XIX IMEKO World Congress
Fundamental and Applied Metrology
September 6−11, 2009, Lisbon, Portugal

# 2ND GENERATION LEAD MEASUREMENT

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**Abstract** − Typically, a car has over 80 dynamically stressed seals. Since many years, the radial seal in combination with the shaft is a critical component for the automotive industry. A helix-like structure due to the manufacturing process (as a fingerprint of said process) can lead to an unacceptable oil consumption. In collaboration with Daimler, an objective method for characterising such function-relevant helix-structures better known as lead was developed. The lead parameters, which are relevant to leak-tightness, are derived from a least square fit of complex exponential functions in both directions radial and circumferential. The basic measurement setup consists of a stylus instrument in combination with a rotation unit.

**Keywords** : lead, leak-tightness, approximation

## 1. INTRODUCTION

Systems with dynamic sealing function are wide-spread in vehicular and mechanical engineering. Lubricants must be present in sufficient quantity, for example on tribologically relevant positions of moving parts, but may not leave the system. A multiplicity of assemblies with different functionality-requirements and different media to be sealed, which can be manufactured by a multiplicity of subcontractors, are affected by this.

This was the reason why Daimler has developed the first generation lead metrology in 1997 [1]. The goal of this development was the impartial evaluation of periodic surface components caused by turning or grinding manufacturing processes which are relevant to leak-tightness between a shaft and its sealing ring. Function relevant parameters were derived to reach specific requirements to the surface depending on the application, such as engine speed, left- or right-handed motion, degree of pollution, working temperature and the medium to be sealed in. One major lack of this approach is the mathematical method detecting the critical periodically components. It based on the areal Discrete Fourier Transformation (DFT) [2] which is simply an areal fourier series. This means that only an integer number of waves can be perfectly reconstructed. Nevertheless, the main advantage is the numerical efficiency of such an algorithm because the fitted functions spanned an orthogonal space. But due to its limitation concerning the approximation properties the results were not always sufficient good.

In 2004 Daimler decided to develop a new detection algorithm to overcome the problems of the first generation approach. The lead parameters have been retained unchanged. The second generation lead algorithm based on a least square fit of superimposed harmonic related complex exponential functions.

The algorithm can be divided into two steps: the first one is the analysis equation to find the best approximation of the lead structure. The second one is the synthesis equation to reconstruct the relevant lead structure. As shown in the following, the approximation problem is separable and can be implemented in a very efficiency manner.

## 2. LEAD MEASUREMENT

Lead describes all surface occurrences, which contribute to a conveying action of the fluid to be sealed in. Lead arises both in turning and grinding. Inevitably due to feed motion, turning results in a usually singly thread-like surface-structure, where the rotary feed determines the lead-gradient. During grinding with dressing-infeed-motion the dressing-infeed creates a helix on the surface of the workpiece of the grinding disc (Fig. 1).

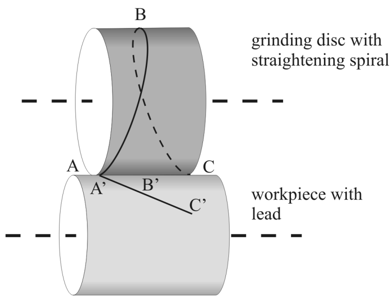


Fig. 1. Projection of the dressing helix to the workpiece.

The lead arises, as this dressing-helix is transferred to the workpiece surface during grinding subject to the ratio of rotational velocities. Therefore, the entire work-piece perimeter shall be considered in order to determine the lead structure.

### *2.1. Measurement procedure*

Lead is a periodical structure in both axial direction and circumferential direction. To characterise the roughness structure in axial direction it is convenient to use a high resolution measurement. In circumferential direction the resolution can be significantly coarser because the frequency of the structure lies typically in a range between 0 and 180 waves per perimeter. Hence, it is adequate to measure lead by a defined number of parallel roughness traces in axial direction. To avoid any influences caused by the clamping of the workpiece (e.g. excentricity) a standard formtester with a surface texture probe is used. But there are other measurement setups possible. As an example, Fig. 2 shows a roughness measurement system with a rotation unit. In this case, a special algorithm has to be adapted to eliminate the radially run out [3].

