XIX IMEKO World Congress
Fundamental and Applied Metrology
September 6−11, 2009, Lisbon, Portugal

DYNAMIC DEVIATION ERROR IN SINGLE FLANK GEAR TESTING

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Abstract - Total dynamic deviation completely describes the influence of geometrical errors of gear on their dynamic behavior during operation in a gear box. At present, commonly used dynamic load factor model is just approximation, it is not precise. A procedure of total dynamic deviation determination takes into consideration the influence of such independent errors as circumferential pitch deviation, basic radius deviation and geometrical eccentricity deviation. The knowledge of total dynamic deviation would enable to construct the gears with better dynamic parameters (wear resistance, quietness and so on).

Keywords: gear measurement, dynamic and kinematic deviations, geometrical errors of gear.

1. THE PURPOSE OF INVESTIGATIONS

The issue of manufacturing accuracy of gears and their inspection is still a subject of interest of many researchers. Many works have been published, and many conferences have been organized on that topic.

However, thorough analysis of the published works reveals the lack of full theoretical description of the gears' accuracy. In the area of the dynamic accuracy, which is the most important topic for all stages from the very project through the manufacturing and measurement up to exploitation, the fundamental disagreement is seen. On the other hand, many methods assume such a high degree of simplification that unable to perform complex analysis and description of the dynamic accuracy of gears dependent on their manufacturing inaccuracy. Large number of works deals with the measurement of gears with uniform measuring tools and two-side testing, while the single-flank testing underwent no analysis. In order to perform a scientific description of that issue, in the Division of Metrology and Measuring Systems following works have been conducted:

- the mathematical model of the single-flank testing has been worked out,
- the independent geometrical errors required for the successful digital simulation of the single-flank testing have been pointed out,
- the computer program for determination of total dynamic deviation has been worked out.

The purpose of the actual investigations was to complete the research project on the gear accuracy analysis through dynamic characteristics. The main thesis of the project is as following: „If the method of determination of the dynamic deviation and total dynamic deviation of the gear is known, and the values of those deviations are known, the aimed projecting, supervision, machining and inspection of the gears become possible in order to ensure the proper exploitation of gears". To prove this thesis, it should be done as following:

- to define the terms of dynamic deviation and total dynamic deviation,
- to work out digital simulation and program for dynamic deviation,
- to verify experimentally the performed theoretical analysis, formulas and programs.

In the model of the gear transmission work, the assumptions were made different from those available in the published papers. Dynamics of the gear transmission underwent analysis from metrological point of view only, which means that the rotational speed of gears is small and the load corresponds with a measurement, not with real work. The load and deformation are not considered as a time variables, and the gear itself is treated as a stiff body. Such an assumption was made based on the definition of the dynamic measurement by prof. A. Metal: "If the measured value changes in time, then the measurement is a dynamic one. (...) Hence, the result of the dynamic measurement is the registration of the changes in time. (...) All the factors that have impact on the correct reproduction of the measured value with the recording device are the subject of interest of the dynamic measurement." [10]

2. MANUFACTURING ACCURACY OF THE GEARS

The manufacturing accuracy of a gear means the degree in which the observed geometrical and functional parameters of the manufactured gear differ from the theoretical values.

The deviation of one of geometric parameters like pitch is called an elementary deviation. The complex deviation, in contrary, means the deviation of a functional parameter like kinematic deviation. The accuracy of a gear is described with the deviations of functional and geometrical parameters of gear. In the Fig. 1, there are presented functional parameters of gears and their indexes.

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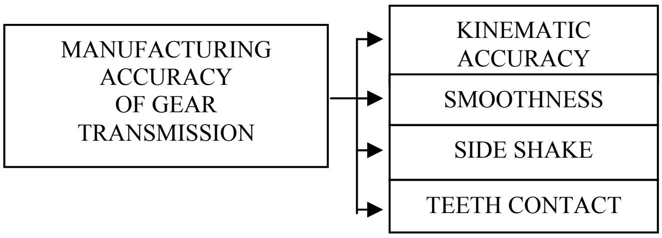


Fig.1. Dependent parameters of the manufacturing accuracy of gears

Standard PN-79/M-88522.01 defines 12 accuracy classes for gears and gear transmissions, and the 1st class is the most accurate one. According to the new standard PN-ISO 1328, two approaches are possible. The first part of the standard describes geometrical approach to the accuracy of a gear. The deviations defined here are called the deviations of the same sides of teeth [9].

