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POSITIONING ACCURACY OF NON-CONVENTIONAL PRODUCTION MACHINES

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Abstract − The paper deals with machines employing parallel-kinematics structures (PKS). They represent a relatively new generation of machine tools. Depending on the number of struts, the machines are referred to as hexapod or tripod machines. Such machines offer several advantages comparing to the conventional machine tools with serial kinematics, such as high flexibility, high stiffness, and high accuracy. It is very suitable for High-Speed-Machining (HSM), light machining and has received a wide interesting in manufacture industry. To achieve a desired positioning accuracy and stability, the static and dynamic properties of the machine must be searched and mathematically described. The calculation of the estimate of positioning deviation, including respective uncertainty and covariances, is much more complicated task comparing to the serial kinematics.

Keywords: parallel kinematic structures, positioning accuracy, measurement uncertainty

1. INTRODUCTION

The technology of the so called high-speed cutting (HSC) recently leaves the research laboratories and is introduced into the industrial practice. To achieve desired cutting speeds, new designs of machines must be developed. The classical three-dimensional structure employing the three commonly perpendicular axes usually avoids reaching the required cutting parameters, proving that HSC machines need to operate under the new designs.

The future trend brings an integration of technological operations in a single machine, thus requiring multiprofessional and multitechnological machines. This brings a need for new principles of production machines including new kinematic solutions. The closed parallel kinematic structure (PKS) of the Tricept types has already proved several advantages [1].

2. TRICEPT – A TYPICAL REPRESENTATIVE OF PKS

Seeing from the kinematics point of view, Tricept has a form of a fixed platform connected to the movable platform by three driven telescopic rods and a central rod without a drive (see Fig. 1). The central rod is rigidly connected to a movable platform, while the fixed platform connection enables a translation movement of a central rod without the possibility to rotate.

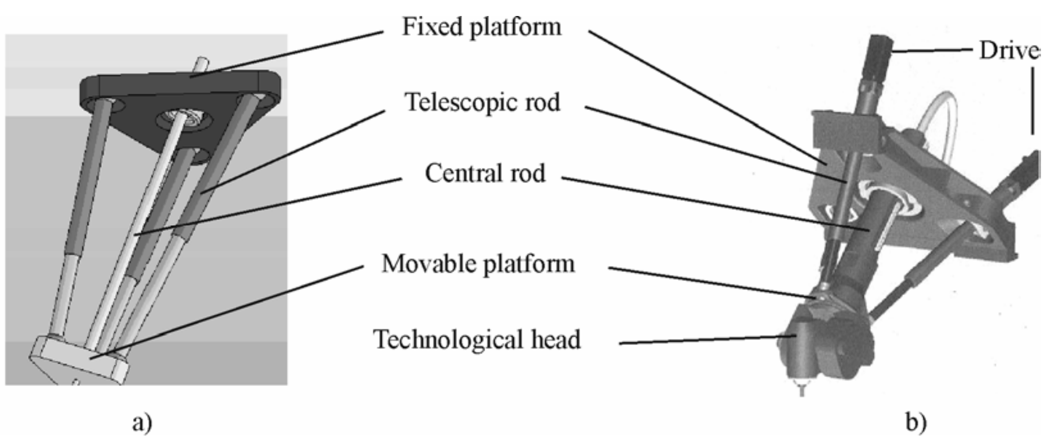

Fig. 1 Parallel kinematic structure of the Tricept type
a) computer model, b) design concept

When analyzing a Tricept work space, two influences were searched – distance of joints and position of joints at a telescopic rod. One must consider that location of a joint at a telescopic rod affects the Tricept design significantly. Such a location can vary from a position at a motor up to a position at an outer part that is farthest from the motor.

The location farthest from a motor gives an advantage of a minimum stress in telescopic rods. When keeping the other dimensions constant, the angle among axes of telescopic rods is biggest comparing to the other designs. On the other hand, this concept significantly decreases a workspace. The other extreme – the connection point at the position closest to the motor – increases a workspace in all axes.