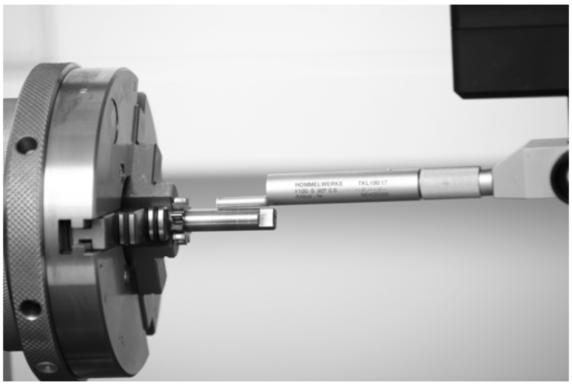

Fig. 2. Lead measurement with a standard roughness measurement system.

An example of a tactile measured workpiece (turned surface) with lead is shown in Fig 3. The data set consists of 72 roughness traces. Each roughness trace is 2mm long with a sampling distance of 0.5µm

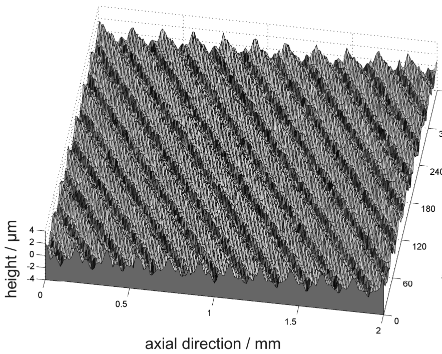


Fig. 3. Grinded surface with lead.

### *2.2. Measurement conditions*

Two different measurement grids have to be distinguished. The first one is defined over the perimeter of the work piece with an angle step of 5°. This initial grid with 72 traces is used to eliminate the said radially run out and to detect helix structures with a frequency lower equal 15 in circumferential direction. If the frequency of the helix structure is higher than 15 a second grid with an angle step of 0.5° over a range of 36° is used (72 traces). The default evaluation length of each roughness trace is 2mm. This means that a total amount of 136 profile traces are needed for the whole data set. Using a standard speed of 0.5 mm/s the total measuring time is approximately 15 minutes. As an alternative, the tactile probing system can be substitute by a confocal chromatic sensor in combination with a high speed drive unit (see Fig 4.). Thereby, the measuring time can be reduced by a factor of five. By default, the tactile measuring system is the reference system.

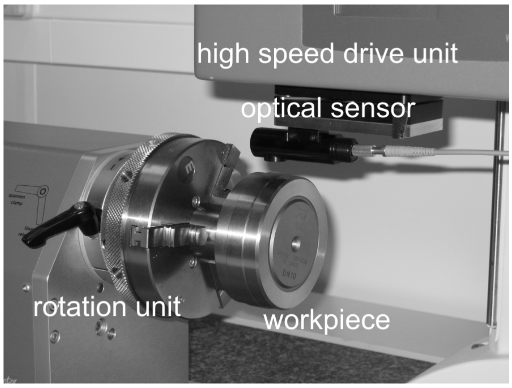


Fig. 4. Lead measurement in combination with an optical confocal chromatic sensor.

## 3. LEAD EVALUATION

### *3.1. Approximation algorithm*

Per definition, lead is a periodically structure in axial ($x$-coordinate) and circumferential direction ($y$-coordinate). Therefore, it is straightforward to use a superposition of harmonic related cosine functions to synthesise the lead surface $w(x, y)$:

$$w(x,y) = \sum_{k=1}^{\infty} A_k \cdot \cos\left(2 \cdot \pi \cdot k \cdot \left(f_x \cdot x + f_y \cdot y\right) + \varphi_k\right), \quad (1)$$

where $A_k$ are the amplitudes, $\varphi_k$ are the phases, $f_x$ is the frequency in axial direction and $f_y$ is the frequency in circumferential direction. To characterise function relevant properties, it is sufficient to use the first three harmonic components of the given series. Taken into account, that a cosine can be expressed by a complex exponential function, equation (1) can be rewrite as follows:

$$w(x,y) = \sum_{k=1}^{3} c_k \cdot e^{i \cdot 2 \cdot \pi \cdot k \cdot \left(f_x \cdot x + f_y \cdot y\right)} + \overline{c}_k \cdot e^{-i \cdot 2 \cdot \pi \cdot k \cdot \left(f_x \cdot x + f_y \cdot y\right)}. \quad (2)$$

The amplitudes and phases are expressed by the coefficients $c_k$ and $\overline{c}_k$, where $\overline{c}_k$ is simply the complex conjugate of $c_k$.