Table 1. The deviations of the same sides of teeth, PN-ISO 1328 [9]

DEVIATIONS OF PITCH	
Pitch deviation	f_{pt}
Summed deviation of *k* pitches	F_{pk}
Total deviation of the pitches	F_p
DEVIATIONS OF PROFILE	
Total profile deviation	F_α
Profile form deviation	$f_{f\alpha}$
Profile position deviation	$f_{H\alpha}$
FLANK PITCH LINE DEVIATIONS	
Total pitch line deviation	F_β
Profile pitch line deviation	$f_{f\beta}$
Profile pitch line deviation	$f_{H\beta}$
KINEMATIC DEVIATIONS	
Kinematic deviation of gear	F_i'
Kinematic deviation in the pitch	f_i'

In the Standard, for the deviations of the same sides of teeth 13 accuracy classes are determined, with 0 class as the most accurate one.

The second part of the standard PN-ISO 1328-2 describes the accuracy of the gear manufacturing, but in functional terms. Here, the defined deviations are called the complex radial deviations and the runout.

The tolerances for the radial deviations are classified into 9 accuracy classes with 4th class as the most accurate and 12th as the less accurate one.

Table 2. The complex radial deviations and the runout, PN-ISO 1328 [9]

COMPLEX RADIAL DEVIATIONS	
Inequality of the measuring distance of a gear axis	F_i''
Inequality of the axis measuring distance in a pitch	f_i''
RUNOUT	
Radial runout of the teeth	F_r

3. SINGLE-FLANK TESTING OF THE GEAR TRANSMISSION

The principle of the single-flank gear testing is presented in the Fig. 2. The distance between the axes of the cooperating gears is constant, as it is in real work conditions of a transmission. During the measurement, one of gears is driven while the other is slightly braked in order to ensure constant contact between the cooperating sides of the gears. The rotational speed of the gears is small, so the geometrical deviations of the gears will reveal themselves through the non-uniform movement of the passive gear [9].

Momentary differences between the observed and nominal rotational angle of the passive gear are being registered. The results are presented as a graph dependent on the rotational angle of the driving gear. The method enables to check a single gear as well as a transmission. In case of a single gear check, the driving gear is a master.

From the single-flank testing, the following deviations could be derived:

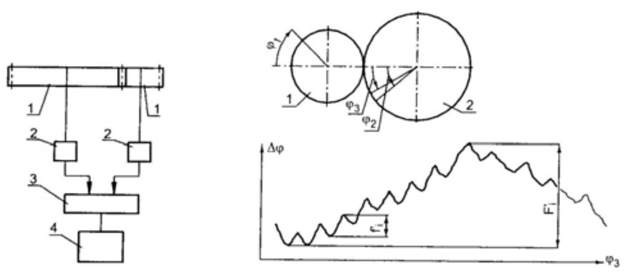
- Kinematic deviation of gear (F_i')

It is a maximal difference between the theoretical and actual circumferential displacement of the tested gear during cooperation with a master gear on the whole rotational cycle.

- Kinematic deviation in the pitch (fi')

It is a value of the kinematic deviation observed in an interval of one pitch.

A graph of the single-flank testing results is presented in the Fig. 3.



1 – cooperating gears
2 – rotation angle sensors
3 – comparing unit for angles
4 - registrator

1 – angle of the master gear rotation
2 – observed rotational angle of the tested gear
3 – nominal angle of rotation of tested gear corresponding with the angle of master gear rotation

Fig. 2. Principle of the single-flank gear testing

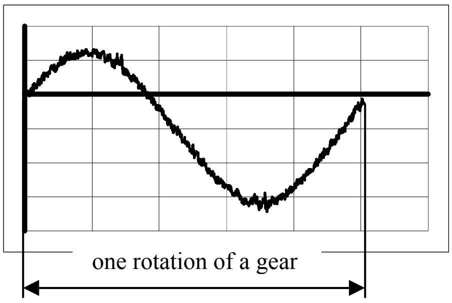


Fig. 3. Graph of the kinematic deviation

The investigations of single-flank testing were performed with the laboratory equipment worked out in the Division of Metrology and Measuring Systems (Poznan University of Technology). The equipment is presented in the Fig. 4, it is unique in Poland.

