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DEVELOPMENT OF ACCURATE WEIGHING SYSTEM USED UNDER THE VIBRATION-LIKE MOVING CONDITIONS
-VERIFICATION OF WEIGHING SYSTEM WITH 3 ACCELEROMETERS-

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Abstract – This research deals with a weighing system used under the conditions in which various movements exist. These various movements are heaving motion, rolling motion, pitching motion, etc. In this paper, these various movements are collectively called as "vibration-like movements". The term of "vibration-like moving conditions" means the conditions in which vibration-like movements exist. In the previous paper[1], the weighing system which has 3 dummy loadcells is discussed. "Dummy loadcell" is the loadcell which observes the vibration-like movements. We manufactured the weighing system and made several experiments with the weighing system. As a result of those experiments, it is confirmed that the proposed method is able to weigh under the vibration-like moving conditions accurately.

The purpose of this research is the practical realization of the weighing system. In order to put the weighing system into practical use, it is needed to reduce the size and weight of the weighing system. Accelerometers are substituted for the dummy loadcells. Generally, accelerometers are lighter and smaller than loadcells. This research verifies the effectiveness of the weighing system with "dummy accelerometers".

Keywords: accurate mass measurement, system identification, vibration-like moving environment, accelerometer

1. INTRODUCTION

It is important to measure the mass value of an object accurately under the vibration-like moving conditions, from the viewpoint of industrial application. Therefore, we proposed the weighing method under the vibration-like moving conditions[1],[2]. The weighing method considers the position of a gravity center of a weighed object and it basically requires the loadcells which observe the vibration-like movements.

Generally, a loadcell is adopted as a weighing sensor in a weighing system. The loadcell vibrates under the vibration-like moving conditions. Therefore, the loadcell is regarded as a vibration system. There are dynamical characteristics in a vibration system. For these reasons, the same type loadcells are adopted as the "dummy sensors" which observe the vibration-like movements in the previous research.

However, the weighing loadcell is not enough to be small. In order to reduce the size and weight of the weighing system, we consider adopting the different type sensors as "dummy sensors". Therefore, we use the accelerometers as the dummy sensors. We manufactured the smaller weighing system. Some experiments are performed using this weighing system. The performance of the weighing system is examined.

As mentioned above, there are the dynamic characteristics in a vibration system. The dynamic characteristics of the accelerometers differ from those of the weighing loadcell. This difference of dynamic characteristics of two type sensors has prospects of influencing the undesirable effect to measuring accuracy. The purpose of this research is to confirm the influence of the dynamic characteristics difference to the weighing method.

2. WEIGHING METHOD USED UNDER THE VIBRATION-LIKE MOVING CONDITIONS

2.1. Weighing method with 4 dummy loadcells

In this section, the weighing method with 4 dummy loadcells is described. Figure 1 shows the weighing system with a weighing loadcell and 4 dummy loadcells. However, in the weighing system which is discussed, 3 accelerometers are installed as the dummy sensors. Figure 2 shows the coordinate system in which these sensors are installed. A sensor detects the force or acceleration in the direction of z-axis. δ_z represents the translational motion in the direction of z-axis. $\theta_i(t)$ represent the rotational motion around i-axis($i = x, y, z$).

Supposing that the position of a loadcell is located at the point $p(x_p, y_p, z_p)$, the following accelerations influence the output of the loadcell.

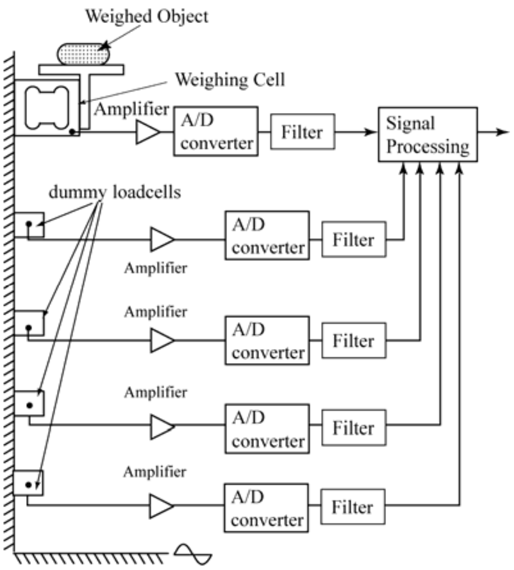

Fig. 1. Weighing system

- The tangential accelerations $y_p\ddot{\theta}_x(t), x_p\ddot{\theta}_y(t)$
- The centrifugal accelerations $z_p\dot{\theta}_x^2(t), z_p\dot{\theta}_y^2(t)$
- The translational acceleration $\ddot{\delta}_z(t)$