Because the surface is sampled, the coordinates must be replaced by $x = m \cdot \Delta x$ with $m = 0,...,M-1$ and $y = n \cdot \Delta y$ with $n = 0,...,N-1$ respectively, where $\Delta x$ and $\Delta y$ are the sampling distances in the given directions. As described above, the number of measuring points in circumferential direction is $N = 72$. $M$ depends on the sampling distance $\Delta x$. Typical values are $M = 4000$ which implies a step size of $\Delta x = 0.5\,\mu$m. To simplify the calculation procedure the sampled surface $z(m\cdot\Delta x, n\cdot\Delta y)$ and the lead structure $w(m\cdot\Delta x, n\cdot\Delta y)$ is reorganized as column vectors $\mathrm{z}$ and $\mathrm{w}$ with vector elements $z_{m+n\cdot M} = z(m\cdot\Delta x, n\cdot\Delta y)$ and $w_{m+n\cdot M} = w(m\cdot\Delta x, n\cdot\Delta y)$.

The unknown parameters are calculated using the a least square approximation:

$$(\mathrm{z}-\mathrm{w})^{H}\cdot(\mathrm{z}-\mathrm{w}) \rightarrow \min_{f_x, f_y, c_k, \bar{c}_k}, \tag{3}$$

where $H$ is the hermitian of the matrix. It is shown in [4], that the solution for the given least square approximation depends only on the unknown frequencies $f_x$ and $f_y$ and results in the expression

$$\mathrm{z}^{H}\cdot\mathrm{E}\cdot\left(\mathrm{E}^{H}\cdot\mathrm{E}\right)^{-1}\cdot\mathrm{E}^{H}\cdot\mathrm{z} \rightarrow \max_{f_x, f_y}. \tag{4}$$

The $M\cdot N\times 6$ matrix $\mathrm{E}$ spanned the vector space of linear independent complex exponential functions according to equation (2). The elements of $\mathrm{E}$ are defined as

$$\mathrm{E}_{m+n\cdot M, 2\cdot k-1} = e^{i\cdot k\cdot\left(\Omega_x\cdot m+\Omega_y\cdot n\right)}, \quad k=1,2,3 \tag{5}$$

and

$$\mathrm{E}_{m+n\cdot M, 2\cdot k} = e^{-i\cdot k\cdot\left(\Omega_x\cdot m+\Omega_y\cdot n\right)}, \quad k=1,2,3, \tag{6}$$

where $\Omega_x = 2\cdot\pi\cdot f_x\cdot\Delta x$ and $\Omega_y = 2\cdot\pi\cdot f_y\cdot\Delta y$.

After calculating the frequencies $f_x$ and $f_y$ which maximise equation (4), the solution for the fitted data is

$$\mathrm{C} = \left(\mathrm{E}^{H}\cdot\mathrm{E}\right)^{-1}\cdot\mathrm{E}^{H}\cdot\mathrm{z}, \quad \mathrm{w} = \mathrm{E}\cdot\mathrm{C}, \tag{7}$$

where

$$\mathrm{C} = \begin{bmatrix}\bar{c}_1 & c_1 & \cdots & \bar{c}_3 & c_3\end{bmatrix}^{H} \tag{8}$$

is the vector with the unknown parameters $c_k$ and $\bar{c}_k$. As an example, Fig. 5 shows the reconstructed helix-structure of the measured surface pictured in Fig. 3.

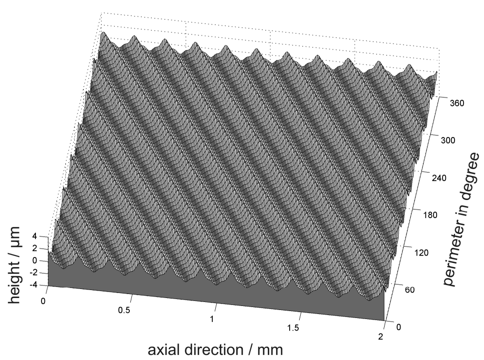


Fig. 5. Reconstructed lead of the measured surface in Fig. 2.