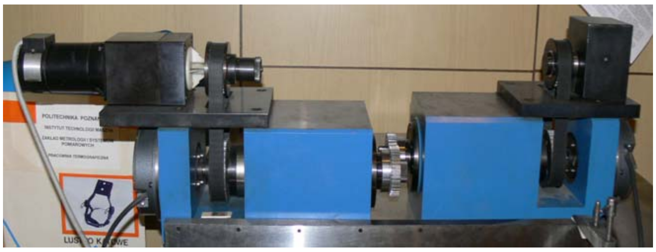


Fig. 4. The single-flank testing equipment worked out in Division of Metrology and Measuring Systems

Measuring signals are generated from the incremental sensors of rotation made by Heidenhain, marked with a symbol ROD-800. The accuracy of the sensors is ±1", with discretization degree of 0.36". The last is ensured by the code disk divided into 36,000 sectors. Ones of them are able to transmit the light, and others are unable to do it. They are placed in a pattern giving 4 times higher resolution, and additional interpolator is applied.

The elementary deviations like pitch deviation (f_{pt}), contact pitch deviation (f_{pb}), base radius deviation ($f_{H\alpha}$), pitch inequality (f_i''), total deviation of the pitches (F_p), summed deviation of *k* pitches (F_{pk}), radial runout of the teeth (F_r) could be measured with uniform measuring tools, as well as with specialized gear measurement tools or with coordinate measuring machines. Measurement of single geometrical deviation enables to correct an accuracy of a produced gear, or to eliminate them according to the requirements determined by the constructor. From the analysis of the values of single geometrical deviations, it is possible to conclude about the work of the gear in the transmission, but in very limited extend. It is impossible to evaluate precisely its future work. The main reason is that the measured deviations cooperate with other gear in random way [4].

Thus, the main demerit of the elementary deviations measurement is the fact that the transmission error remains still unknown. However, it should be a purpose of the measurement itself, because only such a complex parameter could indicate the way of the gear work in future real conditions in a transmission.

Measurement of the elementary deviations is also very time-consuming process, although often still unable to gain the needed data for the quality control system. The diagnostic methods, also often applied to the gear transmission check (like measurement of vibrations or noise) not always give the true source of the irregular work of the transmission.

To analyze the dynamic deviation, the computer program for single-flank testing simulation was worked out in Poznan University of Technology. The input parameters are: pitch deviation f_{pt}, profile angle deviation $F_{H\alpha}$ and the radial runout of the gear F_r [3,4]. Those are independent elementary deviations.

4. THE RESULTS OF KINEMATIC DEVIATION ANALYSIS

The investigations and analysis performed in the Division of Metrology and Measuring Systems have proved that the pitch deviation had had the largest impact, up to 80%, on the kinematic deviation [5]. The impact of other factors had been determined as 0.5% for base radius deviation and 19.5% for eccentricity. The results of analysis are presented in the Fig. 5.

