XIX IMEKO World Congress
Fundamental and Applied Metrology
September 6−11, 2009, Lisbon, Portugal

NEW ALGORITHMS FOR THE OPTIMAL SELECTION OF THE BANDPASS SAMPLING RATE IN MEASUREMENT INSTRUMENTION

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Abstract − The modern measurement instruments involved in telecommunication systems are generally based on suitable digital signal processing methods which provide the desired quantities by elaborating the digitized samples. To meet the accuracy and repeatability required by the telecommunication applications and to warrant the alias-free sampling (*Nyquist-Shannon* theorem), the measurement instruments are usually forced to operate with high sampling frequencies, long observation periods and very fast measurement algorithms. It is worth noting that fixed the observation period, a reduction in the sampling rate directly leads to a reduction in the number of samples to be stored in memory, and consequently in the computational burden and the processing time of the measurement algorithm. If bandpass signals are involved, as it happens in modern telecommunication systems, the bandpass sampling theory could be employed to significantly reduce the sampling rate, without any replica overlapping. This opportunity is very attractive for both instrument designers and users since it allows optimizing the hardware resources through a more efficient employment.

The choice of the bandpass sampling rate is a not trivial task, and wrong values may cause aliasing phenomena and affect the accuracy of measurement results. In this paper, two original algorithms, particularly useful to both instrument designers and users, are proposed to automatically select the sampling rate when bandpass signals have to be measured. To assess and validate the efficiency and the suitability of bandpass sampling criteria proposed, preliminary tests were performed on emulated DVB-T signals.

Keywords: Bandpass sampling, data acquisition system, bandpass communication signal.

1. INTRODUCTION

Nowadays telecommunication technology is subjected to a continuous evolution. The arising and wide diffusion of the innovative communication systems leads to the realization of innovative measurement instruments able to reliably characterize these systems and to evaluate their compliance to new standards and regulations.

Generally these modern instruments are based on digital signal processing methods, that allow to evaluate some specific information by using a suitable processing of the samples of the input signal.

To avoid aliasing and to obtain a good performance in terms of accuracy and repeatability and to they are designed to work with very fast sampling rates, wide acquisition intervals, and very fast measurement algorithms. These features fix hard constraints to the hardware to be used. In particular, to assure wide acquisition intervals in presence of fast sampling rate, it is necessary to have a lot of memory installed on board. In addition, high performance processing units have to be used to achieve adequate measurement rates (in many cases a real-time operating is also required). Obviously, all these characteristics significantly influence the instrument cost.

Fortunately, in modern telecommunication applications, a special class of signals is widely adopted, they are called *bandpass signals* and are characterized by a low ratio of bandwidth B to carrier frequency f_C. In such cases, the necessary condition for the alias-free sampling becomes $f_S>2B$, where B is the bilateral bandwidth of the baseband spectrum and f_S denotes the sampling rate [1]. Then, it is possible to alias-free sample band-pass signals at a rate much lower than twice the upper frequency component of the signal f_u (*Nyquist* rate). Since this condition is not sufficient, the sampling rate has to be chosen very carefully, otherwise aliasing can occur even though $f_S>2B$ [1-3].

In [4], *Angrisani et Al.* proposed two methods for automatic selection of the sample rate for bandpass signals that meet a very common requirement in electronic measurements. In particular, given the signal bandwidth and its centre frequency, the idea is to select the minimum value of f_S among those allowed, which implies the allocation of a replica of the spectrum at a centre frequency chosen by the user.

In the development of measurement instrumentation two other conditions may be very helpful: (a) to estimate the minimum sampling rate allowable; (b) to calculate the minimum admissible f_S that is submultiple of the fixed sampling rate of an given analog-to-digital converter (ADC). These conditions could be very useful to design a cost effective measurement instrument able to warrant a correct signal analysis by using limited hardware resources.

Starting from these considerations in this paper the authors propose two new algorithms for the selection of bandpass sampling rate.