The distance among individual joints at a movable platform has the similar influence as that at a fixed platform. It affects an angle among the rods (thus also the stresses in individual rods), the workspace, gear ratio (angular movement of the motor shaft against the amplitude of joint movement at a fixed platform).

The Tricept's resulting workspace is represented by an intersection of workspaces generated by individual rods. Thus it is an intersection of three similar disc-shaped spaces having the center of rotation at different places within a single cone. Disc-shaped spaces have the same orientation, parallel axes and their spherical points are located at a single defined circle.

The inhomogeneous workspace regarding to the positioning accuracy and the stiffness represents the main disadvantage of Tricept kinematics. The first step in analysis must be then determination of required extension of telescopic rod in relation to a desired position of a searched point (end point of an effector) [2].

3. TRICEPT'S MOTION EQUATIONS

To get a desired position of a technological effector, one must be able to calculate and to set up the required length of each telescopic rod. The created mathematical model for control of individual rods extension is based on analysis of a vector loop.

Figure 2 schematically shows Tricept together with a vector loop. The loop consisting of vectors $\overrightarrow{SP}$, $\overrightarrow{PL}$, $\overrightarrow{LB}$, $\overrightarrow{SB}$ describes the position of a point L that is important for calculation of a vector $\overrightarrow{LB}$ whose length is identical with the extension of a telescopic rod.

To calculate the point L coordinates, one has to know the coordinates of the point P first. The position of an central rod is defined by a pair of points S and Q. Tricept is located in a coordinate system in such a manner that S point is in its beginning. Point Q is the second point defining the position of a central rod and represents one point of the overall curve defining the position of Tricept end effector.

When the carrier moves, common position of points S and Q changes, i.e. the central rod is inserting into a central joint. Its part behind the central joint is not important as seen from the kinematics point of view.

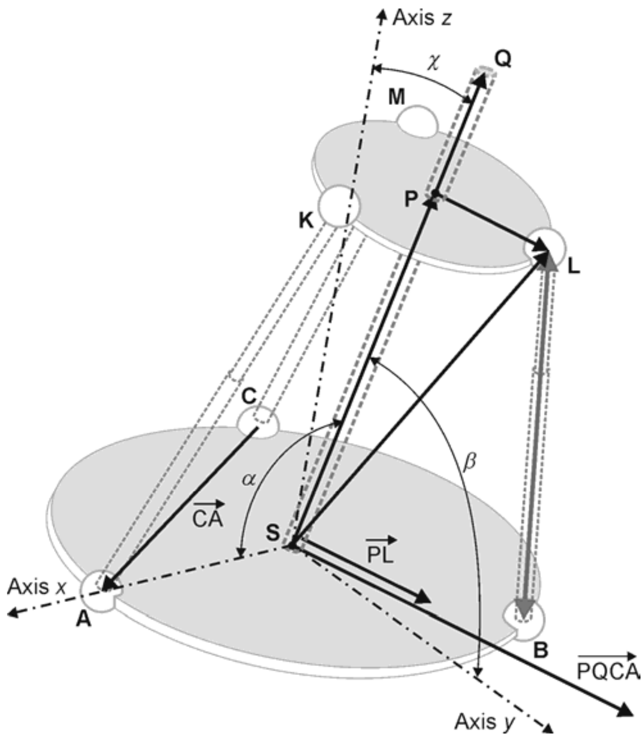

Fig. 2 Loop of kinematic vectors of Tricept

When creating the kinematic functions, the first step is to determine the position of the point P. They can be expressed as:

$$P_x = Q_x + QP_x = Q_x + |QP| \cdot \cos\alpha$$
$$P_y = Q_y + QP_y = Q_y + |QP| \cdot \cos\beta \qquad (1)$$
$$P_z = Q_z + QP_z = Q_z + |QP| \cdot \cos\chi$$

