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ESTIMATION OF RESIDUAL ERROR PARAMETERS FOR VECTOR NETWORK ANALYZERS

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Abstract – Recently, a novel approach for the estimation of the residual error parameters of a calibrated vector network analyzer has been proposed. The method is based on a reflection measurement employing a high precision airline terminated by a short. From this measurement the complex valued residual error parameters are calculated utilizing a sophisticated data analysis scheme. In this work the uncertainty associated with the obtained residual error parameters is evaluated. The uncertainty evaluation is performed by applying the GUM S1 approach employing a Monte-Carlo method. Resulting uncertainties arising due to imperfections of the dimensional parameters of the airline are presented.

Keywords: VNA, calibration, uncertainty

1. INTRODUCTION

The uncertainty of a vector network analyzer (VNA) measurement depends on the applied calibration method and on the uncertainty associated with the available estimates of the utilized calibration standards. Further influences are VNA stability, VNA nonlinearities, noise, and connector repeatability.

High precision airlines are well known as traceable impedance standards used for the verification of a VNA calibration [1]. Recently, a novel approach for the determination of the complex-valued residual error parameters of a calibrated VNA has been proposed [2,3]. The residual error parameters are determined from a reflection measurement employing a high precision airline terminated by a short using signal processing techniques such as low-pass filtering and linear prediction. Apart form the verification of the VNA calibration, the residual error parameters can be used to perform a second order correction of a reflection measurement [3]. The corrected reflection values were shown to be consistent with results obtained by the well known cross-ratio technique [4].

In this work, the novel approach is investigated with respect to the resulting uncertainty associated with the obtained residual error parameters. The analysis is performed according to the *Guide to the Expression of Uncertainty in Measurement* (GUM) [5]. In the recent supplement GUM S1 [6] to the GUM, a Monte Carlo Method (MCM) is proposed for the calculation of uncertainties. The approach is based on the concept of propagation of probability density functions (PDFs), where the PDFs encode the knowledge about the quantities of interest. Using the (joint) PDF of all influencing quantities, the MCM numerically determines a PDF for the output quantity employing the model which relates the measurand to the influencing quantities.

For the uncertainty evaluation of the residual error parameter estimates a model of the airline is considered, which especially takes the dimensional parameters of the airline into account. The resulting uncertainties of the residual error parameter estimates due to imperfections of the airline parameters will be presented.

2. RESIDUAL ERROR PARAMETER ESTIMATION

The approach is based on the commonly applied one-port error model. The corresponding signal flow graph is shown in Fig. 1, including a representation of the airline and short.

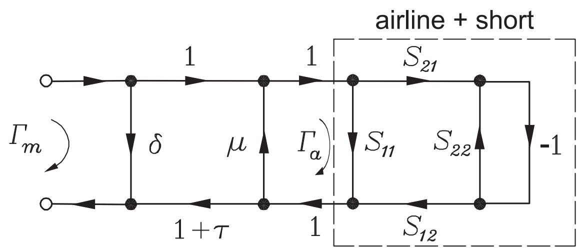

Fig. 1. One-port signal flow graph for a short-circuited airline.

The measured reflection coefficient Γ_m is related to the actual reflection coefficient Γ_a by

$$\Gamma_m = \delta + (1+\tau)\frac{\Gamma_a}{1-\mu\Gamma_a}, \qquad (1)$$

where δ, μ and τ denote the residual error parameters directivity, source match and reflection tracking of the calibrated VNA. The reflection coefficients as well as the error parameters are complex-valued functions of the frequency. Assuming a high precision airline with $S_{11} = S_{22} = 0$ the reflection coefficient Γ_a can be approximated as $\Gamma_a \approx -\lambda \exp(-2j\omega l / v)$ where l denotes the overall lengths of airline and offset short and v denotes the phase velocity. The weak and slowly frequency dependent losses of the airline and short are described by λ.

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Assuming small residual error parameters for a calibrated VNA, i.e. $|\delta|,|\mu|,|\tau| << 1$, (1) can be approximated as

$$\Gamma_m \approx \delta - (1+\tau)\lambda e^{-2j\omega l/v} + \mu(1+\tau)\lambda^2 e^{-4j\omega l/v}. \quad (2)$$

The considered signal processing scheme relies on the assumption, that the variation of the residual error parameters in dependence on the frequency is small as compared to $\exp(-2j\omega\, l/v)$. As described in [3] in detail the residual error parameters are then determined by a sequence of down-conversion and low-pass filtering steps. The approach also includes an extrapolation procedure based on linear prediction in order to reduce filtering artefacts. The determination of the residual reflection tracking τ is based on a further reflection measurement of the utilized short. A brief discussion on method parameters such as the bandwidth of the low-pass filter is given in [3].

