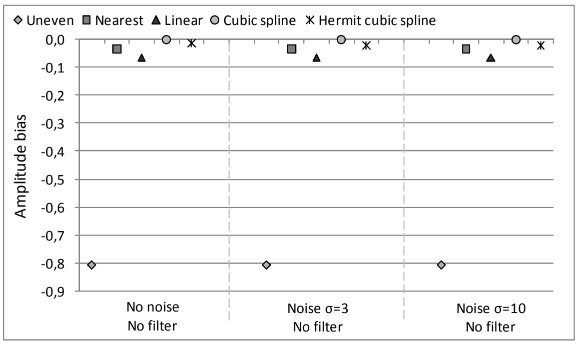

Fig. 6. Comparison of the mean amplitude bias for the 500 UPR component resulting from the uneven sampling and the four interpolation methods after 10.000 iterations.

The Lomb-Scargle transform permitted only the evaluation of the last parameter introduced, as it does not allow inverse transformation due to lack of the phase information. Although not showed here, the use of oversampling factor values other than 1 caused distortions on the lower frequencies of the spectrum, besides increasing the processing time. But mainly, as more than one amplitude value is then possible to be found for each frequency, the comparison became not plausible to be performed. With the modification presented on equation (2), however, the use of oversampling became unnecessary, making possible the comparison with the reference spectrum.

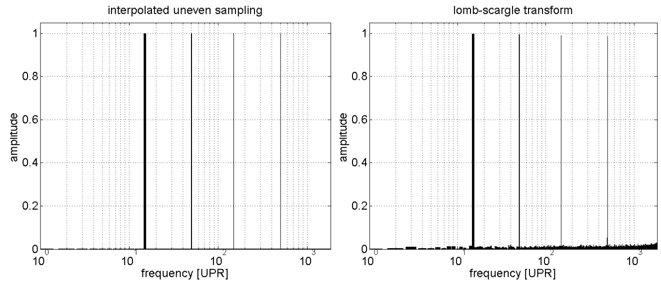

Fig. 7. Frequency spectra (of a noiseless profile) obtained with cubic spline interpolation (left) and the Lomb-Scargle transform with oversampling factor equal to 1 (right).

Nevertheless, as shows Fig. 7, the transformation is still not as good as the cubic spline interpolation, as it adds noise not present on the reference profile and does not recover the amplitudes as much as well as the cubic spline interpolation does. The Lomb-Scargle transform has not been included on the iterative evaluation because of the intensive processing necessary to solve each frequency spectrum, even with the possibility of using no oversampling.

4.2. Real measurements

The two different measurements strategies demanded the CMM controller in different manner. Fig. 8 shows the sampling pattern distributions for both measurement strategies as presented by two of the machines. This dissimilar behaviour produces different results, as will be shown.

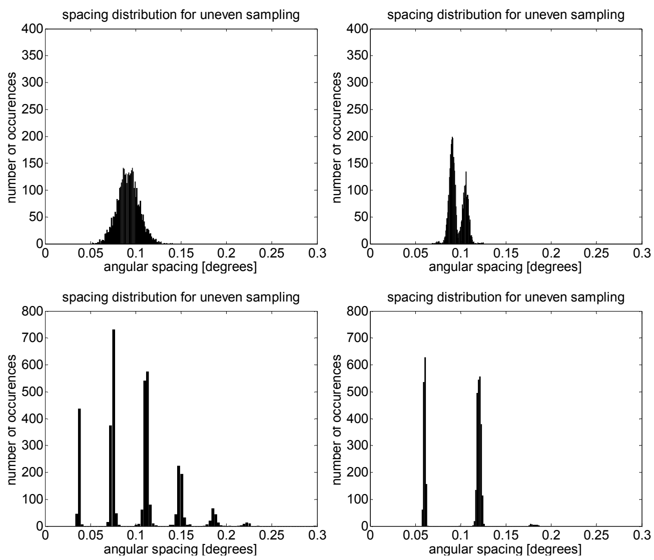

Fig. 8. Distributions of the sampling pattern presented by CMM 1 (left) and CMM 3 (right), at low sampling rate (strategy 1, up) and high sampling rate (strategy 2, down).

The parameters evaluated with the experimental analysis are presented on Table 2. From the harmonic content analysis, it can be noted that for low sampling rates the amplitudes of the frequency spectrum does not suffers notably from the unevenly sampled pattern. For high sampling rates the amplitudes of the higher frequencies are drastically attenuated, as expected. As noticed from the simulations, the cubic spline interpolation provides great improvement on the evaluation of the harmonic content. It can be noted that the CMMs who can perform spectral analysis already use some method to deal with the uneven sampling issue.

