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KINEMATIC METROLOGICAL MODEL OF THE COORDINATE MEASURING ARM (MCMA)

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Abstract - Among mobile systems which are being implemented more generally to the measuring practice such as coordinate measuring arms (CMA), there is a lack not only commonly accepted methods of accuracy assessment but also procedures to determine the accuracy of any realized measurement, what is particularly important and expected by industry. According to the authors application of simulation using virtual CMA would be the effective solution. In this article the model of kinematic, metrological CMA elaborated in Laboratory of Coordinate Metrology in Cracow University of Technology and its research verification in relation to the real CMA were described. Thanks to such elaborated model, there is a possibility to elaborate virtual measuring arm, which should comprise not only elaborated kinematic, metrological model but also application of metrological software in order that the assessment of uncertainty of realized measurement in real and quasi-real time be possible.

Keywords: Coordinate Measuring Arms (CMA), Model, Kinematics.

1. INTRODUCTION

Development of coordinate metrology means not only new measuring systems but also new methods of assessment of uncertainty of realized measurements. One of the most promising method is application of virtual, measuring systems working in time closed to time of realization of a typical task and giving the uncertainty of measurement together with the result of measurement. In case of manual measuring systems such as CMA carrying out this task is difficult because the systems are redundant and their kinematics has a nature of open chain supported by rotational, kinematic pairs. However using the knowledge about manipulators and metrological robots the idea to elaborate this model arose in Laboratory of Coordinate Metrology in Cracow University of Technology [3] and [1,2,3,7]. The base of accepted idea is kinematic description of CMA, thanks to which there is possible to change the space of configurated coordinates to the space of Cartesian coordinates in accordance with Denavit-Hanterberg notation. This notation of determination of kinematics consists in connecting local coordinate system with each joint, and than specifying the sequence of transformation of next coordinate systems and leads to calculation of kinematics of device as a connection of these transformations (Fig. 2).

Simple kinematic task consists in calculation of the position and orientation of the working element towards the reference system of the base, for the given set of configurated coordinates. This task can be treated as a mapping of description of the position of kinematic chain from the space of configurated coordinates to the space of Cartesian coordinates.

Reverse kinematic task consists in determination of each possible sets of values of angular and linear displacements (configurated coordinates) in moving coonections, which enables CMA to achieve the tasked positions and orientations of the measuring pin.

If we accept, that coordinates obtained from the calculation of the simple kinematic task are nominals and the coordinates red from the machine interface are tasked points, we can carry out the kinematic analysis of CMA. Thanks to that determined task we can simulate a work of our device, study its errors and even generate the uncertainty of measurement in quasi-real time.

Thanks to obtained data from kinematic description of CMA, the CMA was modeled in Catia v.5 in order to display the results and verify the correctness of work of the model quickly.

2. DESCRIPTION OF KINEMATICS OF CMA

2.1.Position vector

Assuming, that particular elements of CMA are stiff solids, than their location in reference system is described with the help of their position and orientation. Marking Cartesian coordinate reference system by *xyz*, the origin of coordinates by *O* and unit vectors by *i*, *j*, *k* of the axes *x*, *y*, *z*, system *x'y'z'* was assigned to stiff solid with the origin *O'* and unit vectors *i'*, *j'*, *k'*.

Position of point *O'* towards the system *xyz* describes position vector:

$$\mathbf{p} = \mathbf{p_x i} + \mathbf{p_y j} + \mathbf{p_z k}, \qquad (1)$$

where $\mathbf{p_x}, \mathbf{p_y}, \mathbf{p_z}$ are its components along particular axes of the reference system

$$\mathbf{p} = [\mathbf{p_x}\ \mathbf{p_y}\ \mathbf{p_z}] \qquad (2)$$

2.2. Representation of rotation

To minimize the number of elements describing the rotation, the representation of rotation was introduced., which uses less number of variables.

The rotation is represented by unit vector k of axis of rotation k, described in reference system xyz, and rotation angle θ. In literature this method is known as the representation axis/angle [2,5].

