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A COMPLEX MATHEMATICAL MODEL OF THE RESPIRATORY SYSTEM AS A TOOL FOR THE METROLOGICAL ANALYSIS OF THE INTERRUPTER TECHNIQUE

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Abstract − The paper presents the issue connected with the construction and simulations of the complex mathematical model of the respiratory system for the interrupter technique. Combining morphometric data for the consecutive segments of the object (assumption on symmetrical structure of the bronchial tree) with information on existence and proportions between the processes, proper for the interrupter experiment, resulted in an exact imitation of the occlusion conditions, both in the time and frequency domain. It is expected that the model will be a base for design of the simple and reliable diagnostic test dedicated to the respiratory mechanics measurement. From this point of view, the proposed complex mathematical analog of the respiratory system during airflow interruption is a forward model in the forward-inverse scheme of cognition.

Keywords: respiratory system, mathematical modelling, interrupter technique

1. INTRODUCTION

The paper concerns construction of a complex mathematical model of the respiratory system as an element of the general idea of the forward-inverse modelling [1] (Fig. 1).

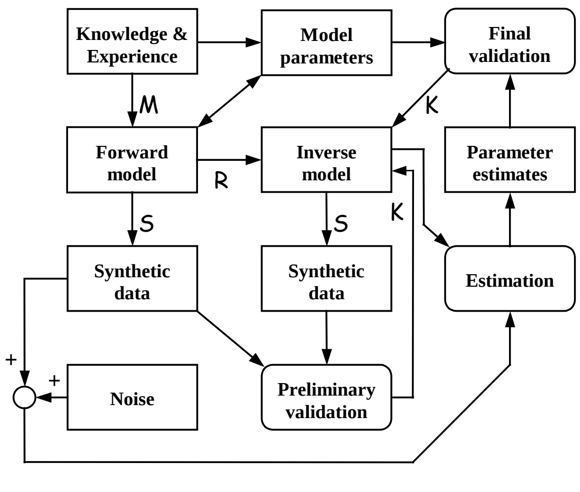

Fig. 1. The scheme of the forward-inverse modelling: M – modelling, R – model reduction, *S* – simulation, *K* – model correction [1].

It was assumed, that prepared linear analog would represent (structurally and parametrically) as wide as possible spectrum of the characteristics and behaviours proper for the object – the respiratory system. Particular attention has been paid to the abilities to reconstruction of the measurement condition during the respiratory mechanics evaluation by the airflow interruption technique (IT). It is worth to note, that despite of numerous propositions (linear or nonlinear, less or more precise, complex) for the other methods of the respiratory system evaluation, the modelling of dynamical interrelations during the interrupter experiment is an area of minimal and required activity, as so far [2, 3, 4, 5]. Meanwhile, as was suggested during the introductory investigations [3, 4, 6], completion with this missing link can contribute to the increasing reliability and repeatability of the interrupter measurement.

The airflow interrupter technique consists of short-term interruption of the airflow at the mouth and simultaneous measurement of the pressure change (ΔP_{ao}) and flow (Q_{ao}) in the mouthpiece. In the classic frame, the value of the diagnostic parameter – interrupter resistance $R_{int} = \Delta P_{ao} / Q_{ao}$ – is calculated, which is interpreted as a measure of the airways resistance. Numerous reports proved limited effectiveness of the conception [7, 8, 9, 10], hence the attempts to modify the basic measurement algorithm have appeared [3, 4, 11]. Attractiveness of the interrupter method is based on various arguments, the most important of which are:

- small invasiveness,
- short time of measurement,
- minimal requirements regarding patient co-operation,
- low hardware requirements, which give possibility to realise a portable device.

2. MATERIALS AND METHODS

The basic quantities that define the respiratory system are: resistance *R*, inertance *L* and compliance *C* (inversely proportional to elastance), often interpreted in the context of the electrical quantities *R*, *L*, *C* and the same circuit *RLC*. Modeling of the respiratory system concerns typically three essential structural areas: upper airways, bronchial tree and tissue segment.

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2.1. Model of the upper airways

During summarising investigations, the upper airways analog have been assumed as showed in Fig. 2.

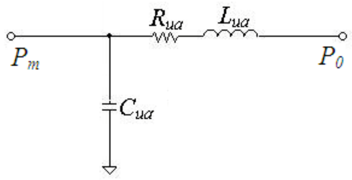

Fig. 2. Model of the upper airways.

The resistance R_{ua} of airflow in the upper airways was described accordingly to the experimental equation [12, 13]:

$$\Delta P_{ua} = R_u \cdot Q_{ua}^r \,, \tag{1}$$

where R_u is a factor which scales airflow resistance and a coefficient r = 1.5÷2 results from nonlinear character of pressure changes ΔP_{ua} in the upper airways. The value of gas compliance C_{ua} was determined as in [14, 15] and its inertance L_{ua} was set as double value of this quantity in description of trachea.

2.2. Model of the bronchial tree

The starting point for the work in this section is an assumption on symmetry of the structural representation of the bronchial tree according to Weibel [16] (Fig. 3).