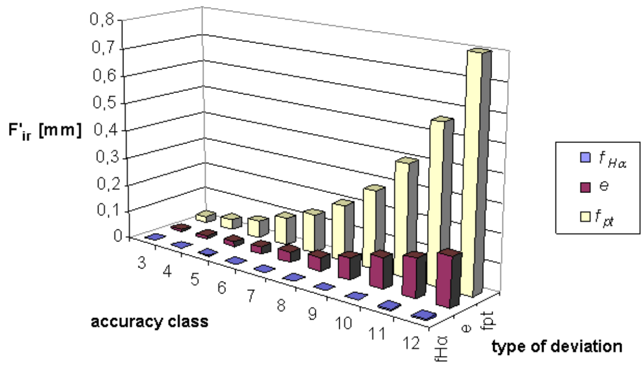


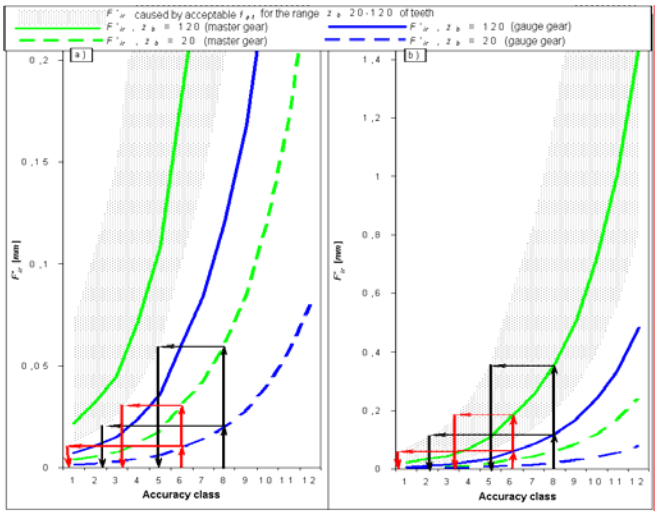
Fig. 5. The impact of the elementary deviations on the kinematic deviation of a gear [6]

It was proved also, that the kinematic accuracy of gear transmission is dependent on the total deviation of pitches F_p as well as summed deviation of pitches in *k* sectors with pitch length *S* – F_{pkS}. Additionally it was revealed that in case of gears of high accuracy and transmissions of high rotational speed, of high importance is also the graph of summed deviation of pitches around the gear [8]. From the analysis, the following recommendations on the gears measurement, related to the hierarchy of gears accuracy have been formulated. The acceptable values of the kinematic deviations were assumed for the hierarchy of tested gear – master gear – gauge gear.

– A master gear – acceptable kinematic deviation of the master gear should not exceed 30% of the acceptable kinematic deviation of tested gear.
– A gauge gear – acceptable kinematic deviation of the gauge gear should not exceed 10% of the acceptable kinematic deviation of tested gear [6].

The proposed values of the acceptable kinematic deviation have been assumed in analogy with the elementary deviations defined by standard PN-ISO 1328. This standard recommends to apply the master gears made in accuracy classes 2 to 3 classes higher than the tested gear. Similarly, the gauge gears should be of 2-3 classes higher than the master gear.

The examples of such hierarchy are presented in the Fig. 6.



a) hierarchy for the tested gear z_b = 20, range $F'_{ir} < 0.2$ mm,
b) hierarchy for the tested gear z_b = 120, range $F'_{ir} < 1.4$ mm,

Fig. 6. Hierarchy based on the kinematic deviation caused by the pitch deviation f_{pt}

From the obtained results it could be stated that in order to perform single-flank testing of a gear made in 8th class of accuracy, the master gear for that purpose should be made in 4-5th class of accuracy, while the gauge gear should be made in 1st-2nd class of accuracy. When the gear is made in lower class of accuracy, e.g. in 6th, the master and gauge gears should be made in lower classes of accuracy, respectively, which means master gear in 2nd-3rd class of accuracy and the gauge gear in the 1st class of accuracy.

5. DYNAMIC MODEL

Very important role in the gear transmission work plays the dynamic deviation, because the exploitation of the gears is dependent on the actual dynamic excesses. They have impact on the noise of the transmission, wear and tear of teeth, as well as their fatigue strength. In the stage of construction, the acceptable values of the dynamic excess should be determined possibly accurately. Therefore, the appropriate parameters should be pointed out which unequivocally determine the influence of the manufacturing accuracy of gears on their dynamical load during the work of transmission.

5.1. Definition of total dynamic deviation

For the first time, the terms of dynamic deviation and total dynamic deviation have been introduced as a result of investigations performed in Division of Metrology and Measuring Systems (Poznan University of Technology). Researchers Jan Chajda, Andrzej Cellary, Andrzej Gazdecki and Waldemar Woliński in frames of the project financed by Central Program of Basic Researches have introduced the term of dynamic deviation and proposed the method with computer program able to determine dynamic deviation in the whole area of the gear operation, with possibility of the harmonic analysis [5]. The investigations revealed that the parameter able to describe the influence of the manufacturing accuracy of gear on the values of dynamic exceedes is so called dynamic deviation.