The second step is calculation of coordinates of the point L. The analytical geometry is employed to determine the coordinates of the point L:

$$\overrightarrow{SP} + \overrightarrow{PL} = \overrightarrow{SL} \qquad (2)$$

Length of one telescopic rod is:

$$\left|\overrightarrow{BL}\right| = \sqrt{\begin{array}{l} \left[\left\{\left[Q_x - |QP| \cdot \cos\alpha\right] + \left(\left[\left(\left(|QP| \cdot \cos\beta\right) \cdot CA_z\right) - \left(\left(|QP| \cdot \cos\chi\right) \cdot CA_y\right)\right] \cdot k_{PQCA} \cdot i\right)\right\} - B_x\right]^2 + \\ \left[\left\{\left[Q_y - |QP| \cdot \cos\beta\right] + \left(\left[\left(\left(|QP| \cdot \cos\chi\right) \cdot CA_x\right) - \left(\left(|QP| \cdot \cos\alpha\right) \cdot CA_z\right)\right] \cdot k_{PQCA} \cdot j\right)\right\} - B_y\right]^2 + \\ \left[\left\{\left[Q_z - |QP| \cdot \cos\chi\right] + \left(\left[\left(\left(|QP| \cdot \cos\alpha\right) \cdot CA_z\right) - \left(\left(|QP| \cdot \cos\beta\right) \cdot CA_x\right)\right] \cdot k_{PQCA} \cdot k\right)\right\} - B_z\right]^2 \end{array}} \qquad (3)$$

To reach the desired position of an end effector, i.e. desired position of the Q point, one must determine the function covering the calculation of lengths of all telescopic rods.

4. INFLUENCES AFFECTING THE POSITIONING ACCURACY

The positioning accuracy of effector of any production machine is defined as a closeness of coincidence between the actually reached position and the programmed position, set up by control system. When comparing the conventional serial kinematics with the parallel one, several basic differences occur. Therefore determination of the positioning accuracy represents a more complex and more difficult problem for the parallel kinematics [3].

Production tolerances, installation errors and displacements of individual joints cause deviations against the nominal kinematics parameters. As a result, if nominal values of those parameters are used in a control system, the actual position of the end effector will not fully correspond to the programmed (desired) one.

The positioning accuracy is significantly affected by geometry errors, flexibility errors and time-varying thermal errors. Geometric errors arise due to imperfect machining of Tricept individual parts, due to imperfect mutual positioning of individual parts during the assembly process or due to wear. Flexibility errors are basically errors of individual joints as well as errors generated by deflection of individual parts of the machine. They depend on actual position of the end effector. Thermal errors are generated by a thermal load and resulting thermal expansion of individual parts.

The flexibility of individual parts and their joints significantly affects the Tricept performance and its stability. The mass of all parts plus the external load cause deflection of individual elements of the device design and they displace the flexible joints thus affecting the overall performance of the device. The flexibility effect manifests itself through six differential changes – the three longitudinal displacements and the three rotational ones.

5. ESTIMATION OF DETERMINATION OF AN END POINT OF THE EFFECTOR

When estimating the positioning accuracy, a crucial problem is to determine function describing the positioning of effector (point Q) in relation to the extension of telescopic rods. Changing the extension of telescopic rods is the only possibility to affect the position of point Q. Unlike the serial kinematics, positioning leads to complex functions containing trigonometric elements thus resulting in non-linear solutions. The practical result can be observed that the positioning accuracy does not depend only on accuracy of extension of telescopic rods but on the position of point Q in a Tricept workspace as well.

To document the complexity connected to calculation of uncertainty of desired position of Q point, let us list individual geometrical parameters that contribute to overall uncertainty of reaching the desired position:

a) common position of joints against the centre of a fixed plate, i.e. distance of points AS, BS, CS,
b) common positions of joints at the fixed plate, i.e. distance of points CA, CB, BA,
c) common position of points against the centre at a movable plate, i.e. distance of points KP, MP, LP,
d) common position of joints at the movable plate, i.e. distance of points KM, KL, ML,
e) distance between the fixed and movable plate at a centre rod, i.e. distance of points SP,
f) distance between the end point of effector and a fixation point of a movable plate at a center rod, i.e. distance of points PQ,
g) lengths of individual telescopic rods, i.e. distance of points KA, MC, LB.