To determine the matrix resultant of the rotation $R_k(\theta)$ it is presented as a connection of elementary rotations around axis of reference system, which we obtain by making the following convert sequences towards reference system xyz (in accordance with Fig. 1):

- Rotation about angle –γ around axis z, and than about angle –β around axis y (by this transformation vector k become compatible with axis z).
- Rotation about angle θ around axis z
- Rotation about angle β around axis y, and than about angle γ around axis *z* y (by this transformation vector k gets original sense).

This sequence is described by the formula:

$$Rot(\theta)=R_k(\theta)=R_z(\gamma)R_y(\beta)R_z(\theta)R_y(-\beta)R_z(-\gamma) \quad (3)$$

The angles β and γ are determined on the basis of the components of unit vector $k=[k_x, k_y, k_z]^T$:

$$\sin\gamma = \frac{k_y}{\sqrt{k_x^2+k_y^2}} \qquad \cos\gamma = \frac{k_x}{\sqrt{k_x^2+k_y^2}}$$

$$\sin\beta = \sqrt{k_x^2+k_y^2} \qquad \cos\beta = k_z \qquad (4)$$

Final formula is:

$$Rot(k,\theta)=\begin{bmatrix} k_x^2(1-\cos\theta)+\cos\theta & k_xk_y(1-\cos\theta)-k_z\sin\theta & k_xk_z(1-\cos\theta)+k_y\sin\theta \\ k_xk_y(1-\cos\theta)+k_z\sin\theta & k_y^2(1-\cos\theta)+\cos\theta & k_yk_z(1-\cos\theta)-k_x\sin\theta \\ k_xk_z(1-\cos\theta)-k_y\sin\theta & k_yk_z(1-\cos\theta)+k_x\sin\theta & k_z^2(1-\cos\theta)+\cos\theta \end{bmatrix} \quad (5)$$

In practice we'll be interested in determination of vector k and angle θ when we have matrix of rotation R:

$$R=\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \qquad (6)$$

and in accordance with [1] we have:

$$\theta = \arccos\left(\frac{r_{11}+r_{22}+r_{33}-1}{2}\right)$$

$$k = \frac{1}{2\sin\theta}\begin{bmatrix} r_{32}-r_{23} \\ r_{13}-r_{31} \\ r_{21}-r_{12} \end{bmatrix} \qquad (7)$$

In a case of representation axis/angle, rotation is described with the help of four parameters it means angle and three components of versor of axis of rotation [1].

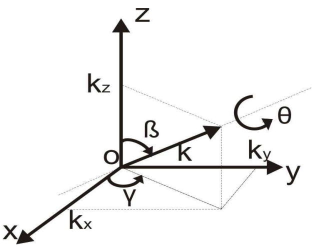

Fig.1 Rotation around optional axis

3. DIMENSIONING AND REPRESENTATION OF CMMA

3.1. Notacja Denavita-Hartenberga

Coordinates of Denavit-Hartenberg are described by four parametres: θ_i – rotation around axis z_{i-1}, λ_i – displacement, stand-off distance along axis z_{i-1}, l_i – distance of axis z_{i-1}, and z_i measured along common perpendicular, it means axis x, α_i – rotation angle, beveling axis z_i towards axis z_{i-1}, as a rotation towards axis x_i (Fig.2). In a case of rotational pair, angle θ_i is variable, and stand-off distance λ_i is constant [2,4,5].

Matrixes of homogeneous conversions, adequate to specified paremetres, can be presented as a formula:

$$A(z_{i-1},\theta_i)=\begin{bmatrix} \cos\theta_i & \sin\theta_i & 0 & 0 \\ -\sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A(z_{i-1},\lambda_i)=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\lambda_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (8)$$

$$A(x_i,{}_i)=\begin{bmatrix} 1 & 0 & 0 & -l_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A(x_i,\alpha_i)=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha_i & \sin\alpha_i & 0 \\ 0 & -\sin\alpha_i & \cos\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

By multiplication of these matrixes we obtain the matrix of position and orientation which concert i coordinate system to i-1 system.