Definition of the dynamic deviation is based on the function of the kinematic deviation of the gear $\Theta(\varphi)$. This function is defined as the difference between observed and minimal angles of the rotation of the tested gear driven by the master gear, while the distance between the gears' axes remain nominal. The function $\Theta(\varphi)$ of the kinematic deviation of the gear is a periodic function with the period of 2π rad. The differentiation of the function by the time gives the formula of the accelerations caused by the kinematic inaccuracy of the gear [3].

$$\varepsilon_\omega = \frac{d^2\Theta}{d\varphi^2} \qquad [\text{rad/s}^2] \qquad (1)$$

Similarly to the kinematic deviation, the obtained function (ε_ω) was called a dynamic deviation. Its parameter, which describe maximal values of the function, was called the total dynamic deviation defined in the following way:

$$\gamma = \max(\varepsilon_\omega) - \min(\varepsilon_\omega) \qquad (2)$$

where: $\varphi \in A$

$$A = \left\langle \frac{2\pi}{z} \cdot n, \frac{2\pi}{z} \cdot (n+1) \right\rangle \qquad (3)$$

$n = 1, 2, ..., z$
z – number of teeth.

Thus, the total dynamic deviation is the maximal difference between the observed values of general acceleration in the area of the gear's pitch.

5.2. Methodology of the determination of the dynamic deviation

The value of the dynamic deviation like the values of kinematic deviations is dependent on geometrical inaccuracy, i.e. manufacturing errors of the produced gear. The main influence on the dynamic deviation reveal the following elementary geometrical deviations:

- Pitch deviation f_{pt},
- Total profile deviation $F_{H\alpha}$,
- Eccentricity of a gear axis *e*.

It should be emphasized that the kinematic deviation $\Theta(\varphi)$, which is the initial point for the further calculations, is determined from the data obtained from the single-flank testing equipment or obtained from the digital simulation. In case of digital simulation, it is assumed that the gear is an ideally stiff body. This way the load conditions are omitted in the calculations of the gear work. Then the dynamic deviation could be determined through simple analysis of the dynamic deviation. The procedure is presented and explained in the Fig. 7.