Table 2. Results for measurements on the multiwave-standard with the three CMMs and the two measurement strategies. The CMM values were obtained from the measurement software and the uneven sampled and cubic spline interpolated values were obtained by externally processing the raw data.

STRATEGY 1 - LOW SAMPLING RATE				STRATEGY 2 - HIGH SAMPLING RATE			
Form Error (mm) - 500 UPR Gauss Filter				**Form Error (mm) - 500 UPR Gauss Filter**			
CMM #	CMM Value	Uneven	Cubic Spline	CMM #	CMM Value	Uneven	Cubic Spline
1	0.0140	0.0141	0.0141	1	0.0226	0.0226	0.0226
2	0.0139	0.0139	0.0139	2	0.0159	0.0159	0.0159
3	0.0136	0.0136	0.0136	3	0.0140	0.0139	0.0139
500 UPR Amplitude (µm)				**500 UPR Amplitude (µm)**			
CMM #	CMM Value	Uneven	Cubic Spline	CMM #	CMM Value	Uneven	Cubic Spline
1	1.87	1.95	1.94	1	3.34	0.51	3.88
2	2.03	2.13	2.13	2	0.34	0.05	0.37
3	N/A	1.85	1.84	3	N/A	0.96	2.06
150 UPR Amplitude (µm)				**150 UPR Amplitude (µm)**			
CMM #	CMM Value	Uneven	Cubic Spline	CMM #	CMM Value	Uneven	Cubic Spline
1	1.99	1.99	1.99	1	1.83	0.88	1.82
2	1.98	1.99	1.98	2	2.18	1.09	2.19
3	N/A	2.01	2.01	3	N/A	1.87	2.06

The roundness error did not show to be different for the two strategies. One possible explanation is that, for the machines evaluated, the spacing gaps that generate the multimodal distribution occur in a more periodic basis than the simulated ones, which were randomly generated.

5. CONCLUSIONS

This paper presented an analysis on the effects of the uneven sampling patterns at high sampling rates roundness measurements by CMMs. A comparison of methods to minimize these effects was carried out. From the simulated data, it was possible to recognize that interpolation methods are preferable over the Lomb-Scargle transform to evaluate roundness measurements for several reasons, such as possibility of profile reconstruction, accuracy of the resulting periodogram, computation time, etc. Nevertheless, the modification introduced on the Lomb-Scargle algorithm produced great improvement on transformation, and may be also applicable for other purposes.

Among the interpolation methods, the cubic spline proved to be the more suitable for improving not only the spectral analysis, but also to reconstruct the profiles and to provide comparability between them on space domain. The real measurements agree with the simulation results, with the cubic spline presenting enhancement of the Fourier analysis. In fact, the analysis showed that some CMMs already use mathematical resources to handle the uneven sampling subject. An overall examination of the results reveals that the use of an adequate method to manage the uneven sampling matter on CMMs is essential to the quality of profile evaluations.

ACKOWLEDGEMENTS

The authors would like to thank the German Research Foundation (DFG) for the financial support to this project and the companies which have collaborated with the experiments.

REFERENCES

[1] T. Pfeifer, A. Napierala, "Scanning on coordinate measuring machines", *XVI IMEKO World Congress*, Austria, Wien, 2000.

[2] O. Jusko, F. Lüdicke, F. Wäldele, "High Precision Form Measurements with Coordinate Measurement Machines". Chemnitz. p. 341-351, 2000.

[3] ISO/TS 12181-2:2006, *Geometrical Product Specifications (GPS) – Roundness – Part 2: Specification operators*. International Organization for Standardization, Geneva, Switzerland.

[4] F. Marvasti, *Nonuniform sampling: Theory and practice*, Kluwer/Plenum, New York, 2001.

[5] D. Potts, G. Steidl, M. Tasche, *Fast Fourier transforms for nonequispaced data: A tutorial*, in: Modern Sampling Theory: Mathematics and Applications, edited by J. J. Benedetto and P. J. S.G. Ferreira (Birkhäuser, Boston, 2001), chap. 12, pp. 247 – 270.

[6] O. Jusko, F. Lüdicke, "Novel multi-wave standards for the calibration of form measuring instruments", Proc. 1st EuSPEN, Bremen, Vol. 2, ISBN 3-8265-6085-X, 1999, 299-302.

[7] C. de Boor, *A Practical Guide to Splines*, Springer-Verlag, 1978.

[8] D. Kahaner, C. Moler, S. Nash, *Numerical Methods and Software*, Prentice Hall, 1988.

[9] W. H. Press, S. A. Teukolsky, W. T. Vetterling and B. P. Flannery, *Numerical recipes in C: the art of scientific computing*, 3rd Ed., Cambridge University Press, 2007.

[10] B. Muralikrishnan, J. Raja, *Computational Surface and Roundness Metrology*, Springer-Verlag London Limited, 2009.