The former, given the input signal characteristics in terms of bandwidth and center frequency, selects the minimum admissible f_S. This condition minimizes the overall hardware resources and optimizes the memory buffer over the observation time. This algorithm can be particularly useful for the instrument designer that at the design stage, on the basis of the specific application, can minimize the hardware resources required (in terms of ADC rate, memory buffer, processing unit performance).The latter, given the input signal characteristics in terms of bandwidth and center frequency, selects the minimum admissible f_S that is submultiple of the fixed sampling rate of an ADC. This second algorithm permits minimizing the memory resources and processing unit performance when the ADC and the sampling frequency are given. In particular, it is useful in such cases where, for an existing ADC stage, the user can select the sampling rate only by using a simple prescaler factor. It often happens in the development or in the use of measurement instruments which are not equipped with sophisticated phase locked loop (PLL) oscillator.

In the following a summary of bandpass sampling theory is presented in Section 2; analytical details of the two algorithms are given in Section 3 and the results of some experiments, carried out on emulated DVB-T telecommunication signals are presented in Section 4. At least in Section 5 some conclusive considerations are reported.

2. SUMMARY OF BANDPASS SAMPLING THEORY

As well known, when a band-limited analogue signal $s(t)$ is sampled, the spectrum of its sampled version is composed by an infinite set of replicas of the original spectrum centered at integer multiples of the sampling rate f_S, as described by the following relation

$$S_\delta(f) = f_S \sum_{k\in\mathbb{Z}} S(f - kf_S), \tag{1}$$

where $S_\delta(f)$ denoted the spectrum of the sampled signal and $S(f)$ the spectrum of $s(t)$. To avoid the superimposition of the replicas, phenomenon known as aliasing, the *Nyquist-Shannon* theorem imposes a lower limit to f_S. This limit is called *Nyquist rate* and is equal to $2f_u$, where f_u is the upper frequency component of $S(f)$.

Typical telecommunication signals are characterized by a high carrier frequency to bandwidth ratio (f_c/B>>1) with the spectrum null in the bandwidth [0-f_c-B/2] (see Fig. 1a). These peculiarities allow resorting to the bandpass sampling theory to select a sampling rate much lower than the Nyquist one, without any replicas overlapping in the abovementioned bandwidth.

In general, the infinite set of replicas are centered at the frequencies [4]

$$f_{\lambda,\nu} = \lambda f_c + \nu f_S \quad \nu \in \mathbb{Z} \qquad ,1\}. \tag{2}$$

Thanks to the periodicity of the spectrum of the sampled signal, the analysis can be limited to the interval [0-f_S]. Only two frequencies that belong to (2) are inside this interval. Their values are respectively

$$f_{\lambda_1,\nu_1} = \mathrm{mod}(f_c, f_S) \tag{3}$$

and

$$f_{\lambda_2,\nu_2} = f_S - \mathrm{mod}(f_c, f_S), \tag{4}$$

where mod(x,y) denotes the remainder after division x/y.

Considering that (3) and (4) are symmetrical respect to $f_S/2$ and ignoring for the moment the case $f_{\lambda1,\nu1}=0$ (i. e. f_S submultiple of f_c), to avoid aliasing the following condition has to be respected

$$\frac{B}{2} < f^* < \frac{f_S - B}{2}, \tag{5}$$

where $f^*=\min(f_{\lambda1,\nu1}, f_{\lambda2,\nu2})$. This condition implies that the replica is totally inside the interval [0-f_S/2]. An example of this condition is represented in Fig. 1b.

The particular case that verifies $f_{\lambda1,\nu1}=0$ produces a replica, characterized by an amplitude twice $S(f)$ and a phase spectrum identically null, due to the superimposition of the positive and negative replicas, permitting only measurement on the power spectrum.

If the condition (5) is verified, it implies that $S(f)$ is totally included in a f_S/2 wide interval i.e.

$$\begin{cases} f_l \ge (n-1)\dfrac{f_S}{2} \\ f_u \le n\dfrac{f_S}{2} \end{cases} \tag{6}$$

where f_l is the lower frequency component of $S(f)$ and n is an integer value greater than one.