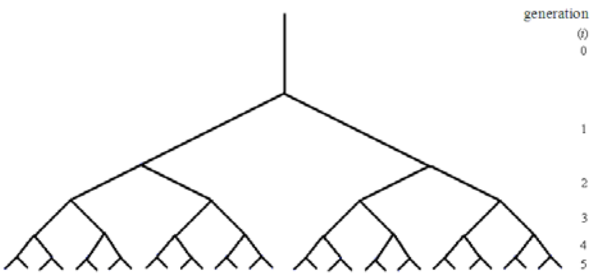

Fig. 3. Symmetrical dychotomy in the Weibel model of the bronchial tree.

In this frame, the model consists of 24 generations of the airways, where generation 0 correspond to the trachea and the last – 23 generation, it is the model of alveolar space. The whole number N of the airways in a given i-th generation equals to 2^i. The assumption on symmetrical dychotomy of the respiratory system enables to aggregate the quantities which describe airways tree in the following generations according to the formulas [17]:

$$R_i = R_i^a / N \,, \tag{2}$$

$$L_i = L_i^a / N \,, \tag{3}$$

$$C_i = C_i^a \cdot N \,, \tag{4}$$

where R_i^a is an analog of resistance connected with friction during medium moving through the individual airway, L_i^a depicts dynamical properties resulted from inertia of the gas in the airway and C_i^a is a measure of gas compression contained in a single airway of i-th segment.

Each of the quantities was calculated individually for the airways in the following generations appropriately to [18-24]. Distributions of R_i, L_i and C_i in the bronchial tree were presented in Fig. 4–Fig. 6.

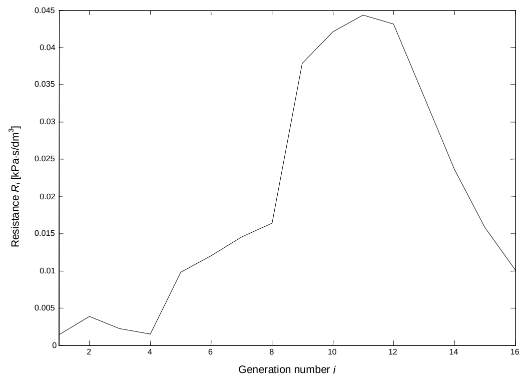

Fig. 4. Resistance R_i in the following generations of the airways tree.

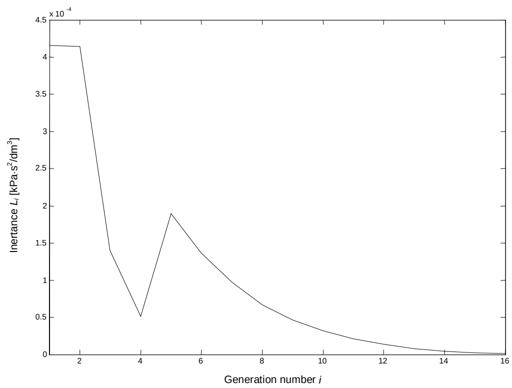

Fig. 5. Inertance L_i along the bronchial tree.

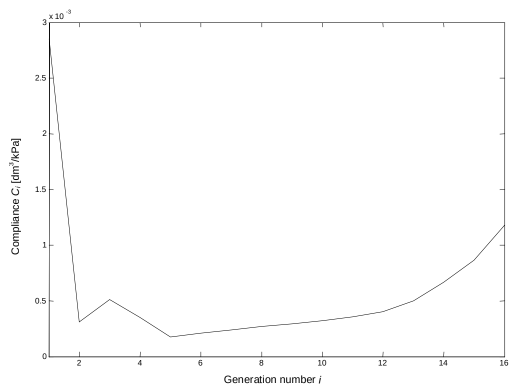

Fig. 6. Compliance in the i-th generations of the bronchial tree.

2.3. Model of lung tissue and chest wall

In an electrical formulation, the models of lung tissue and chest wall were showed in Fig. 7 and Fig. 8, respectively.

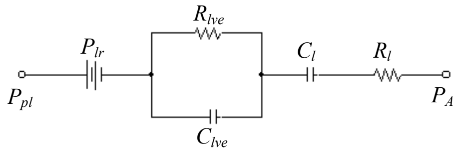

Fig. 7. Electrical equivalent of the lung tissue segment.

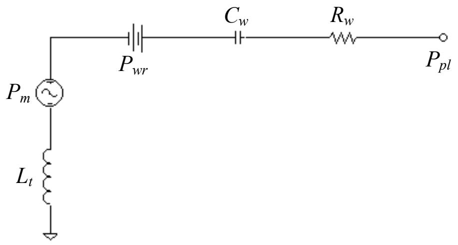

Fig. 8. Model of the chest wall.

