XIX IMEKO World Congress
Fundamental and Applied Metrology
September 6−11, 2009, Lisbon, Portugal

MEASUREMENT IN A POINT VERSUS MEASUREMENT OVER AN INTERVAL

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Abstract − The paper reviews and discusses two strategies of digital measurements: measurement in a point and measurement over an interval. The first strategy, widely employed, has represented the backbone in measurement evolution and has become a standard method. For high accuracy measurements, it presents demanding requirements on technology regarding processing of measured signals. The second strategy, recently developed, carries clear advantages in three challenging areas of measurement theory and practice: measurement at high frequencies, measurement of noisy signals and measurements that require high linearity and high accuracy. These three advantages can be combined, as can be seen in the reviewed literature. This paper highlights the most important characteristics of the two measurement strategies.

Keywords: sampling, measurements, stochastic systems.

1. INTRODUCTION

Advanced measurement instrumentation is digital nowadays. Time-continuous signals are sampled and converted into discrete digital variables. In the conversion process, accuracy and speed are opposing requirements. Accurate measurements of low-level, noisy and distorted signals have been a challenging problem in the theory and practice of measurement science and technology.

A possibility for reliable operation of instruments with inherent random error has been researched since 1956 [1]. An inherent property of such an approach is a very simple hardware, which can operate very fast. It has been shown that adding a random uniform dither to an A/D converter input decouples measurement error from the input signal [2]. This dither also suppresses the measurement error due to both coarse A/D conversion and the external additive noise in the input signal.

Following this generic approach, several specific methods has been developed for measuring average DC inputs, AC inputs and/or distorted AC inputs. Several prototype and small-series commercial instruments has been realized and their measurement uncertainty can be extremely low [3-5].

Section 2 discusses issues of measurement in a point strategy, while section 3 presents the principles of measurement over an interval strategy.

2. MEASUREMENT IN A POINT

Measurement in a point is a separable approach to the measurement of the signal values and parameters.

2.1. Analog to digital conversion

The term “measurement” today is considered to be a discrete digital measurement, i.e. measurement in a point. In metrology jargon this is called sampling measurement method. The sampling method has two inherent sources of systematic measurement error: discretization in time and discretization in value. If the sampling theorem conditions are satisfied, discretization in time is eliminated as a source of measurement error. However, discretization in value always causes a systematic measurement error – it cannot be eliminated, but only can be, under certain conditions, reduced to an acceptable value.

The essence of the sampling method is as follows: in a theoretically infinitely short time interval (practically in an instant), a sample of an analogue measured variable is taken and in a time interval Δt this sample is converted into a number in a device called A/D converter. This time interval is related to a sampling frequency f_s , which is related to the upper frequency

$$\frac{1}{\Delta t} = f_s = 2 f_g \qquad (1)$$

where f_g is the highest frequency which appears in the measured signal, i.e. the upper limit of the frequency band of the measured signal.

As the aim is to capture fast-changing signals, it is important to have the shortest possible interval Δt. The fastest devices are the flash A/D converters, which perform the conversion within a single clock cycle and hence achieve $\Delta t \tilde{\ } 1\ ns$. The problem of flash A/D converters is the compromise between the resolution (number of bits) and hardware complexity. Resolution of flash A/D converters is up to 10 bits and the measurement uncertainty follows this. On the other hand, every additional bit of resolution doubles the hardware complexity. From that point of view, it is desirable to have a lower resolution. However, if the resolution is below 7 bits, the Bennett model of quantisation error does not apply any more [2]. In such a case the quantisation error cannot be treated as a white noise, which becomes a serious theoretical and practical problem.

To sum up this issue, precise and accurate A/D converters are slow, while fast A/D converters are imprecise and inaccurate. This is the central problem of measurement in a point - pronounced inaccuracy at high frequencies.

The second problem of measurement in a point is the treatment of noisy signals. The theory of discrete signals does not consider the quantisation error at all, but estimates the signal within the noise [6]. It has been shown that the signal is better estimated in the noise if the sampling frequency is higher. Hence fast A/D converters are crucial in this case as well.

The trend in the progress of measurements in a point is the development of fast high-resolution A/D converters. This is a big technology aim and task. It is a clear concept and a backbone of development not only in measurements, but also in telecommunications, control, power electronics and other branches of science and technology. The math that describes the measurement in a point is the discrete math, i.e. the theory of discrete signals and systems. The key mathematical tool is the algebra. For measurement of noisy signals, the theory of stochastic processes needs to be applied as well [6].

2.2. Processing

For the progress of measurement in a point strategy, development of A/D converters is not enough. The discrete values of signals need to be processed in order to obtain various signal parameters. The technology component that enables the fast and efficient processing of discrete values is the digital signal processor (DSP). When developed, the DSP was a big technology step for the improvement of measurement in a point. To eliminate the accumulation of the processing error, DSPs work with high-bit formats in floating point arithmetic. The newest commercially available Texas Instrument DSP, can achieve processing at an excellent 2.1 GFlops (2.1 billion of floating-point operations per second).

2.3. Concluding statement

The above indicates that methods and hardware are becoming standardised in discrete digital measurements, while the progress in signal parameter measurements is to follow. It seems that there is little to research but a lot to apply and standardise the already-discovered knowledge. In other words, the methodology progress is exhausted – there is only the technology evolution.