$$A_i^{i-1} = \begin{bmatrix} \cos\theta_i & -\cos\alpha_i \sin\theta_i & \sin\alpha_i \sin\theta_i & l_i \cos\theta_i \\ \sin\theta_i & \cos\alpha_i \cos\theta_i & -\sin\alpha_i \cos\theta_i & l_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & \lambda_i \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{where} \quad (9)$$

$$A_i^{i-1} = \begin{bmatrix} B & & & p \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{where}$$

$$B_i = [n_i \quad o_i \quad a_i] = \begin{bmatrix} n_{ix} & o_{ix} & a_{ix} \\ n_{iy} & o_{iy} & a_{iy} \\ n_{iz} & o_{iz} & a_{iz} \end{bmatrix}$$

$$p_i = [p_{ix} \quad p_{iy} \quad p_{iz}]^T, \text{where} \quad (10)$$

B_i- matrix of orientation, p_i- matrix of position [2,4]

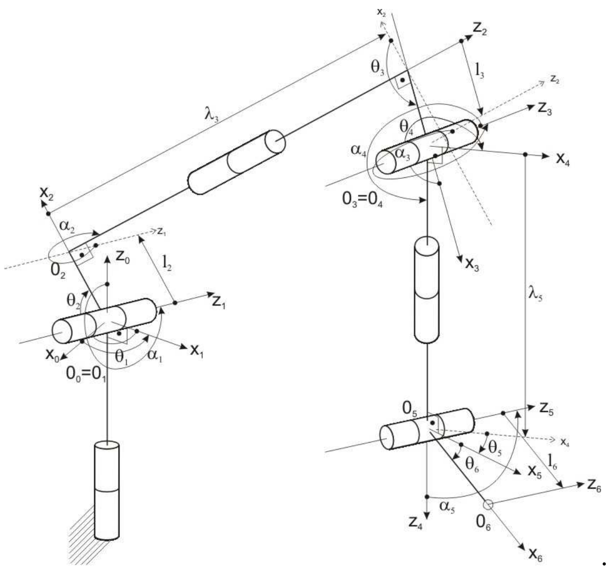

Fig.2. Kinematic diagram CMA

3.2. Simple kinematic task

Simple kinematic task consists in calculation of position and orientation of coordinate system (i+1) towards reference system.

Using the definition of the matrix of homogeneous transformation we write:

$$A_i^{i-1} = A_1^0 A_2^1 A_3^2 \ldots A_i^{i-1} \qquad (11)$$

3.3. Reverse kinematic task

This method is based on the equation (11). Left side of this equation shows the position and orientation of the measuring pin towards basic system and is tasked it means we know the position and orientation of the tool in the given moment. Right side is the product of the homogeneous matrixes multiplied from the base of CMA to their measuring pin.

This task consists in determination of the generalized positions (means connections of particular degrees of freedom CMA) and is very difficult to realize because the equations are nonlinear and it is hard to find the solution in clear form. In a case of CMA, which are redundant systems, the solution can have infinitesimal large number of solutions.

4. DESCRIPTION OF THE ERRORS OF POSITION AND ORIENTATION

4.1. Error of position

Determination of error of position is expressed by the formula:

$$\overline{p_p} = \overline{p_d} - \overline{p} \ , \qquad (12)$$

what is seen in Fig. 3a, where vector $\overline{p_p}$ is the difference between tasked vector $\overline{p_d}$,and $\overline{p}$ tasked by the machine.