One has to consider that nominal values of individual parameters are affected by geometrical errors, thermal errors and stiffness errors. Let us designate f_Q the function used for determination of the position of point Q in dependence on parameters AS, BS to LB according to a previous list, i.e.

$$Q = f_Q(\mathrm{AS, BS, CS, CA, CB, BA, KP, MP, LP, KM, KL, ML, SP, PQ, KA, MC, LB}) \tag{4}$$

Uncertainty u_Q of determination of the Q point position is subjected to a law on uncertainty propagation that means [4]

$$u_Q^2 = \left(\frac{\partial f_Q}{\partial \mathrm{AS}}\right)^2 u_{\mathrm{AS}}^2 + \left(\frac{\partial f_Q}{\partial \mathrm{BS}}\right)^2 u_{\mathrm{BS}}^2 + \ldots + \left(\frac{\partial f_Q}{\partial \mathrm{L}B}\right)^2 u_{\mathrm{LB}}^2 + + \left(\frac{\partial f_Q}{\partial \mathrm{AS}}\right)\left(\frac{\partial f_Q}{\partial \mathrm{BS}}\right) u_{\mathrm{ASBS}} + \ldots \left(\frac{\partial f_Q}{\partial \mathrm{M}C}\right)\left(\frac{\partial f_Q}{\partial \mathrm{L}B}\right) u_{\mathrm{MCLB}} \tag{5}$$

where

AS, BS, .. LB are respective parameters,

$u_{\mathrm{AS}}, u_{\mathrm{BS}}, \ldots, u_{\mathrm{LB}}$ are uncertainties of particular parameters,

$u_{\mathrm{ASBS}}, \ldots, u_{\mathrm{MCLB}}$ are covariances among particular parameters.

One can observe that analytical representation of partial derivations of f_Q function must be found. Uncertainties u_{AS}, u_{BS}, ..., u_{LB} of respective parameters consist of individual partial sources, as stated in part 4. The problem is to determine the analytical function among the input parameters and the output point Q so that the uncertainty could be evaluated. To do so, a simplified matrix model is introduced.

6. MATRIX MODEL

When determining the position of an end effector, one has to consider individual factors affecting the resulting uncertainty of positioning (see part 5). The desired position is result of a function (4). The function uses individual distance parameters from which only lengths of the individual rods - KA, LB and MC - are adjustable during operation. The rest of parameters are represented by constant distances of individual elements of the Tricept. Those distances are adjusted (fixed) during the manufacturing of the Tricept and they remain constant during operation. Anyhow, accuracies of their determining (i.e. distances among the joints and the centre of fixed respectively movable platform) affect the overall uncertainty of the positioning of end effector.

When taking only adjustable parameters into account, following model can be written:

$$\mathrm{SQ} = f_1(\mathrm{KA, LB, MC}),$$

$$\alpha = f_2(\mathrm{KA, LB, MC}),$$

$$\beta = f_3(\mathrm{KA, LB, MC}), \tag{6}$$

$$\chi = f_4(\mathrm{KA, LB, MC}),$$

where individual parameters are designated according to the Fig. 2.

When transferring the model (6) into a matrix form, we obtain:

$$\boldsymbol{Y} = \boldsymbol{f}(\boldsymbol{X}) \tag{7}$$

where

$\boldsymbol{Y}$ is the matrix of output parameters,

$\boldsymbol{X}$ is the matrix of input parameters,

$\boldsymbol{f}$ is the design matrix, determining the relations among the input and output parameters,

$$\boldsymbol{Y} = \begin{pmatrix} SQ \\ \alpha \\ \beta \\ \chi \end{pmatrix}$$

$$\boldsymbol{X} = \begin{pmatrix} \mathrm{KA} \\ \mathrm{LB} \\ \mathrm{MC} \end{pmatrix}$$

$$\boldsymbol{f} = \begin{pmatrix} f_1(\mathrm{KA, LB, MC}) \\ f_2(\mathrm{KA, LB, MC}) \\ f_3(\mathrm{KA, LB, MC}) \\ f_4(\mathrm{KA, LB, MC}) \end{pmatrix}$$