For each circuit mathematical equations have been written. R_l and C_l=1/E_l in Fig. 7, it is viscous resistance and quasi-static measure of elastance E_l of the lung tissue [25, 26], respectively. Furthermore, the description (R_{lve}, C_{lve}) of the viscoelastic behaviour of the lung segment [25, 27, 28] has been included in the structure of lung tissue model. Additionally, the source P_{lr} has been introduced which points at occurrence of a pressure component, connected with tendency of the tissue segment to the elastic recoil between inspiration and expiration – the plastoelastic forces of retraction. Similarly, viscous resistance R_w, compliance C_w, adequate to the quasi-static elastance E_w, and the source P_{wr}, which models retractive forces in this part of the lung structure, have been defined for the chest wall section. The inertance L_t depicts the inertia, which depends on acceleration of the chest wall mass, and the excitation P_m reconstructs work of the inspiratory muscles in the chest wall model.

3. SIMULATIONAL COMPLEX MODEL

The analog of the interrupter valve (“shutter” type) – R_v resistance (6) – and the equivalent of the transducers unit (constant resistance R_{td}) have been additionally introduced in the simulational complex model.

$$R_v = \frac{k(l)}{\pi^2 r^4 \left[1 - \sin\left(\frac{2\pi}{T}\cdot t\right)\right]^2} \tag{6}$$

where:

$k(l)$ – coefficient proportional to lenght,
r – valve radius,
t – time of valve turning,
T – time.

The construction of the valve-transducer unit produced by Jaeger (Germany) has been reconstructed during research by fitting conductive output of the model $G = 1/(R_v + R_{td})$ to the conductance characteristics from [29].

Model of the structure of the respiratory system with defined properties has been applied in Matlab/Simulink. Some subsystems has been distinguished in it: model of the valve-transducer unit, upper airways model, model of the trachea (zero extra-thoracic generation), model of the conductive airways (generations 1-15), model of the acinar airways (generations 16-23), model of the lung tissue and the chest wall analog.

4. SIMULATIONAL INVESTIGATIONS OF THE COMPLEX MODEL

Wide spectrum of simulations has been conducted which verified model adequacy, both during spontaneous breathing and interrupter test. Results were analysed in the time and frequency domain. Example plots were showed in Fig. 9 and Fig. 10.

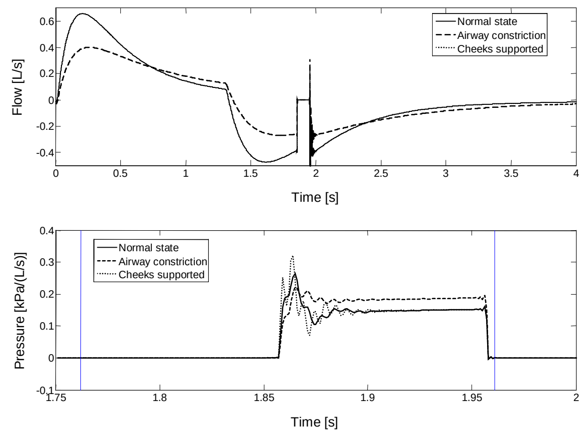

Fig. 9. Pressure P_{ao} and airflow Q_{ao} in the complex model of the respiratory system during simulations of the interrupter manoeuvre.

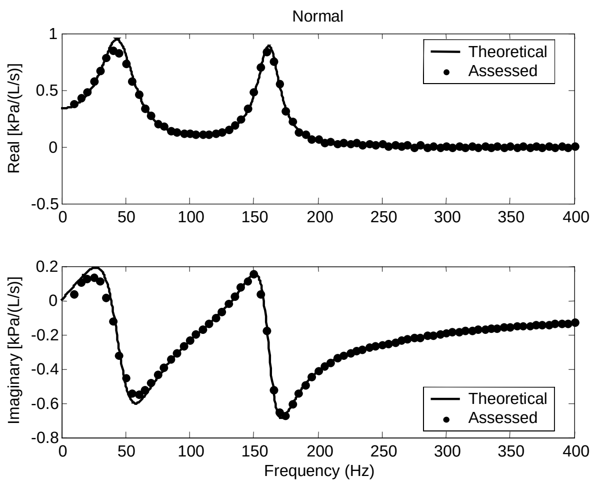

Fig. 10. Input impedance of the complex model measured in computer interrupter experiment and calculated theoretically.

5. SUMMARY

The procedure of the mathematical model construction of the respiratory system has been discussed in the paper. This complex structure, consisting of more than 100 parameters, is thorough documented by experimental facts. Adequacy of reconstruction of the interrupter experiment has been verified in computer simulations. Thereby, its usefulness to the metrological evaluation of the interrupter technique has been proved in the area of linear systems. It is worth to note that postulated analog completes missing link in the works on IT [3, 6] and can be a starting point for the nonlinear mathematical representations of the respiratory system, at the same time. Presented studies is a part of a developmental project aimed at construction of a tele-medical distributed system for the monitoring of patients suffering from airway diseases. To fulfil the preliminary assumptions, the research in the area of post-interrupter data processing, both in the time and frequency domain, is needed. In the context of forward-inverse problem, the complex mathematical model constructed in the paper is a valuable tool for the future computer and experimental analysis.

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