### *3.2. Fast approximation algorithm*

The lead algorithm described before can be calculated by a numerical high efficient algorithm. To get a fast solution for our optimisation problem, we define two basis vectors

$$\mathrm{e}_{x,k} = \begin{bmatrix} 1 \\ e^{i\cdot\Omega_x\cdot k} \\ \vdots \\ e^{i\cdot\Omega_x\cdot k\cdot(M-1)} \end{bmatrix} \text{ and } \mathrm{e}_{y,k} = \begin{bmatrix} 1 \\ e^{i\cdot\Omega_y\cdot k} \\ \vdots \\ e^{i\cdot\Omega_y\cdot k\cdot(N-1)} \end{bmatrix} \tag{9}$$

Because the complex exponential function is separable in $x$ and $y$ we define a matrix with three linear independent cosine functions for each direction

$$\mathrm{E}_x = \begin{bmatrix}\mathrm{e}_{x,1} & \bar{\mathrm{e}}_{x,1} & \mathrm{e}_{x,2} & \bar{\mathrm{e}}_{x,2} & \bar{\mathrm{e}}_{x,3} & \bar{\mathrm{e}}_{x,3}\end{bmatrix} \tag{10}$$

and

$$\mathrm{E}_y = \begin{bmatrix}\mathrm{e}_{y,1} & \bar{\mathrm{e}}_{y,1} & \mathrm{e}_{y,2} & \bar{\mathrm{e}}_{y,2} & \bar{\mathrm{e}}_{y,3} & \bar{\mathrm{e}}_{y,3}\end{bmatrix}. \tag{11}$$

For the further considerations, we define a matrix $\mathrm{Z}$ for the sampled measured data and a matrix $\mathrm{W}$ for the approximated lead structure. The parameter vector $\mathrm{C}$ in equation (7) can be now calculated by

$$\mathrm{C} = \left(\left(\mathrm{E}_x^{H}\cdot\mathrm{E}_x\right)\cdot\left(\mathrm{E}_y^{H}\cdot\mathrm{E}_y\right)\right)^{-1}\cdot\operatorname{diag}\left(\mathrm{E}_x^{H}\cdot\mathrm{Z}\cdot\bar{\mathrm{E}}_y\right). \tag{12}$$

Following equation (4), we have to maximise the expression

$$\operatorname{diag}\left(\mathrm{E}_x^{H}\cdot\mathrm{Z}\cdot\bar{\mathrm{E}}_y\right)^{H}\cdot\mathrm{C} \rightarrow \max_{f_x, f_y}. \tag{13}$$

The conditions (12) and (13) are valid for an arbitrary frequency grid. But in the case of $f_y$, we know that the number of threads in circumferential direction can only be an integer. Moreover, in the case of the 360° data set it can be easily shown that the optimisation problem is reduced to

$$\operatorname{diag}\left(\mathrm{E}_x^{H}\cdot\mathrm{Z}\cdot\bar{\mathrm{E}}_y\right)^{H}\cdot\operatorname{diag}\left(\mathrm{E}_x^{H}\cdot\mathrm{Z}\cdot\bar{\mathrm{E}}_y\right) \rightarrow \max_{f_x, f_y} \tag{14}$$

without calculating the inverse according to equation (12). After solving equation (13) and equation (14) respectively, the reconstructed lead structure can be finally determined by

$$\mathrm{W} = \mathrm{E}_x\cdot\begin{bmatrix}\bar{\mathrm{e}}_{y,1}\cdot\bar{c}_1 & \mathrm{e}_{y,1}\cdot c_1 & \cdots & \bar{\mathrm{e}}_{y,3}\cdot\bar{c}_3 & \mathrm{e}_{y,3}\cdot c_3\end{bmatrix}^{H}. \tag{15}$$

To find the optimal parameters for the reconstructed surface, we can summarise the procedure as follows: Define a set of frequencies $f_y$. Make a $M_{FFT}$-point FFT [5,6] in $x$ direction for each column in $\mathrm{Z}$ (with zero padding if necessary) and build the $M_{FFT}\times N$ matrix $\mathrm{M}$ with the matrix elements $\mathrm{M}_{p,n}$ and the discrete frequencies $f_x = 2\cdot\pi/M_{FFT}\cdot p$, where $p\le M_{FFT}/6$ and $p\in\mathbb{N}$. For a first guess the inverse in equation (12) can be negligible to calculate the unknown frequencies $f_x$ and $f_y$. Therefore, the optimisation problem in equation (14) can be expressed by

$$\left\| \left[ M_p \cdot \bar{e}_{y,1} \quad M_{p\cdot 2} \cdot \bar{e}_{y,2} \quad M_{p\cdot 3} \cdot \bar{e}_{y,3} \right] \right\|^2 \to \max_{p,\, f_y} \qquad (16)$$

With the given start values, calculate the true frequencies $f_x$ and $f_y$ using a Golden Section Search [7] for equation (13) (36° grid) or equation (14) (360° grid).