Solving (6) respect f_S the following condition can be obtained

$$2\frac{f_u}{n} \le f_S \le 2\frac{f_l}{n-1}. \tag{7}$$

As previous said, the lower value of n is 1 and it is the special case analyzed in the *Nyquist-Shannon* theorem, instead the higher value of n admissible can be easily derived from (7)

$$n \le \frac{f_u}{f_u - f_l} = \frac{f_u}{B}. \tag{8}$$

Generally (8) is not an integer number, for this reason n is limited in the following range

$$1 \le n \le I_g\left[\frac{f_u}{B}\right], \tag{9}$$

where I_g denotes the integer part. The condition (7) combined with (9) is called *condition for uniform bandpass*

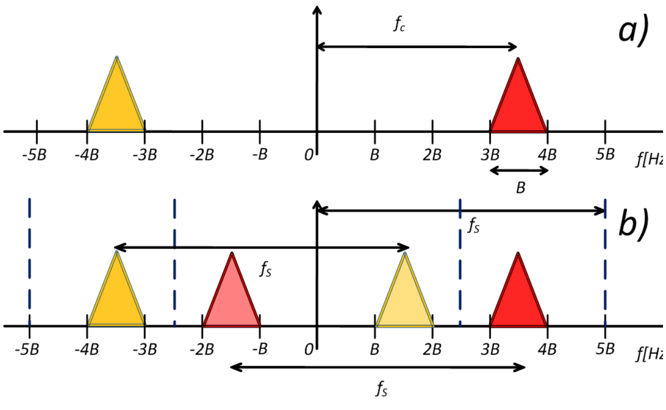

Fig. 1. a) Spectrum of a bandpass signal, in red the positive side and in orange the negative side; b) effects of bandpass sampling, in light red and in light orange are the replicas of the positive side and the negative side of the spectrum respectively.

sampling rate [1].

Supposing that the ratio $n=f_u/B$ is integer and substituting it in the left side of (7), the theoretical minimum sampling rate $f_S=2B$ can be found [1].

When f_l and f_u are fixed, this condition indicates that there are n sets of admissible f_S. Chosen an allowable value of n means specify a set in which several values of f_S can be selected. The f_S choice is a very important task for an instrumentation designer, because it will fix the spectral allocation of the replicas and the distances between them. These distances usually are called *guard bands* and are denoted as

$$B_{GT} = B_{GL} + B_{GU}, \tag{10}$$

where B_{GL} and B_{GU} are the lower and the upper guard bands respectively.

Called Δf_S the n-th range width, it can be divided into values above and below the selected operating point f^*_S as specified by the following relations

$$\begin{aligned} \Delta f_{Su} &= f_{S,\max} - f_S^* \\ \Delta f_{Sl} &= f_S^* - f_{S,\min}, \end{aligned} \tag{11}$$

with $f_{S,\min}$ and $f_{S,\max}$ the lower and upper bound of the range respectively. The upper and lower guard bands can be easily derived as follows [1]

$$B_{GL} = \Delta f_{Su} \frac{n-1}{2} \tag{12}$$

$$B_{GU} = \Delta f_{Sl} \frac{n}{2}. \tag{13}$$

3. THE PROPOSED ALGORITHMS

As shown in the previous section, the sampling rate can be selected in several sets of values and its choice influences the spectral allocations of the replicas.

Generally, the instrumentation designer has to select the sampling rate according to the following very common constraints:

(i) the measurement section of the instrument is often optimized to operate in a specified range of frequencies. Therefore it could be useful to select a sampling rate which allows the replica of $S(f)$ to be allocated inside that range;

(ii) cost effective instruments are usually made with hardware characterized by limited resources mainly in terms of on board memory and clock rates, thus demanding to select the minimum allowable sampling rate;

(iii) the ADC is often driven by the unique clock present on the measurement instrument and the operating frequencies have to be chosen by applying suitable integer decimation procedure. Therefore these values can be chosen only as submultiples of the fixed operating sampling rate.

Angrisani et al. in [4] have proposed two methods for automatic selection of the sample rate for bandpass signals that satisfy the first above cited constraint. In this paper the authors propose two original algorithms that allow satisfying also the constraints (ii) and (iii).

