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REAL-TIME DYNAMIC ERROR COMPENSATION OF ACCELEROMETERS BY DIGITAL FILTERING

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Abstract – The output signal of an accelerometer typically contains dynamic errors when a broadband acceleration is applied. In order to retrieve the applied acceleration, post-processing of the accelerometer's output signal is required. To this end, we propose the application of a digital filter. We describe the construction of an appropriate filter and consider the uncertainty associated with the filtered output signal. Explicit formulae can be employed to calculate both, the filtered output signal and its associated uncertainty, in real-time. We illustrate the need and benefit of the proposed approach in terms of an example.

Keywords: Accelerometer, Uncertainty, Digital Filter

1. INTRODUCTION

Accelerometers are electromechanical transducers whose dynamic behavior can be described in terms of a linear time-invariant (LTI) system [1] within certain amplitude and frequency ranges. Often, a second-order model is appropriate [2], whose frequency response can be determined by sinusoidal excitations [3]. One goal in constructing an accelerometer is to obtain a frequency response whose magnitude is constant over a large range of frequencies and which has a linear phase. In this case, and when the spectrum of the applied acceleration is restricted to this frequency range, the accelerometer's output signal is – up to a time shift - proportional to the applied acceleration. However, for larger frequencies such as $f \gtrsim$ 10 kHz, say, such an ideal behavior is often not met. In this case, the accelerometer's output signal depends also on past values of the applied accelerations which induces dynamic errors such as ringing.

For the compensation of the non-perfect dynamic behavior of an accelerometer, application of a digital filter has been suggested [4,5]. To obtain a complete measurement result, also the uncertainty [6,7] associated with the output signal of the digital filter is required. Ideally, all this should be made possible by real-time capable algorithms which could then be implemented into a sensor.

We propose the design of an appropriate FIR-type compensation filter. Furthermore, we extend the uncertainty calculation scheme proposed in [5] to account for non-perfect compensation. In this way, the proposed uncertainty evaluation could also be applied for a static analysis which assumes a frequency response with constant magnitude and linear phase.

We illustrate the benefit of the proposed post-processing in terms of simulated accelerometer measurements which allow for an assessment of the applied analysis. We show that reliable uncertainty evaluation for a static analysis yields for broadband accelerations large uncertainties, correctly reflecting the size of the dynamic errors. After compensation, these dynamic errors are eliminated and the corresponding uncertainties highly reduced.

2. ACCELEROMETER MODEL

The input-output behavior of an accelerometer within a certain amplitude and frequency range can be modeled by the differential equation

$$\ddot{x}(t) + 2\delta\omega_0\dot{x}(t) + \omega_0^2 x(t) = \rho\, a(t)\,, \tag{1}$$

cf. [2]. The output signal $x(t)$ of the accelerometer is passed through a charge amplifier and undergoes an analogue-to-digital conversion (ADC). For the results illustrated in this paper we used the following values for the parameters: $\delta = 0.0055$, $S_0 = \rho/\omega_0^2 = 0.124\,\mathrm{pC/(m/s^2)}$ and $f_0 = 35.97\,\mathrm{kHz}$, which resulted from calibration measurements of a Brüel & Kjær type 8305 accelerometer, cf. [2]. Fig. 1 shows the frequency response of this model.

The frequency response of a charge amplifier is usually flat (except for the DC region) and shows a linear phase. When the dynamic behavior of the charge amplifier is more involved, its dynamics could be accounted for by constructing an additional compensation filter. For simplicity, we omit the treatment of the charge amplifier here.

3. DIGITAL COMPENSATION FILTER

We consider the application of a digital compensation filter $g[n]$ to the discrete-time output signal $x[n]$ of the accelerometer for discrete-time estimation of the applied acceleration $a[n] = a(n \cdot T_s)$; $f_s = 1/T_s$ denotes the sampling frequency. Here we propose to design the compensation filter as a cascade of an FIR filter whose frequency response approximates the inverse of the frequency response of the accelerometer and a linear-phase lowpass filter to suppress high-frequency noise. The coefficients of the FIR filter can be determined by applying a least-squares procedure, cf. [5] for details.