In Fig. 2, a typical reflection measurement of a short circuited airline is presented showing the well known ripples indicating an imperfect VNA calibration.

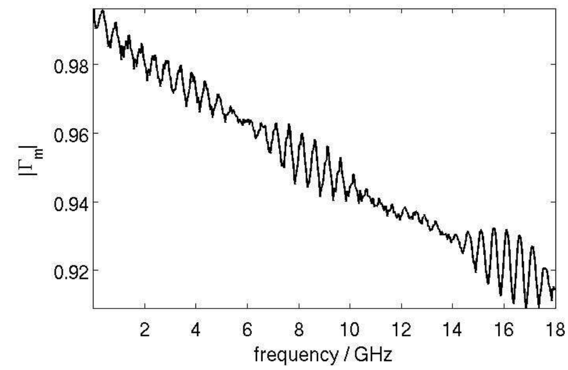

Fig. 2. Reflection coefficient of a short-circuited airline.

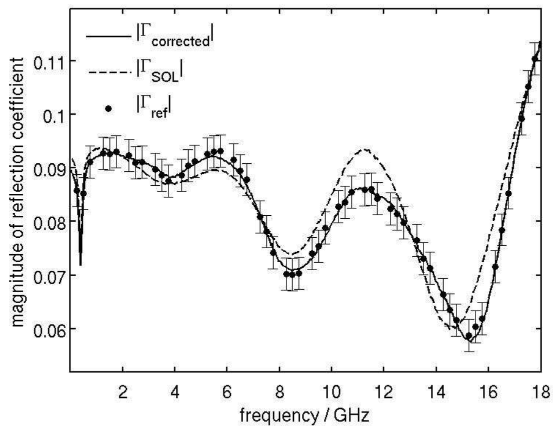

Fig. 3. Reflection coefficient of a mismatch before (Γ_{SOL}) and after ($\Gamma_{corrected}$) second-order correction. Comparison to reference reflection values (Γ_{REF}) obtained by the cross-ratio technique.

The impact of a second-order correction is illustrated in Fig. 3. The correction has been performed using residual error parameter estimates obtained by the proposed signal-processing scheme. While the reflection coefficient resulting from a standard short-open-load calibration (Γ_{SOL}) clearly displays deviations from the cross-ratio reference values [4], the second-order corrected reflection coefficient appears to be an improved estimate.

3. UNCERTAINTY EVALUATION

The measured reflection coefficients as well as the error parameters are complex-valued quantities whose treatment is not explicitly covered by the GUM. The analysis described here follows a proposal for the uncertainty treatment of S-parameters which represents an extension to the GUM [7]. To this end all calculations are performed using a Cartesian representation of the complex-valued quantities. As proposed in [7] the resulting uncertainties are evaluated and presented separately for the real and the imaginary part.

The uncertainty evaluation is realized as a two-step scheme. First, the reflection coefficient Γ_a of the short circuited airline is modelled based on the corresponding physical parameters of the airline such as the length and the inner and outer diameters [8]. The model includes a description of the short using the calibration kit standard definition. For the measured reflection coefficient follows from (1)

$$\Gamma_m = \delta + (1+\tau)\frac{\Gamma_a(\boldsymbol{z})}{1-\mu\Gamma_a(\boldsymbol{z})}, \quad (3)$$

where $\boldsymbol{z}$ denotes a vector of physical parameters of the model $\Gamma_a(\boldsymbol{z})$. The residual error parameters are determined by the proposed signal processing scheme which is denoted here as $g(\Gamma_m)$

$$(\delta,\mu,\tau) = g(\Gamma_m) + \Delta_{(\delta,\mu,\tau)}. \quad (4)$$

The term $\Delta_{(\delta,\mu,\tau)}$ represents (frequency dependent) corrections of $g(\Gamma_m)$.

The uncertainty associated with the residual error parameter estimates is evaluated according to the "propagation of distributions" approach of GUM S1. To this end the PDFs of the two contributing terms in (4) are required. The distribution of first term in (4) is obtained by MCM using the estimates and uncertainties of the dimensional parameters of the airline.