3.1. Algorithm I

This algorithm makes the selection of the minimum

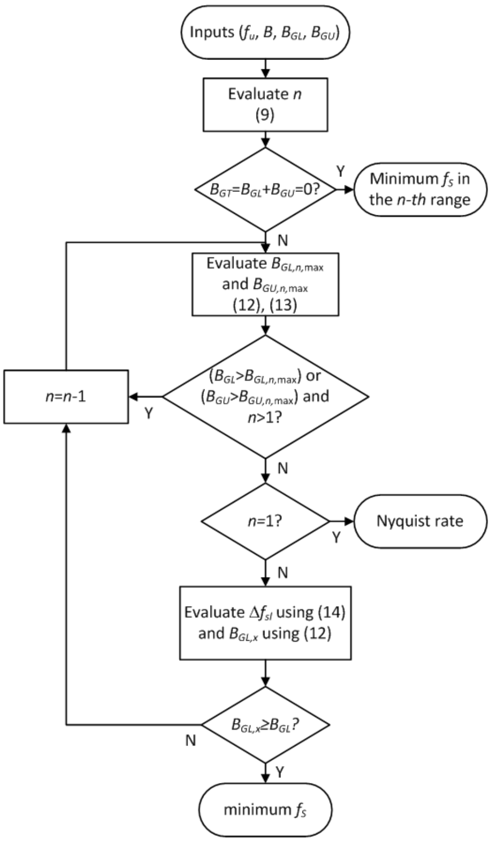

Fig. 2. Block diagram of the algorithm I.

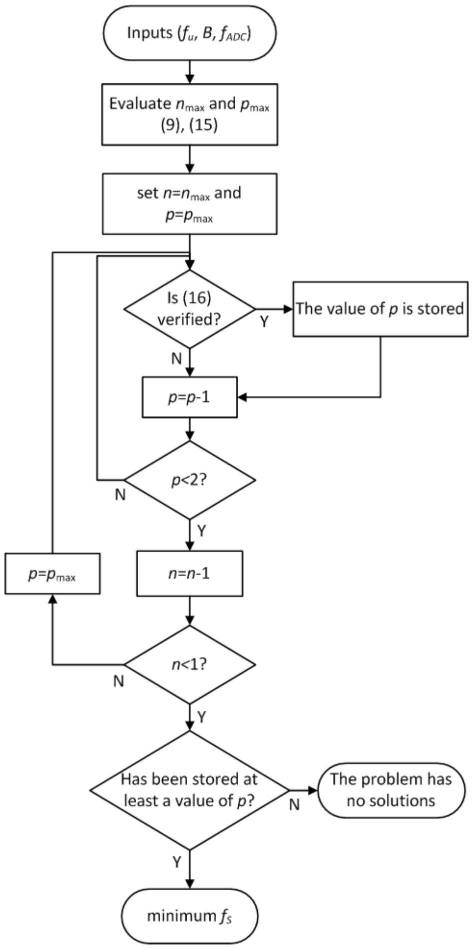

Fig. 3. Block diagram of the algorithm II.

admissible sample rate and informs about the position of f^*, for a given signal characterized by an upper frequency component f_u and a bandwidth B. The user can also specify the guard bands B_{GL} and B_{GU}. A block diagram that illustrates the operative steps executed by this algorithm is depicted in Fig. 2.

At first it evaluates the value of n to estimate how many sets of f_S are available. Successively it verifies if the guard band (B_{GT}) requested by the user is equal to zero. If this hypothesis is true, the algorithm output is the minimum f_S in the set of order n, otherwise it starts to verify if the maximum lower and upper guard band available in the n-th set ($B_{GL,n,\max}$, $B_{GU,n,\max}$) are compatible with the guard bands specified by the user. If this check is negative the algorithm makes an iterative analysis for the n-1 remaining sets, until a set that satisfies these conditions is found.