Fig. 1 shows the frequency response of the resulting compensation filter.

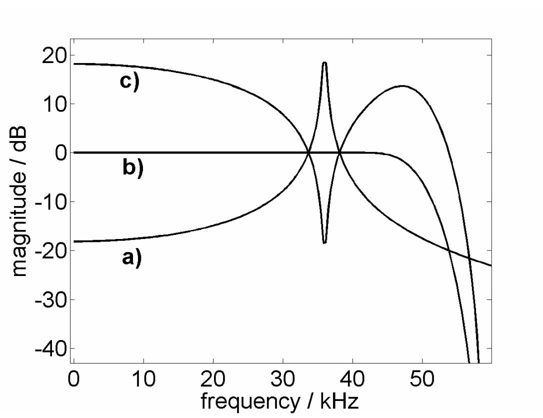

Fig. 1 Dimensionless magnitude response of the model (a), the compensation filter (c) and the model followed by the compensation filter (b).

4. DYNAMIC ERROR COMPENSATION AND UNCERTAINTY EVALUATION

Estimates $\hat{a}[n]$ of the applied acceleration are obtained from the available accelerometer output signal $\hat{x}[n]$ according to

$$\hat{a}[n-n_0] = (\hat{g} * \hat{x})[n], \qquad (2)$$

where n_0 accounts for a possible (small) time delay introduced for the construction of the compensation filter $\hat{g}[n]$, cf. [5]. The uncertainty associated with these estimates is given by

$$u^2(\hat{a}[n-n_0]) = u^2\left((\hat{g} * \hat{x})\,[n]\right) + \gamma^2 / 3\,. \qquad (3)$$

The second term on the right-hand side of (3) accounts for possible remaining dynamic errors and it results from the following approximate bound on the dynamic error $\Delta[n]$:

$$\begin{aligned} &|\Delta[n]| \\ &\lesssim \frac{1}{2\pi} \int_{-\pi f_s}^{\pi f_s} \left| e^{j\omega n_0 / f_s} G(e^{j\omega / f_s}) H(j\omega) - 1 \right| \cdot \overline{A}(\omega)\, \mathrm{d}\omega \;=: \gamma \end{aligned} \qquad (4)$$

where $H(j\omega)$ denotes the frequency response of model (1), $G(e^{j\omega / f_s})$ that of the compensation filter, and $\overline{A}(\omega)$ is an assumed known upper bound on the magnitude spectrum of $a(t)$. The bound in (4) can be derived using Fourier techniques. By assigning a uniform probability density function for the dynamic error within the bounds given in (4), the second term on the right-hand side of (3) results [6].

The first term on the right-hand side of (3) accounts for the uncertainty of the constructed compensation filter $\hat{g}[n]$ and the uncertainty in the available output signal $\hat{x}[n]$. It can be explicitly evaluated according to

$$\begin{aligned} &u^2\left((\hat{g} * \hat{x})\,[n]\right) = \\ &\hat{\boldsymbol{c}}^{\mathrm{T}} \boldsymbol{U}_{\hat{x}_{\mathrm{low}}[n]} \hat{\boldsymbol{c}} + \hat{\boldsymbol{x}}_{\mathrm{low}}^{\mathrm{T}}[n] \boldsymbol{U}_{\hat{c}} \hat{\boldsymbol{x}}_{\mathrm{low}}[n] + \mathrm{Tr}(\boldsymbol{U}_{\hat{x}_{\mathrm{low}}[n]} \boldsymbol{U}_{\hat{c}}) \end{aligned} \qquad (5)$$