The estimate of the output parameters is calculated as

$$\boldsymbol{y} = (\boldsymbol{A}^{\mathrm{T}}\boldsymbol{U}_x^{-1}\boldsymbol{A})^{-1}\boldsymbol{A}^{\mathrm{T}}\boldsymbol{U}_x^{-1}\boldsymbol{x} \tag{8}$$

Then the uncertainty of estimates of the output parameters (the covariance matrix $\boldsymbol{U}_y$) is calculated as

$$\boldsymbol{U}_y = \boldsymbol{A}\boldsymbol{U}_x\boldsymbol{A}^{\mathrm{T}} \tag{9}$$

where

$\boldsymbol{U}_y$ is the covariance matrix of output parameters,

$\boldsymbol{U}_x$ is the covariance matrix of input parameters,

$\boldsymbol{A}$ is the matrix of sensitive coeeficients,

$$\boldsymbol{U}_y = \begin{pmatrix} u^2_{\mathrm{SQ}} & u_{\mathrm{SQ},\alpha} & u_{\mathrm{SQ},\beta} & u_{\mathrm{SQ},\chi} \\ u_{\alpha,\mathrm{SQ}} & u^2_{\alpha} & u_{\alpha,\beta} & u_{\alpha,\chi} \\ u_{\beta,\mathrm{SQ}} & u_{\beta,\alpha} & u^2_{\beta} & u_{\beta,\chi} \\ u_{\chi,\mathrm{SQ}} & u_{\chi,\alpha} & u_{\chi,\beta} & u^2_{\chi} \end{pmatrix}$$

$$\boldsymbol{U}_x = \begin{pmatrix} u^2_{\mathrm{KA}} & u_{\mathrm{KA,LB}} & u_{\mathrm{KA,MC}} \\ u_{\mathrm{LB,KA}} & u^2_{\mathrm{LB}} & u_{\mathrm{LB,MC}} \\ u_{\mathrm{MC,KA}} & u_{\mathrm{MC,LB}} & u^2_{\mathrm{MC}} \end{pmatrix}$$

$$\boldsymbol{A} = \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}} = \begin{pmatrix} \frac{\partial f_1}{\partial \mathrm{KA}} & \frac{\partial f_1}{\partial \mathrm{LB}} & \frac{\partial f_1}{\partial \mathrm{MC}} \\ \frac{\partial f_2}{\partial \mathrm{KA}} & \frac{\partial f_2}{\partial \mathrm{LB}} & \frac{\partial f_2}{\partial \mathrm{MC}} \\ \frac{\partial f_3}{\partial \mathrm{KA}} & \frac{\partial f_3}{\partial \mathrm{LB}} & \frac{\partial f_3}{\partial \mathrm{MC}} \\ \frac{\partial f_4}{\partial \mathrm{KA}} & \frac{\partial f_4}{\partial \mathrm{LB}} & \frac{\partial f_4}{\partial \mathrm{MC}} \end{pmatrix}$$

7. CONCLUSIONS

The paper introduces several theoretical problems connected with determination of positioning accuracy of a special parallel kinematic structure – Tricept. Based on analysis of a vector loop, the sample calculation of a telescopic rod extension was presented, showing the dependence between the length of the telescopic rod and the position of an end effector. When introducing the law on uncertainty propagation, several problems occur during the calculation of uncertainty of the positioning deviation.

The presented procedure enables to simulate the uncertainty of positioning deviation of the end effector in any point Q of the Tricept workspace. This enables to calculate the theoretical capability of the machine design to reach the desired positioning accuracy, as prescribed in the design phase. To do so, one must know the geometry of the machine kinematics, properties of telescopic rods, manufacturing tolerances, etc. The performed analysis can help the designer to adjust precisely selected parameters to enhance the machine positioning accuracy [5].

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