When a candidate set is found, it checks the contemporary compliance of the two guard band conditions in the following way; at first it calculates Δf_{SI} related to the operative point characterized by an upper guard band equal to that specified by the user

$$\Delta f_{SI} = \Delta f_S \frac{B_{GU}}{B_{GU,n,\max}}, \tag{14}$$

Then it evaluates the lower guard band related to the selected operative point ($B_{GL,x}$) (equation 12). If $B_{GL,x}$ is greater or equal than B_{GL} the algorithm outputs f_S and f^*. Otherwise it restart the analysis for the remaining set until n=1. If no sets can satisfy the conditions imposed by the user the algorithm outputs the Nyquist rate and gives a warning in reducing the guard bands. A Matlab™ implementation of this algorithm can be found in [5].

3.2. Algorithm II

This second algorithm has been developed to give a passband answer when integer submultiples of the fixed operating sampling rate are available (constrain iii). It accepts as inputs the upper frequency component f_u, the signal bandwidth B, and the ADC sampling rate f_{ADC}, and makes the selection of the minimum admissible sampling rate that is a submultiple of f_{ADC} such as

$$f_S = \frac{f_{ADC}}{p}, \quad 2 \le p \le I_g\left[\frac{f_{ADC}}{2B}\right]. \tag{15}$$

The maximum value of p has been fixed to avoid the selection of sample rate lower than $2B$ and p=1 has been neglected because it coincides with the obvious case f_S=f_{ADC}.

Substituting (15) in (7) the following condition is obtained

$$2\frac{f_u}{n} \le \frac{f_{ADC}}{p} \le 2\frac{f_l}{n-1} \quad \Rightarrow \quad 2p\frac{f_u}{n} \le f_{ADC} \le 2p\frac{f_l}{n-1}, \tag{16}$$

that in combination with (9) and (15) allows to choose the minimum admissible sampling rate submultiple of f_{ADC}.

We can note that (16) depends on p and n, as a consequence the algorithm analyzes all possible combination of these factors. If more combination verify the (16), it returns the solution that involves the highest value of p, granting the lowest value of f_S. The operative steps executed by this algorithm are depicted in Fig. 3, and its Matlab implementation is available in [5].

4. ALGHORITMS ASSESSMENT

Preliminary experiments have been executed to assess and validate the efficiency of the proposed bandpass sampling rate algorithms. Emulated DVB-T signals have been involved.

To these aims a measurement station, sketched in Fig. 4, was setup and properly characterized. It involves a RF signal generator (Agilent Technologies™ E4438C) equipped with DVB-T personalities and used to provide DVB-T signals having the following transmission settings: 8k transmission mode (k=6817 and T_u=896 µs), 1/4 (Δ=224 µs) and 1/32 (Δ=28 µs) guard intervals, center frequency equal to 610 MHz, nominal total power of -10 dBm (100 µW), 64-QAM modulation scheme, code rate equal to 1/2, and 7.61 MHz bandwidth [6].

The generated DVB-T signal was sampled by a DAS, namely a LeCroy™ WavePro 7300A. The sampling circuit of the DAS was driven by a reference clock signal provided by a Rohde & Schwarz™ SM300 RF signal generator. All the devices that compose the measurement station were controlled by a suitable LabVIEW driver that runs on the control unit, namely a personal computer, by using three different bus, an IEEE-488 bus, an Ethernet bus and an USB bus.

The incoming signal was sampled considering three different sampling rates: 2.5 GS/s, 33.33 MS/s, and 16.158 MS/s. The first one respects the limit imposed by the *Nyquist-Shannon* theorem, the other two are the bandpass sampling rates provided by the proposed algorithms. In particular, 33.33 MS/s is the slowest bandpass sampling rate that is an integer submultiple of f_{ADC}=100 MS/s, whereas 16.158 MS/s is the minimum bandpass sampling rate allowable. These values are obtained imposing the following inputs: f_u=614 MHz, B=8 MHz and B_{GL}=B_{GU} equal to null. This hypothesis can be done because the imposed bandwidth value overestimates the DVB-T nominal bandwidth.

The influence of the proposed bandpass sampling algorithms on the metrological performance were tested on some measurements typical in radio frequency systems.