where $\hat{\boldsymbol{c}}$ denotes the estimates of the parameters of the FIR filter whose frequency response approximates the inverse of the frequency response of the accelerometer (up to some chosen upper frequency), $\boldsymbol{U}_{\hat{c}}$ their variance-covariance matrix and $\hat{\boldsymbol{x}}_{\mathrm{low}}^{\mathrm{T}}[n] = (\hat{x}_{\mathrm{low}}[n], \hat{x}_{\mathrm{low}}[n-1], \ldots)$ a vector with length equal to that of the FIR filter; the sequence $\hat{x}_{\mathrm{low}}[n]$ is obtained by application of the lowpass filter to the sensor output signal $\hat{x}[n]$. Recall that the compensation filter was chosen as a cascade of an approximate inverse filter of FIR type and an FIR lowpass filter. $\boldsymbol{U}_{\hat{x}_{\mathrm{low}}[n]}$, finally, denotes the variance-covariance matrix of $\hat{\boldsymbol{x}}_{\mathrm{low}}[n]$. While the variance-covariance matrix $\boldsymbol{U}_{\hat{c}}$ dates from the uncertainty of the estimates of the parameters of the dynamic model (1), the variance-covariance matrix $\boldsymbol{U}_{\hat{x}_{\mathrm{low}}[n]}$ is caused by uncertainties of the sensor output signal due to, e.g., noise. For details on the determination of these vectors and matrices we refer to [5].

Application of the filter in (2) can be carried out in real-time. Since typically a possible time delay introduced by some chosen $n_0 > 0$ is small, the estimates $\hat{a}[n]$ in (2) of the applied acceleration are available in real-time. Furthermore, when $\boldsymbol{U}_{\hat{x}_{\mathrm{low}}[n]}$ is known as is, for instance, the case when the sensor output signal is corrupted by additive stationary noise with known autocovariance (cf. [5]), then the first and the last term on the right-hand side of (5) can be calculated in advance, thereby providing a lower bound on the uncertainties. The remaining quadratic form in (5) may be calculated in real-time when the FIR filter has not been chosen too large. In this case the proposed post-processing could in principle be integrated into a measurement device, thereby allowing dynamic error compensation and uncertainty evaluation to be carried out during the measurement.

5. RESULTS

We applied the compensation filter of figure 1 to simulated accelerometer measurements. The measurements were simulated using two Gaussian accelerations with spectra shown in Fig. 2. The accelerometer output signals were calculated using (1), and white noise was added (with standard deviation 0.0001 times the maximum of the accelerometer output signal). A sampling frequency of 200 kHz was used. The time shift n_0 in (2) was chosen as $n_0 = 35$. The linear-phase lowpass filter with cut-off frequency 50 kHz was designed using the Kaiser window technique. For the analysis, parameter estimates of the model parameters slightly different than those used for simulating the measurements were taken, this difference being in accordance with the chosen uncertainties of the parameters; these uncertainties were set equal to those reported in table 2 in [2]. In order to apply the bound in (4), we assumed the actual magnitude spectra of the measurands as their available upper bounds.

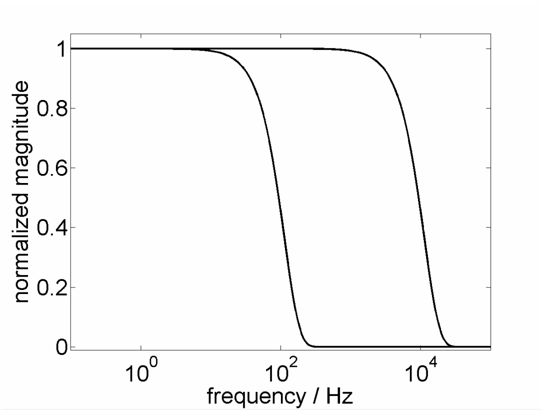

Fig. 2 Normalized spectra of long duration acceleration (left curve) and broadband acceleration (right curve) used for the simulations.