### *3.3. Lead parameters*

The lead parameters (see Table 1) are derived from the reconstructed lead structure. Fig. 6 shows all relevant parameters to characterise the function relevant properties. The lead parameters are defined as follows: Dγ in ° is the lead angle between the structure orientation and the $y$-coordinate (perpendicular to the axis of the shaft). Dt in µm is the totally depth of the reconstructed structure. DG is the number of threads in circumferential direction. DF in µm$^2$ is the theoretical supply cross section e. g. to describe the oil volume between the sealing ring and the shaft. DFu is the total supply cross section and is defined as DF times DG. DP in mm is simply the periodically wave length in axial direction. Finally DLu is the material ratio in % of 80% the peak to valley height of the lead structure.

Table 1. Parameters to characterise lead.

| Parameter | Definition |
|---|---|
| Dγ | lead angle in ° |
| Dt | lead depth in µm |
| DG | number of threads |
| DF | theoretical supply cross section in µm$^2$ |
| DFu | total supply cross section in µm$^2$ |
| DP | period length in mm |
| DLu | material ratio in % of 80% the peak to valley height |

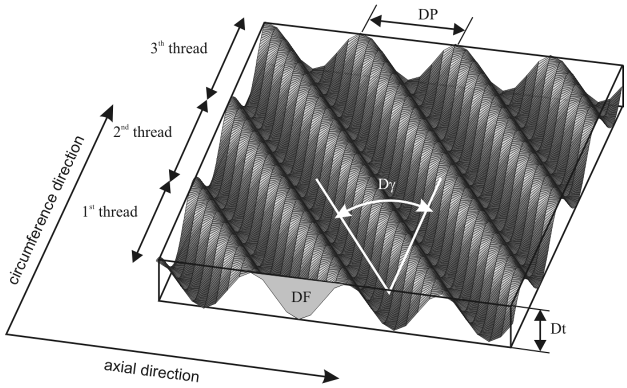


Fig. 6. Visualisation of the lead parameters.

### *3.4. Drawing specification*

Drawing specifications for the lead parameters ensure an efficient manufacturing process. Two examples are given:

*Example 1 (see Fig. 7):*

First row: *Rz* shall be in the range of 1µm up to 4µm. The evaluation length is 4mm, 5 times the sampling length of 0.8mm.

Second row: if the lead angle is equal to 0°, the lead depth shall be lower than 0.8µm between the axial wavelength range of 0.02mm and 0.4mm.

Third row: if the lead angle is unequal 0°, the lead depth shall be lower than 0.5µm between the axial wavelength of 0.02mm and 0.4mm.

The axial measuring length for the lead evaluation is equal to 0.4mm x 5 = 2mm

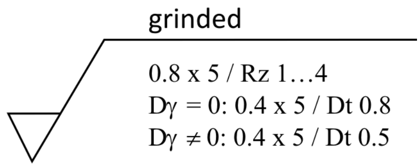


Fig. 7. 1st example for the drawing specification for lead.

*Example 2 (see Fig. 8):*

First row: *Rz* shall be in the range of 1µm up to 4µm. The evaluation length is 4mm, 5 times the sampling length of 0.8mm.

Second row: if the lead angle is equal to 0°, the lead depth shall be lower than 0.8µm between the axial wavelength range of 0.02mm and 0.25mm.

Third row: if the lead angle is equal to 0°, the lead depth shall be lower than 1µm between the axial wavelength range of 0.25mm and 0.5mm.

Fourth row: if the lead angle is unequal 0°, the lead depth shall be lower than 0.3µm between the axial wavelength of 0.02mm and 0.25mm.

Fifth row: if the lead angle is unequal 0°, the lead depth shall be lower than 0.8µm between the axial wavelength of 0.25mm and 0.5mm.

The axial measuring length for the lead evaluation is equal to 0.5mm x 5 = 2.5mm

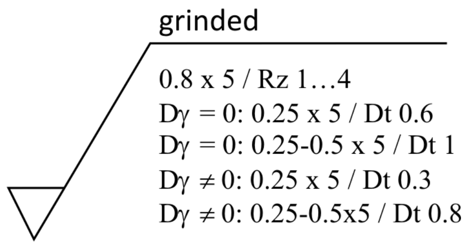


Fig. 8. 2nd example for the drawing specification for lead.

### *3.1. Examples*

The performance of the proposed algorithm is demonstrated in the next two examples. On top of each figure, the original surface with the measured lead structure is pictured. Below the original surface, the reconstructed lead structure calculated according to equation (15) is shown. The goodness of the fit is visualised in the profile plot at the bottom. Shown is one trace of the original surface and the corresponding trace of the reconstructed surface.