The output signal of the loadcell $u_p(t)$ is written as

$$u_p(t) = Em\boldsymbol{P}^T\boldsymbol{B}(t). \tag{1}$$

where,

$$\boldsymbol{P}^T(t) = \begin{bmatrix} x_p & y_p & z_p & 1 \end{bmatrix}, \tag{2}$$

$$\boldsymbol{B}^T(t) = \begin{bmatrix} \ddot{\theta}_y(t) & \ddot{\theta}_x(t) & \dot{\theta}_x{}^2(t)+\dot{\theta}_y{}^2(t) & g_{xy}(t)+\ddot{\delta}_z(t) \end{bmatrix} \tag{3}$$

E and m is the output sensitivity and the mass value of the loadcell, respectively, and ^{T} expresses transposition. $g_{xy}(t)$ is the vertical component to xy plane of the acceleration due to gravity g. $\hat{\boldsymbol{B}}(t)$ is the estimated value of the matrix $\boldsymbol{B}$ and is derived from Eq. (4) . The output sensitivities and mass value of all dummy loadcells are equal and those are represented as E_2 and m_2.

$$\hat{\boldsymbol{B}}(t) = (\boldsymbol{D}^{-1}\boldsymbol{U}_d(t)/E_2m_2) \tag{4}$$

where, $^{-1}$ represents an inverse matrix,

$$\boldsymbol{D} = \begin{bmatrix} x_{d1} & y_{d1} & z_{d1} & 1 \\ x_{d2} & y_{d2} & z_{d2} & 1 \\ x_{d3} & y_{d3} & z_{d3} & 1 \\ x_{d4} & y_{d4} & z_{d4} & 1 \end{bmatrix}, \tag{5}$$

$$\boldsymbol{U}_d = \begin{bmatrix} u_{d1}(t) & u_{d2}(t) & u_{d3}(t) & u_{d4}(t) \end{bmatrix}. \tag{6}$$

d_i (x_{di}, y_{di}, z_{di}) (i = 1, 2, 3, 4) represent the positions of GCDL and u_{di} (i = 1, 2, 3, 4) represent the output of the dummy loadcells. The term of "GCDL" means the "Gravity Center of Dummy Loadcell". Suppose that the output sensitivities and mass value of the dummy loadcells are the same. As a result, $\hat{\boldsymbol{B}}(t)$ is derived from the matrix $\boldsymbol{D}$ and the output signal of the dummy loadcells. The output signal of the weighing loadcell $u_k(t)$ is represented Eq. (7) as follows;

$$u_k(t) = \hat{\boldsymbol{B}}^T(t)\,\boldsymbol{C}, \tag{7}$$

$$\boldsymbol{C} = \begin{bmatrix} a & b & c & d \end{bmatrix}^T. \tag{8}$$

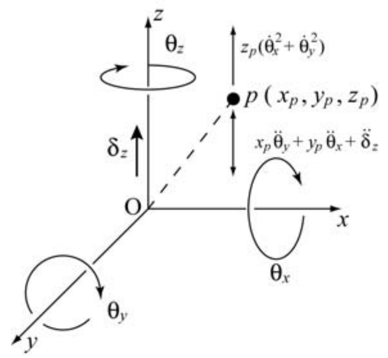

Fig. 2. Coordinate system.

$$a = E_1m_1x_k,\ b = E_1m_1y_k$$
$$c = E_1m_1z_k,\ d = E_1m_1$$

Here E_1 and m_1 is the output sensitivity and the mass value of the weighing loadcell, respectively. The position of GCWL is (x_k, y_k, z_k). Regarding $u_k(t)$ and $\boldsymbol{B}(t)$ as an output signal and input signals of a linear system, the vector $\boldsymbol{C}$ is estimated by means of system identification algorithm [3].