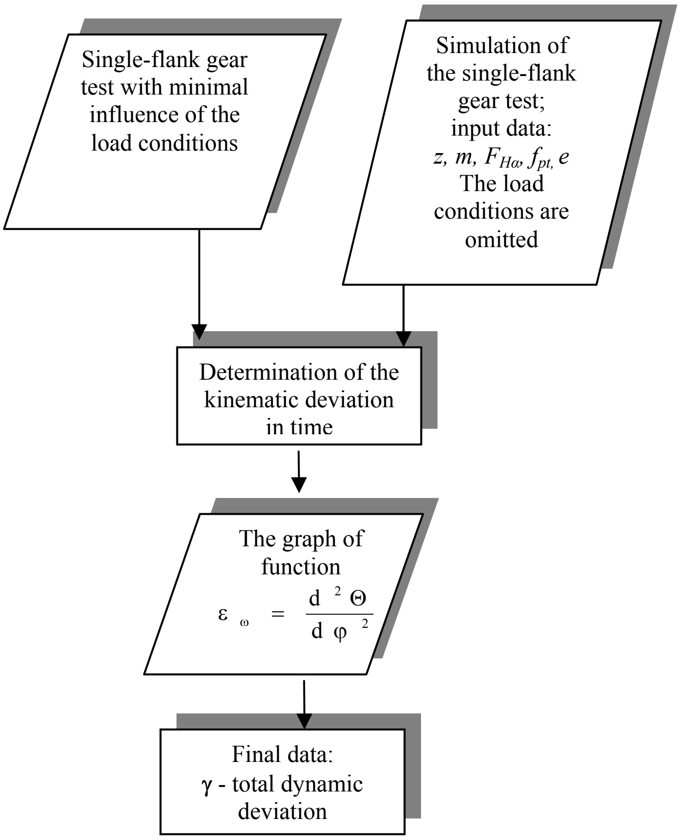


Fig. 7 Algorithm of the dynamic deviation

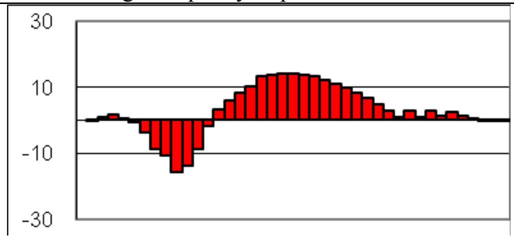
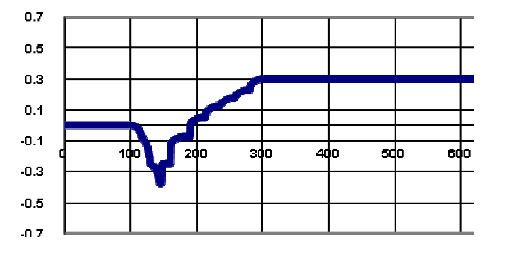
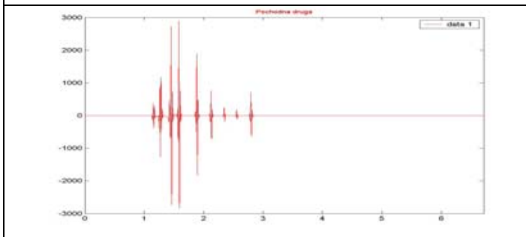
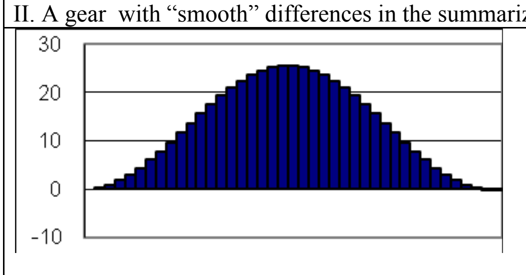
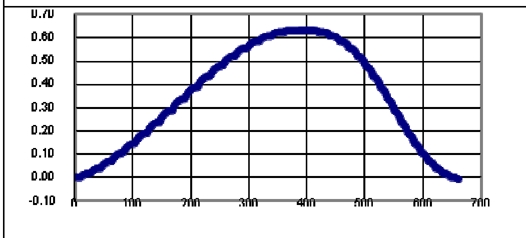
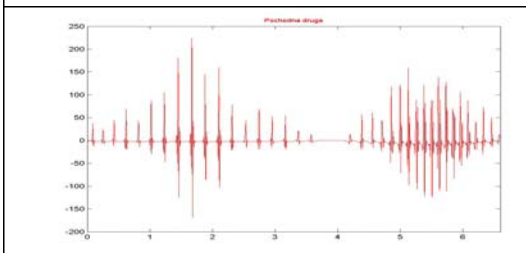
6. IMPACT OF THE MANUFACTURING ERRORS ON THE DYNAMIC DEVIATION

The performed simulations and analysis deal with the gears with a pitch deviations only. Here, the impact of the deviations of the profile and runout on the dynamic deviation were considered negligibly small.

The investigations revealed that the key importance for the dynamic behavior of the working gear transmission has got the inequality of the pitch. In the graph of the summarized pitches they are represented with vertical lines. In the Table 3, there are shown two cases, represented with the graphs. First three graphs represent the gear with high degree of the summarized pitch inequality, while the next three represent the one with "smooth" differences in the pitch F_p. In both cases, the pitch F_p has got almost similar value, as well as the value of kinematic deviation F_i'. Nevertheless, the difference between their total dynamic deviations is almost 20 times [6,8].

Those observations led to the following proposed recommendations on the inspection of the gears. The recommendations enable to predict the accelerations and the dynamical forces in the real gear transmission. Apart of the basic parameters like pitch deviation f_{pt} or total pitch deviation F_p, the particular attention must be paid to the summarized pitch of *k* teeth F_{pk}.

Table 3. The example of the dynamic deviation of a gear

I. Gear of high inequality of pitches	
	Summarized pitch deviation Fp z_I = 40 z_{II} = 40 *m* =2 mm *Fp* =27 µm
	Kinematic deviation F_i' Fi' = 27 µm
	Dynamic deviation
II. A gear with "smooth" differences in the summarized pitch F_p	
	Summarized pitch deviation Fp z_I = 40 z_{II} = 40 m=2 mm Fp=25,41 µm
	Kinematic deviation F_i' Fi' = 25,41 µm
	Dynamic deviation

The programs for digital simulations for single-flank gear testing and the further analysis of the dynamic deviation are designed for the operators who want to predict the behavior the real work of a gear transmission. The results of measurement obtained with e.g. coordinate measuring machine are processed and the results of simulation give the needed information.

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