*Example 1 grinded surface (see Fig. 9)*

In this example a lead structure with thirty threads is detected. Because the number of threads is greater than 15 the finer grid over 36° has to be used. The lead depth is Dt = 1.78µm. The structure is a result of the dressing-procedure of the grinding disc. The high number of threads and the high lead depth leads to a conveying action of the fluid to be sealed in. The surface is not tight and the work piece must be rejected.

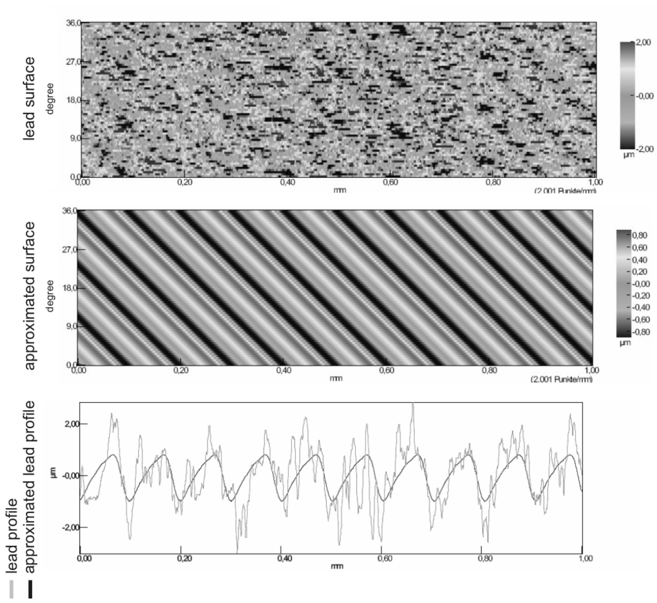


Fig. 9. Original and reconstructed lead structure of a grinded surface.

*Example 2 turned surface (see Fig. 10)*

In this case a turned surface is analysed by the 2nd generation lead algorithm.

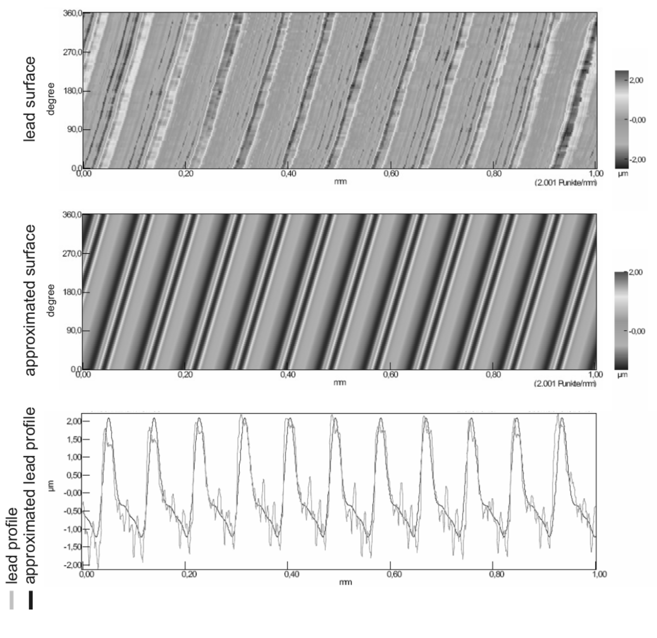


Fig. 10. Original and reconstructed lead structure of a turned surface.

The structure has one thread in circumferential direction. The lead depth is Dt = 3.34µm. As shown in the profile picture the lead structure is well approximated. In spite of its distinct structure the surface is tight because of the occurrence of only one thread.

## 4. CONCLUSIONS

In this paper the 2nd generation lead measurement was introduced. Lead is a periodically structure on the surface which leads to leak-tightness between a shaft and its sealing ring. The basis for the lead evaluation is formed by 136 profile traces per perimeter of the work piece. The measurement can be carried out by using a standard form tester with a surface texture probe or optical point sensor e.g. a confocal chromatic sensor. The function relevant periodical structures are detected and reconstructed by a fast algorithm based on superimposed complex exponential functions. It is shown that the projection matrices are separable. The function relevant structure is characterised by 6 parameters derived from the reconstructed surface. The 2nd generation lead measurement will be implemented in the standard measurement software of BMT Breitmeier Messtechnik, Mahr and Hommel-Etamic in 2009.

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