In particular, for each guard interval and sampling rate, 50 tests were executed and the following figures of merit have been analyzed:

a) the mean value of the channel power measurement results (P_C) computed by integrating the power spectrum density (PSD) in the nominal channel bandwidth;

b) the experimental standard deviation of the channel

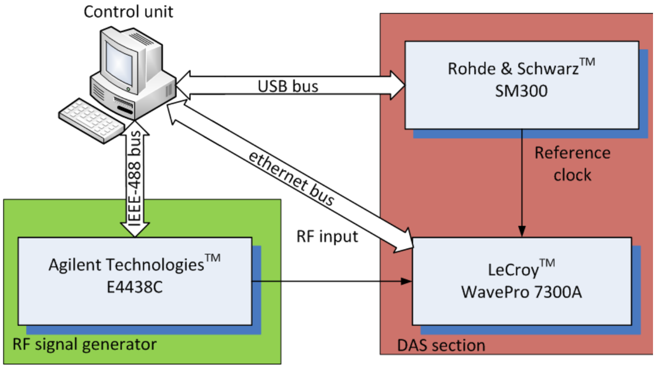

Fig. 4. The designed measurement station.

power measurement results (σ_C);

c) the difference (Δ_C) between the channel power measured on the Nyquist sampled signal and the channel power measured on the bandpass sampled signal;

d) the mean value of the occupied bandwidth measurement results (B) computed as the frequency range in which is collocated the 99% of the input signal power;

e) the experimental standard deviation of the occupied bandwidth measurement results (σ_B);

f) the difference (Δ_B) between the occupied bandwidth measured on the Nyquist sampled signal and the occupied bandwidth measured on the bandpass sampled signal.

For each signal acquired its PSD estimate was evaluated using two different PSD estimators: the modified periodogram [7], [8] and Burg [9].

The former is a nonparametric estimator of the PSD, that computes the modified periodogram of the input signal. It weighs the samples of the signal by using a window with the aim of reducing the estimation bias. The latter belongs to the parametric estimator class; it supposes that the analyzing signal is the output of a *m*-order autoregressive model.

For the evaluation of the modified periodogram a Hamming window was used. As far as the Burg estimator is concerned, a model order (*m*) equal to 3000 and 300 [10] was considered for the sampled signals at 2.5 GS/s and at the selected bandpass sampling rates, respectively.

The measurement results reported in Table. 1 are achieved by considering a guard interval equal to 224 µs and an acquired record length equivalent to a time interval equal to 1/4 of DVB-T symbol. It is possible to highlight that:

1. power measurement results evaluated by using both the proposed bandpass sampling rates differ from that obtained by adopting a sampling rate equal to 2.5 GS/s of a quantity ever lower than 2.66%, that corresponds to 0.11 dB;

2. the repeatability of the channel power measurement results does not seem to be influenced by the three sampling rates;

3. the measurement results of the occupied bandwidth obtained by using the modified periodogram estimator show a little bias that in the worst case (16.158 MS/s) is equal to 3.90%. This experienced bias might be dependent by the hypothesis made on the signal bandwidth. As previous said, this rate has been obtained by imposing a total guard band equal to zero. Even though this condition theoretically assures that the replicas are not overlapped, practically it does not warrant an adequate gap between two adjacent replicas, thus affecting the measurement results. The PSD obtained by using the modified periodogram estimator on the sampled signal at the above mentioned three sampling rates are depicted in Fig. 5;

4. as far as the measurement results of the occupied bandwidth obtained using the Burg estimator is concerned, it is possible to report similar considerations obtained for the modified periodogram case. It is worth nothing that the reference case (sampling rate equal to 2.5 GS/s) has been evaluated by using a model order equal to 3000, instead in the other cases *m*=300 has been adopted. In fact passband sampling allows to reduce the number of samples stored in the memory, in this way it is possible to use a lower model

Table 1. Measurement results.