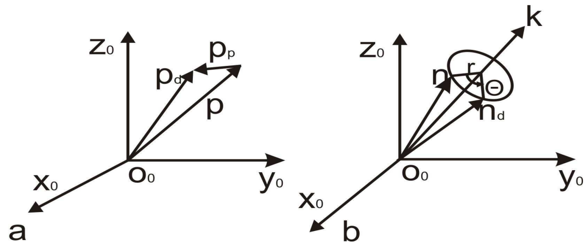

Fig.3a. Error of position
Fig.3b. Definition of error of orientation

4.2 Error of orientation

In case of determination of error of orientation (Fig. 3a) we can use a formula (11), and thanks to it we can obtain matrix of orientation of tip of measuring pin towards basic system. It is necessary to find the formula to count over the errors of orientation it means the difference between the tasked and actual rotation. In general case to determine the rotation there is a need to use three vectors n, o, a, defined in chapter 2.1 in the formula (10). The tasked orientation will be marked by R_d, and actual by R. In our case we are able to obtain only vector $\overline{n}$ from the device (CMA), which is compatible with the position of the measuring pin.
Error of orientation is calculated on the basis of the formula (1):

$$\overline{e} = \overline{k} \sin\theta \qquad (13)$$

The way of determination the orientation presents Figure 3b. [2,4]. In the figure, the basic coordinate system, vectors $\overline{n}$ and $\overline{n_d}$, elementary vector k and rotation angle θ around him are marked. It is also assumed, that vectors $\overline{n_d}$ and $\overline{n}$ have the beginnings in point O (Fig. 3a). The error need to be determined with the help of matrixes of orientation R_d and R. Because k is elementary vector which determines the rotation axis, the angle θ is the equivalent rotation angle between the systems *n o a* and $n_d\ o_d\ a_d$,that is between vectors $\overline{n}$ and $\overline{n_d}$, the error of orientation can be calculated with the help of the formula:

$$Rot\ (k, \theta) = R_d\ R^T \qquad (14)$$

This formula describes what conversion need to be done to determine the coinciding the systems R and R_d, and in our case vectors $\overline{n}$ i $\overline{n_d}$. To calculate this conversion the right side of the formula need to be determined (14), with the vectors in matrix form (3x1). In accordance with vectorial and matrix calculus, the systems or vectors, to coincide, need to fulfil the assumption:

$$R_d R^T = \mathrm{I} \qquad (15)$$

Marking tasked vector by $\overline{n} = \lfloor n_x \quad n_y \quad n_z \rfloor$, and actual vector from the machine by $\overline{n_m} = \lfloor n_{xm} \quad n_{ym} \quad n_{zm} \rfloor$, after multiplication we obtain matrix (3x3).

$$nn_m{}^T = \begin{bmatrix} n_x n_{xm} & n_x n_{ym} & n_x n_{zm} \\ n_y n_{xm} & n_y n_{ym} & n_y n_{zm} \\ n_z n_{xm} & n_z n_{ym} & n_z n_{zm} \end{bmatrix} \qquad (16)$$

Next step is presenting the formula (14) with the help of the formulas (5) – left side and (16) – right side. Comparing the sum of the diagonal terms of the both matrixes, and using the assumption:

$$k_x^2 + k_y^2 + k_z^2 = 1 \qquad (17)$$

we obtain:

$$(1-\cos\theta)(k_x^2 + k_y^2 + k_z^2) + 3\cos\theta = n_x n_{xm} + n_y n_{ym} + n_z n_{zm}$$
(18a)

where after simplifying we have:

$$\cos\theta = \frac{n_x n_{xm} + n_y n_{ym} + n_z n_{zm} - 1}{2} \qquad (18b)$$

we subtruct the outdiagonal terms (1,2) and (2,1) of the both matrixes and we obtain:

$$k_x k_y (1-\cos\theta) + k_z \sin\theta - k_x k_y (1-\cos\theta) + k_z \sin\theta = n_y n_{xm} - n_x n_{ym}$$
(19a)

where after simplifying we have:

$$k_z \sin\theta = \frac{n_y n_{xm} - n_x n_{ym}}{2} \qquad (19b).$$

next we subtruct the outdiagonal terms (3,1) and (1,3) and we obtain:

$$k_x k_z (1-\cos\theta) + k_y \sin\theta - k_x k_z (1-\cos\theta) + k_y \sin\theta = n_x n_{zm} - n_z n_{xm}$$
(19c)

where after simplifying we have:

$$k_y \sin\theta = \frac{n_x n_{zm} - n_z n_{xm}}{2} \qquad (19d).$$

next we subtruct the outdiagonal terms (2,3) and (3,2) of the both matrixes and we obtain:

$$k_y k_z (1-\cos\theta) + k_x \sin\theta - k_y k_z (1-\cos\theta) + k_x \sin\theta = n_z n_{ym} - n_y n_{zm}$$
(19e)

where after simplifying we have:

$$k_x \sin\theta = \frac{n_z n_{ym} - n_y n_{zm}}{2} \qquad (19f).$$

Korzystając z zależności:

$$\sin^2\theta + \cos^2\theta = 1 \qquad (20)$$

we obtain:

$$\sin\theta = \sqrt{1 - \frac{1}{4}(n_x n_{xm} + n_y n_{ym} + n_z n_{zm})^2} \qquad (21).$$

After preliminary research and in accordance with [1] we can state, that error of orientation is expressed with the help of rotation pseudo-vector and can be written by the formula:

$$\overline{e} = \overline{k}\sin\theta = \begin{bmatrix} k_x \sin\theta \\ k_y \sin\theta \\ k_z \sin\theta \end{bmatrix} = \begin{bmatrix} \frac{n_z n_{ym} - n_y n_{zm}}{2} \\ \frac{n_x n_{zm} - n_z n_{xm}}{2} \\ \frac{n_y n_{xm} - n_x n_{ym}}{2} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} n_z n_{ym} - n_y n_{zm} \\ n_z n_{xm} - n_x n_{zm} \\ n_y n_{xm} - n_x n_{ym} \end{bmatrix} \qquad (22)$$

,what is wanted relationship in error of orientation. Let's also notice, that angle limitations implicate fallowing conditions to scalar product $n^T n_m \geq 0$. Certainly, if the vector $\overline{n}$ coincides with vector $\overline{n_m}$, the error of orientation equals 0.

5. DETERMINATION OF KINEMATIC ERRORS

Computing the simple task we obtain not only the matrix with informations about the orientation, but also the location of measuring pin. Thanks to Romosoft software we are able to obtain directly not only vector compatible with measuring pin, but also coordinates x, y, z and angles between particular parts of CMA.

5.1. Model WRP in CATIA V5

The best way to verify the correctness of the model and compare it with the real object is model of a measuring arm 3D built in program CATIA V5R17 in module DMU Kinematics (Fig. 4).

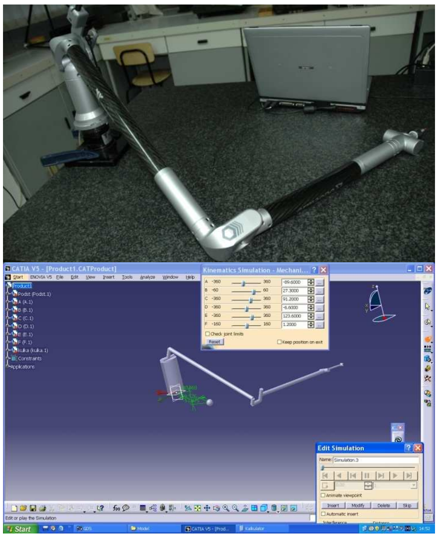

Fig.4.Photo of the real position of CMA and simulated in program Catia v5

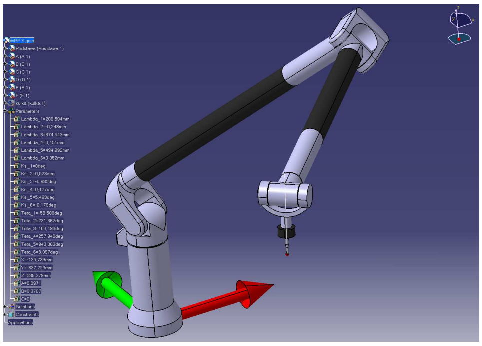

Fig.5. Parametric model of CMA

As the model 3D was improved it became necessary to rebuild simple model and widen its construction in order to give each component in coordinates D-H in Catia.