As shown in Eq. (8), the parameter d of $\boldsymbol{C}$ does not depend on the position of GCL. The estimated mass value of the weighed object $\hat{m}_1$ is obtained from this estimated parameter d as in the following equation;

$$\hat{m}_1 = d/E_1. \tag{9}$$

2.2. Weighing method with 3 accelerometers

Giving our attention to Eq. (1), the output signal of a loadcell $u_p(t)$ is a linear combination of the four components of $\boldsymbol{B}(t)$. The element in row 3 of $\boldsymbol{B}(t)$ is the sum of the angular velocities squared ($\dot{\theta}_x^2(t)+\dot{\theta}_y^2(t)$) and the element in row 1 and 2 of $\boldsymbol{B}(t)$ are angular accelerations ($\ddot{\theta}_x(t)$, $\ddot{\theta}_y(t)$). Therefore, if the following conditions are satisfied, 3 dummy sensors are enough to estimate the vibration-like movement. The conditions are as follows;

- z_{di} of 3 sensors are equal to 0.
- $\dot{\theta}_x(0)$ and $\dot{\theta}_y(0)$ are estimated from the output of the weighing loadcell and 3 dummy sensors.

Supposing that z_{di} is equal to 0, the 3rd element of $\boldsymbol{B}(t)$ does not influence the output of the accelerometers. Therefore, Eq. (10) estimates the following vector $\boldsymbol{B}_3(t)$.

$$\boldsymbol{B}_3{}^T(t) = (\boldsymbol{D}_3{}^{-1}\boldsymbol{U}_3(t)/E_a) \tag{10}$$

$$\boldsymbol{B}_3{}^T(t) = \begin{bmatrix} \ddot{\theta}_y(t) & \ddot{\theta}_x(t) & g_{xy}(t)+\ddot{\delta}_z(t) \end{bmatrix} \tag{11}$$

Here, the elements of $\boldsymbol{B}_3(t)$ is the vector which has the elements of $\boldsymbol{B}(t)$ except row 3(See Eq. (11)). E_a is an output sensitivity of the accelerometers. It is assumed that the dummy accelerometers are installed at the points of (x_{di}, y_{di}, 0) (i = 1, 2, 4), matrix $\boldsymbol{D}_3$ is defined as follows;

$$\boldsymbol{D}_3 = \begin{bmatrix} x_{d1} & y_{d1} & 1 \\ x_{d2} & y_{d2} & 1 \\ x_{d4} & y_{d4} & 1 \end{bmatrix}$$

The vector $\boldsymbol{U}_{d3}(t)$ represents the output vector of 3 accelerometers.

$$\boldsymbol{U}_{d3}(t) = [u_{d1}(t) \quad u_{d2}(t) \quad u_{d4}(t)]^T \tag{12}$$

On the other hand, the position of z_k changes in each mass measurement, because the shape and mass value of each weighed object changes. As a result, $\left(\dot{\theta}_x^2(t) + \dot{\theta}_y^2(t)\right)$ influences the output of the weighing loadcell. Therefore, estimation of $\boldsymbol{C}$ in Eq. (7) requires derivation of $\hat{\boldsymbol{B}}(t)$. Since $\dot{\theta}_i(t)$ (i = x, y) are the integral of $\ddot{\theta}_i(t)$ in continuous time, $\left(\dot{\theta}_x^2(t) + \dot{\theta}_y^2(t)\right)$ is written as follows;

$$\dot{\theta}_x^{\,2}(t) + \dot{\theta}_y^{\,2}(t) = \left\{ \int_0^t \ddot{\theta}_x(t)\ \mathrm{d}t \right\}^2 + \left\{ \int_0^t \ddot{\theta}_y(t)\ \mathrm{d}t \right\}^2 . \tag{13}$$