PSD estimators	Figures of merit	Sampling rate [MS/s] 2500	33.33 Algorithm II	16.158 Algorithm I
Modified Periodogram	P_C [µW]	85.65	84.56	87.88
	σ_C [µW]	0.53	0.66	0.60
	Δ_C [µW]		-1.09	2.61
	B [MHz]	7.5275	7.5353	7.8213
	σ_B [MHz]	0.0041	0.0044	0.0087
	Δ_B [MHz]		0.0079	0.2938
Burg	P_C [µW]	85.64	84.68	87.92
	σ_C [µW]	0.46	0.37	0.45
	Δ_C [µW]		-0.96	2.28
	B [MHz]	7.5435	7.5388	7.8179
	σ_B [MHz]	0.0034	0.0029	0.0059
	Δ_B [MHz]		-0.0045	0.2744

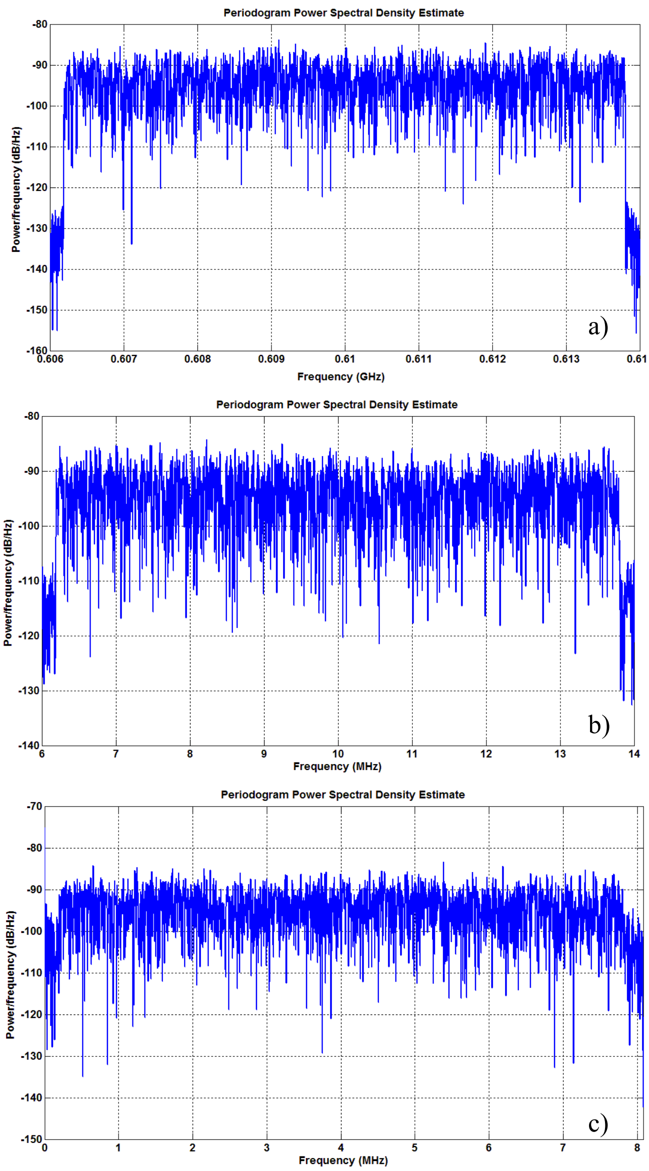

Fig. 5. PSD estimates at different sampling rates: a) 2.5 GS/s, b) 33.33 MS/s and c) 16.158 MS/s. Modified periodogram estimator is involved.