5.2. Error of position

On the basis of first three datas from GDS software we can determine vector p_d, which will show the characteristic point of the measuring tip CMA in global coordinate system. This error is very intuitive and easy to calculate, that's why it was used earlier to comparing the accuracy of the mathematical model in accordance with indications od CMA. In order to calculate the error of position, which will show the CMA accuracy in arbitrary configuration, it is necessary to determine the error of position for relatively large set of configuration, enough dispersed in measuring space to be for it representative.

Particular components of the error of position are calculated on the basis of the solutions of simple task in Mathcad program among each registered configurations for the actual calibration of the arm. The results are represented:

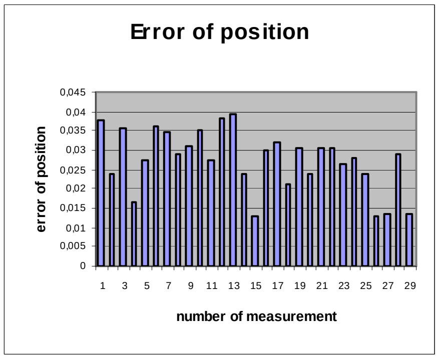

Fig.5.

Fig.6 Graph of error of position

5.3. Error of orientation

Particular components of the error of orientation are calculated on the basis of the solutions of simple task in Mathcad program among each registered configurations for the actual calibration of the arm. The results are represented:

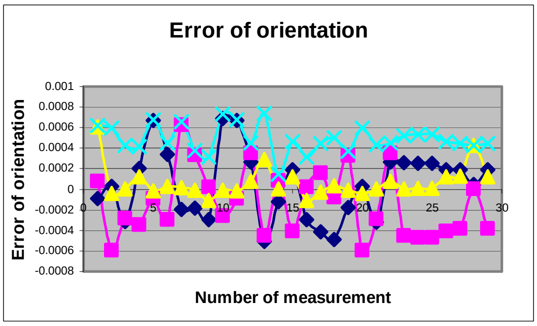

Fig.7. Graph of error of orientation (rhomb-variable x, square- variable y ,triangle- variable z, cross- error of orientation)

6. ASSESSMENT OF THE OBTAINED RESULTS

1) Error of orientation is small, from 0,000106 mm do 0,000687 mm FOR the particular axes (results obtained from 29 measurements), for the device which MPE is ±0.035 mm, what is a satisfactory result,
2) If vectors $\bar{n}$ and $\bar{n}_d$ coincide, the angle between them equals 0 and the error of orientation also equals 0,
3) Derived formula of error of orientation is based only on directional vector of the measuring pin, not on the whole matrix as was earlier, what minimize the number of variables in the formula,
4) Errors of position and orientation of CMA were determined in 29 configurations, which took the whole measuring space of the machine. Results obtained from these configurations are contained in MPE given by the producer.

7. CONCLUSIONS

Determination of the metrological model of CMA consists in identifications of the kinematic errors. These error can be simulated in accordance with kinematics of manipulators. Moreover the essence of the problem is coinciding the component errors on the basis of taken mathematical model, which determines the vector of deviation for arbitrary point in measuring space in CMA model.

In order to determine the error of orientation there is a need to find the conversion to be done because the rotation obtained from the machine needs to coincide with tasked rotation, calculated from simple task. From the software added by producer we can directly read the vector compatible with the direction of measuring pin, coordinates x, y, z, of measuring tip and also readouts from encoders that are configurated coordinates.

On the basis of them we determined the algorithm of simple task, where there is the information about orientation of measuring pin and also the information about the position of the measuring tip and next in this research we found the configuration, thanks to which we obtained the same value of the tasked rotation and obtained rotation.

3D model of a measuring arm built in program CATIA V5R17 in module DMU Kinematics turned out the best way to verify the correctness of kinematic, metrological model and compare it with the real object.

Thanks to module DMU Kinematics quick comparison of movements between model and real arm and comparison between obtained and nominal orientation became possible. Thanks to that, such sources of the biggest component errors were isolated that their correction became possible.

8. LITERATURE

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