Information regarding the distribution of the corrections $\Delta_{(\delta,\mu,\tau)}$ of the estimation scheme may be obtained by a simulation approach. These corrections may dependent on the actual residual error parameters as the signal processing scheme relies on specific assumptions and approximations, e.g. regarding their magnitude and their frequency dependence. For the results presented in this work both, the expectation values and the associated uncertainties of the corrections are assumed to be zero.

4. RESULTS

Starting from the stated values and standard uncertainties of the length and inner and outer diameter of the airline Gaussian PDFs are assigned to these quantities. The presented results are based on the residual error parameters as obtained in [3] for a SOL calibrated VNA using a 300 mm airline.

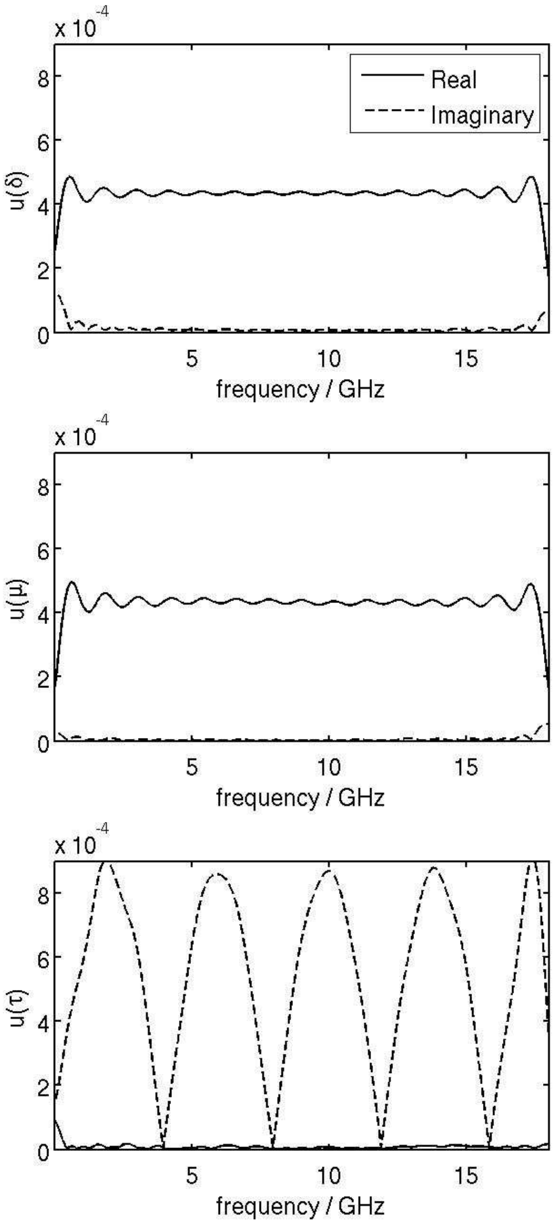

Fig. 4. Uncertainties of the estimated residual error parameters vs. frequency. For each parameter, the uncertainty of the real (solid line) and of the imaginary part (dashed line) is shown.

Applying the MCM results in the (joint) PDF for the first term in (4). From this PDF, the according uncertainties of the residual error parameters are evaluated assuming the expectation values and associated uncertainties of the corrections to be zero.

Fig. 4 shows that the resulting uncertainties of the estimates of the residual directivity δ and of the residual source match μ are nearly independent of the frequency. The same holds for the uncertainty of the real part of the reflection tracking τ. In contrast, the uncertainty of the imaginary part of the reflection tracking displays an oscillation with respect to the frequency. The determined uncertainties are roughly in accordance with results presented in [3], where a typical deviation of 0.001 between second order corrected reflection values and results obtained by the cross ratio technique has been observed.

4. CONCLUSIONS

In this work, an uncertainty evaluation for a recently introduced estimation scheme for the complex-valued residual error parameters of a calibrated VNA is presented. The uncertainty evaluation follows the GUM S1 approach using the dimensional parameters of the airline as input quantities. In addition the uncertainty evaluation scheme allows to take into account systematic corrections. A simulation approach is proposed to acquire information on the distribution of these correction quantities. As a result, the obtained uncertainties of the residual error parameters can now be utilized in order to evaluate the uncertainty of a second order corrected reflection measurement.

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