Equation (13) is rewritten as the following equation in discrete time;

$$\dot{\theta}_x^{\,2}(j) + \dot{\theta}_y^{\,2}(j) = \left\{ \dot{\theta}_x(0) + \sum_{k=1}^{j} \ddot{\theta}_x(k)T \right\}^2 + \left\{ \dot{\theta}_y(0) + \sum_{k=1}^{j} \ddot{\theta}_y(k)T \right\}^2$$
$$= \Theta_{ini} + 2\dot{\theta}_x(0)\,\Theta_x(j) + 2\dot{\theta}_y(0)\,\Theta_y(j) + \Theta_{sum}(j) \cdot \tag{14}$$

where, T is sampling period and

$$\Theta_{ini} = \dot{\theta}_x^{\,2}(0) + \dot{\theta}_y^{\,2}(0)\,, \tag{15}$$

$$\Theta_i(j) = \sum_{k=1}^{j} \ddot{\theta}_i(k)T,\ i = x, y\,, \tag{16}$$

$$\Theta_{sum}(j) = \Theta_x^{\,2}(j) + \Theta_y^{\,2}(j)\,. \tag{17}$$

Seeing Eq. (14), the estimation of $\left(\dot{\theta}_x^2(t) + \dot{\theta}_y^2(t)\right)$ requires to compute Θ_{ini}, $\Theta_{sum}(j)$, $\dot{\theta}_i(0)$ (i = x, y).

Firstly, since $\Theta_i(j)$ are derived easily from the product of $\ddot{\theta}_i(j)$ and T. $\Theta_{sum}(j)$ is calculated from the estimates of $\Theta_i(j)$, easily. We consider the estimation of the Θ_{ini}. Let us set a preparation time for estimation of several parameters. During this preparation time, a weighed object is not loaded on the weighing loadcell. Under such condition, we know the mass value of the weighing loadcell and the position of GCWL. Therefore, it is possible to regard the weighing loadcell as 4th dummy loadcell. (In this case, we should not make the z_k equal to the z_{di}, because $\boldsymbol{D}$ in Eq. (5) becomes singular.)

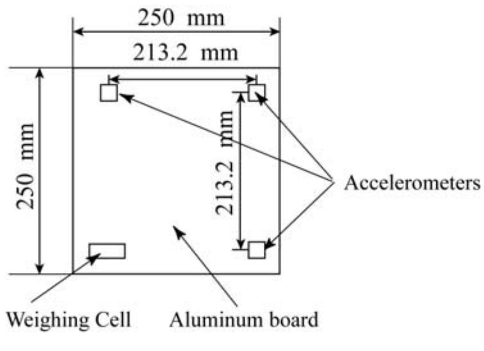

Fig. 3 Schematic of weighing system.

Table 1. Parts of weighing system.

Device	Model
Weighing loadcell	UH-56-3 (Yamato Scale Ltd.)
Accelerometer	3801 (PCB)
DC Amplifier	AL1203 (YOKOGAWA)
Signal Conditioner	478A05(PCB)
A/D board	6036E(NI)
Personal Computer (CPU)	ThinkPad R61e(Lenovo) (Celeron 540, 1.86 GHz)

Regarding the weighing loadcell as 4th dummy sensor, all elements of $\boldsymbol{B}(0)$ in Eq. (4) are computed by using the output signals of 4 sensors at the starting time. Since Θ_{ini} is equal to the element in row 3 of $\boldsymbol{B}(0)$, we obtain Θ_{ini} by using the estimating algorithm mentioned above.

Finally, we must compute $\dot{\theta}_i(0)$,(i = x, y). During the preparation time, Eq. (4) is rewritten as follows;

$$\boldsymbol{A}_o(j) = \boldsymbol{E}^T(j)\boldsymbol{F}\,, \tag{18}$$

where,

$$\boldsymbol{A}_o(j) = \dot{\theta}_x^{\,2}(j) + \dot{\theta}_y^{\,2}(j) - \Theta_{ini} - \Theta_{sum}(j)\,,$$
$$\boldsymbol{E}^T(j) = [2\Theta_x(j) \quad 2\Theta_y(j)],$$
$$\boldsymbol{F}^T = [\dot{\theta}_x(0) \quad \dot{\theta}_y(0)].$$

As mentioned above, $\boldsymbol{A}_o$ and $\boldsymbol{E}$ are computed by using the output signals of the weighing loadcell and 3 accelerometers. Therefore, it is possible for system identification algorithm to compute the estimate value of $\boldsymbol{F}$, regarding $\boldsymbol{A}_o$ and $\boldsymbol{E}$ as output signal and input signals of a linear system, respectively.