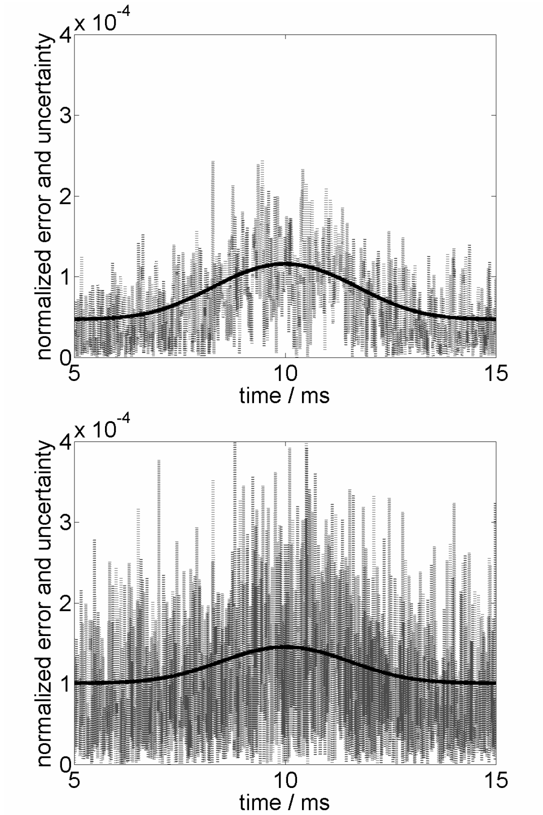

Fig. 3 Normalized errors (dotted lines) of the estimated acceleration with (upper figure) and without (lower figure) compensation for the long duration acceleration together with normalized uncertainties of the estimates (solid lines).

In order to illustrate the benefit of compensation, we considered a static analysis in addition. For this, the accelerometer output signal was simply rescaled, cf. [5].

Fig. 3 shows typical example results obtained with and without compensation for the long duration acceleration; the estimation errors appear to be of similar size for the two analyses. Note that the uncertainties well reflect the size of the estimation errors in both cases. Normalized errors and uncertainties are shown which were obtained by dividing errors and uncertainties by the maximum of the acceleration signal. Consider next in Fig. 4 the results for the broadband acceleration. In this case, the static analysis yields only poor results which is due to the large dynamic error present in this analysis. But note that the (large) uncertainty appears to well reflect the size of the actual errors made by the static analysis. Fig. 4 also shows the results obtained after application of the compensation filter. For this analysis, the estimation errors are again small, and they are well reflected by the calculated uncertainties.

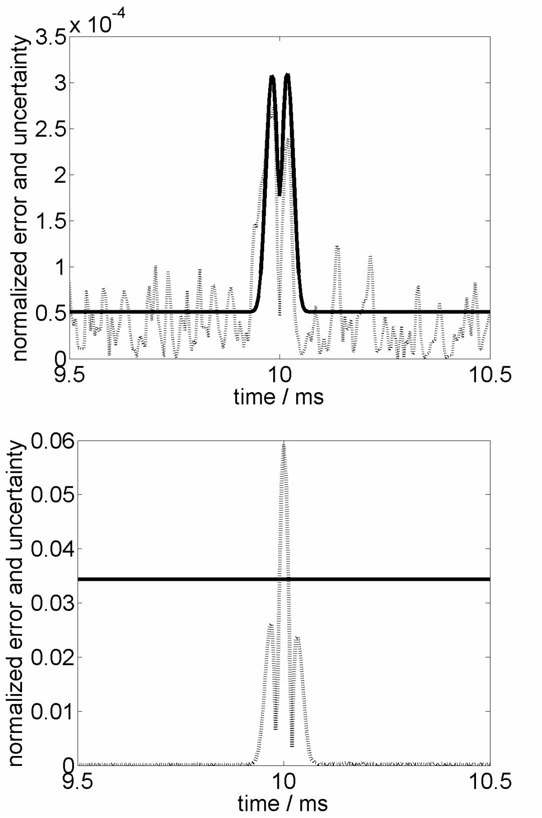

Fig. 4 Normalized errors (dotted lines) of the estimated acceleration with (upper figure) and without (lower figure) compensation for the broadband acceleration together with normalized uncertainties of the estimates (solid lines).

6. CONCLUSIONS

Post-processing is required to determine the applied acceleration from the output signal of an accelerometer when the spectrum of the acceleration has considerable amount in the high frequency region. Digital filtering is a suitable tool for this task. The resulting filtered accelerometer output signal needs to be aligned with an uncertainty, and a procedure in line with the current guidelines in metrology has been given. The proposed algorithms could be suitable for real-time calculation and may be integrated into a sensor.

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