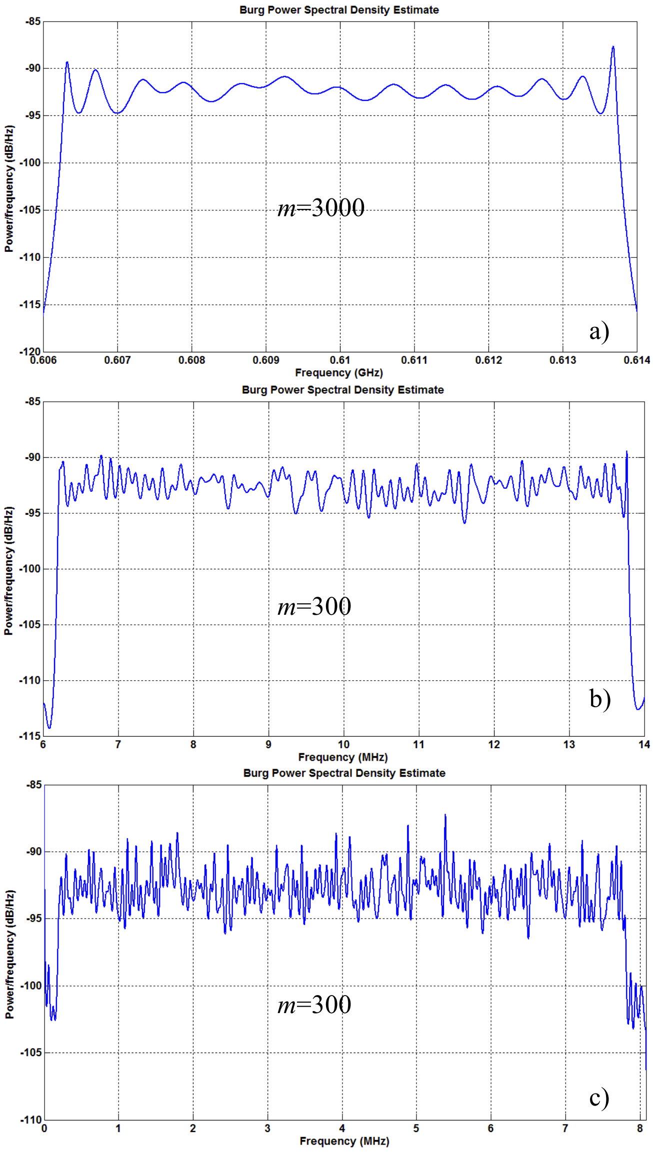

Fig. 6. PSD estimates at different sampling rates: a) 2.5 GS/s, b) 33.33 MS/s and c) 16.158 MS/s. Burg estimator is involved.

order of the estimator, increasing the processing speed, without worsening the repeatability. The obtained PSD by using the Burg estimator on the sampled signal at the above mentioned three sampling rates are depicted in Fig. 6 and Fig. 7.

The same tests have been performed by considering a record length equivalent to a time interval equal to 1/4 of DVB-T symbol and Δ=28 µs achieving similar results.

5. CONCLUSIONS

Two methods for the optimal bandpass sampling rate selection in RF measurement instrumentation has been developed and applied with success on channel power measurement and occupied bandwidth measurements on DVB-T signals.

The measurement results, obtained by the tests performed with real hardware and involving emulated signals, have shown that the employment of the bandpass sampling do not worsen the measures of typical RF and telecommunications parameters.

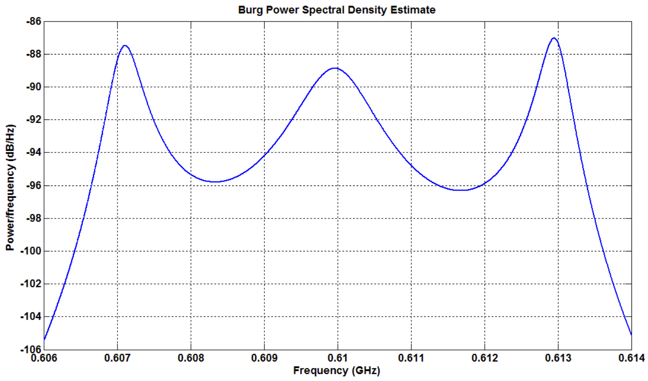

Fig. 7. PSD estimate at 2.5 GS/s. Burg estimator and a model order *m*=300 are involved.

In particular, as the channel power measurement results concern, a tiny bias has been experienced.

As far as the measurement results of the occupied bandwidth is concerned, they seem to be influenced by the bandpass sampling rates, in particular even though the minimum sampling rate, obtained by imposing a total guard band equal to null, should assure that the replicas are not superimposed, it does not warrant an adequate gap between two adjacent replicas, thus affecting the measurement results. Moreover the bandpass sampling allows to use Burg model order lower than the optimum value for the 2.5 GS/s case, without worsening the repeatability.

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