As discussed previously, taking the preparation time for estimating $\dot{\theta}_x(0)$, $\dot{\theta}_y(0)$, we are able to use estimated values of $\dot{\theta}_i(j)$ (i = x, y). As a result, all element of vector $\boldsymbol{B}(t)$ are estimated with 3 accelerometers and the weighing loadcell.

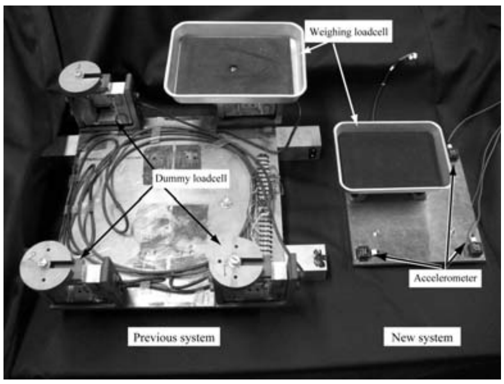

Fig. 4 Photo of the previous system and the new system.

3. WEIGHING SYSTEM WITH 3 ACCELEROMETERS

Figure 3 shows the manufactured weighing system with 3 accelerometers. The weighing loadcell and accelerometers are installed on the square aluminum board. In order to confirm the feasibility of the weighing system with 3 accelerometers, the accelerometers are installed at the vertexes of the 213.2 mm square. The small distance of the accelerometers makes the estimating accuracy of the accelerations worse, as shown in Eq. (10). The size of aluminum board is 250×250 mm. After confirming the efficiency of the weighing system, the distance between accelerometers is decreased and the efficiency of the weighing system is verified.

Figure 4 is the photograph of the previous weighing system and new weighing system. The left aluminium board is the previous system and the right one is the new system.

4. NUMERICAL SIMULATIONS AND CONSIDERATION

In this section, several numerical simulations verify the efficiency of the weighing method. Table 2 shows the simulation conditions. “g” is gravity acceleration. Figure 5 shows the simulated results. The solid line shows the result of the proposed method and the dashed line shows the result which the weighing loadcell output is divided by E g.

Table 2. Simulation conditions

Position of the weighing loadcell	(0.05, 0.05, 0.05) m
Position of the accelerometers	(0.0, 0.2, 0.0) m (0.2, 0.2, 0.0) m (0.2, 0.0, 0.0) m
$\theta_x(t)$	$0.1\times\sin(2\pi\ 0.3t)$ rad
$\ddot{\delta}_z(t)$	$0.3\times g\times\sin(2\ \pi\ t)$ m/s^2
Mass value of the loadcell (Equivalent mass value)	0.6 kg (0.5 kg)
Preparation time	0.8 s
Sampling period	1 ms

Under various conditions, simulations become comparable results. Consequently, the proposed method is feasible to weigh under the vibration-like moving conditions. In simulations, the time required to weigh is about 0.2 seconds.

In the case that a measuring system consists of some kinds of sensors, it is known that the dynamic characteristics difference between the sensors influence the measuring result[4].

Supposing that difference of two sensors’ dynamic characteristics exist, some weighing simulations were carried out. The difference of the dynamic characteristics $G(s)$ is represented as Eq. (19). That is to say, the following filter processes the output of accelerometers $u_{di}(t)$.

$$G(s) = \frac{1}{1+\tau s} \tag{19}$$

By using bilinear transform, $G(s)$ is translated to $G(z)$ as follows:

$$G(z) = \frac{T + T z^{-1}}{(T+2\tau)+(T-2\tau)z^{-1}} \quad . \tag{20}$$

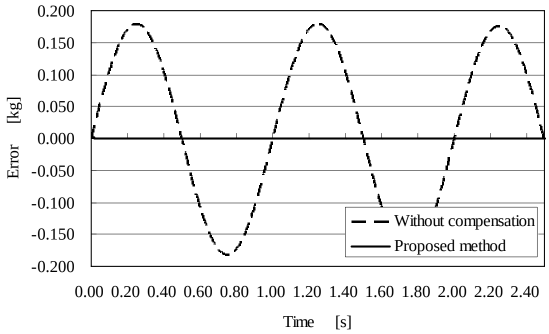

Fig. 5 Simulated Results.

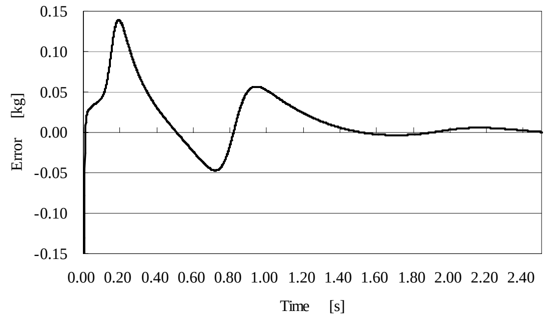

Fig. 6 Simulated Result.

where, T is sampling period. In the case that ‘τ’ is 0.01, simulated result is shown in Fig. 6. The simulated result shows that the dynamic difference of two sensors is considered carefully in this weighing system.

5. EXPERIMENTS AND CONSIDERATION

5.1. Experimental procedure

In this section, the experimental procedure is explained in detail.

1. The data acquisition starts. About 1 second later, the weight is loaded on the tray of the weighing loadcell. The data is acquired for 3.5 seconds.
2. The preparation time is set to 0.8 seconds. During this preparation time, $\dot{\theta}_x(j)$ and $\dot{\theta}_y(j)$ are estimated by processing the output signals of the loadcell and 3 accelerometers.
3. Between 0.8 and 1.5 seconds, the weighing calculation is not conducted, because the influence of loading the object remains. During this time period, the estimation of the angular velocities $\dot{\theta}_x(j)$ and $\dot{\theta}_y(j)$ is continued.
4. After 1.5 seconds elapsed from the start of the data acquisition, the calculation of the mass value starts. At the starting moment of this weighing calculation, angular velocities $\dot{\theta}_x(j)$ and $\dot{\theta}_y(j)$ are substituted for the initial angular velocities $\dot{\theta}_x(0)$ and $\dot{\theta}_y(0)$, respectively

5.2. Experimental conditions

The equivalent mass of the loadcell is about 0.358 kg. In the experiments, the sampling period *T* is 1 ms and the resolution of A/D conversion is 16 bits. Table 3 shows the position of the loadcell and accelerometers.

Table. 3 Position of the loadcell and accelerometers

Position of the loadcell	(0.045, 0.075, 0.03) m
Position of the accelerometers	(0.0, 0.2132, 0.0) m (0.2132, 0.2132, 0.0) m (0.2132, 0.0, 0.0) m

5.3. Exploratory experiment

Figure 7 shows the output signal of the loadcell in the case of following the procedure mentioned above. After 1.0 seconds elapsed from the start of the data acquisition, the weight of 0.1 kg is loaded on the loadcell. The weighing system in static condition.

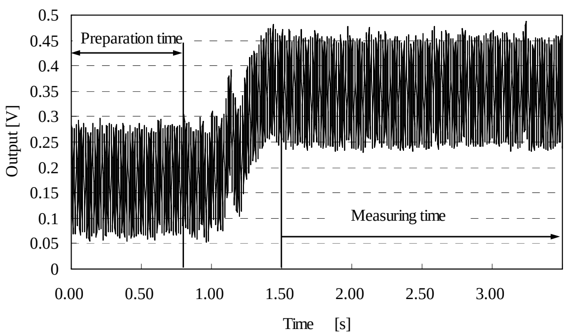

Fig. 7 Output signal of the loadcell.

6. CONCLUSIONS

This research deals with the dynamical mass measurement. From the viewpoint of the practical use, it is intended to reduce the size and weight of the weighing system. Therefore, the accelerometers are used as the dummy sensors. Currently, the difference of the dynamic characteristics is verified. The efficiency of this weighing system is discussed in detail, in conference.

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