3. MEASUREMENT OVER AN INTERVAL

Measurement over an interval is an integral approach to measure the signal values and parameters.

3.1. Advantages

The measurement on a finite time interval can overcome some drawbacks and limitations of the measurement in a point approach. At the same time, it can preserve almost all the advantages of the measurement in a point, especially the huge amount of software developed over the years in all branches of science and technology.

The advantages of measurements on an interval are:

- measurement at high frequencies,
- measurement of noisy signals,
- high linearity and low uncertainty of measurement.

These features often appear simultaneously and can provide high-accuracy results in areas where it was not possible before [7-9].

As measurements on an interval represent a complement to the measurements in a point, they can be performed by low-resolution flash A/D converters. Therefore the sampling frequency can be the maximal frequency that the technology has achieved - currently around 500 MHz is possible.

3.2. One-channel instrument

The basic structure of Fig.1 can measure the integral or the average value of the input signal *y*. It is suitable for measurement of DC or slowly-varying quantities.

As the word length of the flash A/D converter quite short, the quantization error is rather large. In order to reduce the influence of the quantization error, a uniform random noise (dither, denoted *h* in Fig.1), is added to the measured signal *y* [2,10]. Dither is in the range of one quantum of the A/D flash converter and its average value is zero. Then, if the average value of the signal within the interval is measured, the quantization error satisfies the conditions of the Central limit theorem and for the Theory of samples [3]. The standard deviation of the quantization error decays with the increase in the number of samples. This means that the measurement uncertainty due to A/D conversion process reduces with the increase of the measurement time interval and/or sampling frequency [3-8, 11].

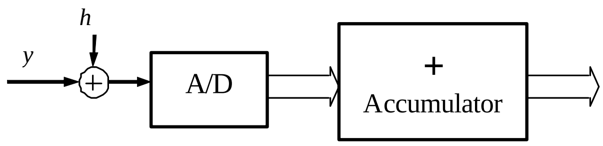

Figure 1: Basic structure for measuring the average value of the signal.

3.3. Two-channel instrument

The hardware can be extended by an additional channel, (a flash A/D converter and a random noise generator), and a multiplier, Fig. 2 [3,5].

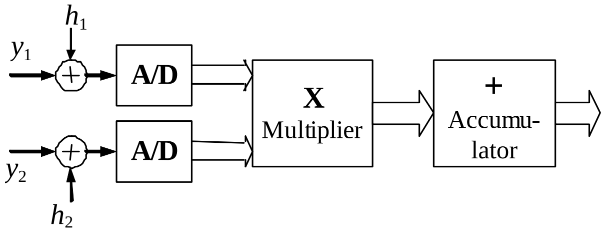

Figure 2: The principal structure of a 2-channel instrument

This opens up the possibility of measurement of the integral (i.e. the mean value) of a product of two physical quantities. This is directly applicable if energy (the integral) or average power (the mean value) are to be measured. The two signals that are multiplied can be voltage & current [4], flow & temperature, flow & pressure, force & velocity, etc.

If the same input signal is brought to the two input channels ($y_1 = y_2$), the signal r.m.s. value is measured [3,5]. If the dither signals h_1 and h_2 are uncorrelated, the ratio of the accumulator value and the number of samples gives the mean square value of the input signal.

As in the one-channel instrument, the standard deviation of the quantization error satisfies the conditions of the Central limit theorem and for the Theory of samples. For a very large number of samples, the measurement uncertainty can be extremely low [5].

3.4. Measuring signals in a transform domain

The input to the other channel of the multiplier can be a basic function from an orthonormed function set [7], say a function from the Fourier set [8]. In such a case, the average value of the accumulator represents the value of an appropriate coefficient of the signal decomposition in the orthonormed set. In this way, coefficients of the orthonormed transformation can be measured very accurately. The problem analysis that supports this statement is given in [5] and the main results are as given below. The only limitation is that the measured signal $y_1 = f_1(t)$ is band limited. Due to the introduced dither, for a large number of samples N in the measurement interval $T = t_2 - t_1$, the average error $\bar{e}$ due to quantization is zero and its variance is:

$$s_{\bar{e}}^2 \le \frac{1}{N}\left\{\frac{\Delta^2}{4}\frac{1}{t_2-t_1}\int_{t_1}^{t_2}\left[f_1^2(t)+f_2^2(t)\right]dt+\frac{\Delta^4}{16}\right\} \qquad (2)$$

When the A/D resolution of the two channels is different, and then (2) becomes:

$$s_{\bar{e}}^2 \le \frac{1}{N}\left\{\frac{\Delta_1^2}{4}\frac{1}{t_2-t_1}\int_{t_1}^{t_2} f_2^2(t)\,dt + \frac{\Delta_2^2}{4}\frac{1}{t_2-t_1}\int_{t_1}^{t_2} f_1^2(t)\,dt\right\}+\frac{\Delta_1^2\,\Delta_2^2}{16} \qquad (3)$$

If the resolution of channel 2 is at least 2 bits greater than in channel 1, then $\Delta_1^2 \ge 16\,\Delta_2^2$ and (3) reduces to:

$$s_{\bar{e}}^2 \le \frac{1}{N}\frac{\Delta_1^2}{4}\frac{1}{t_2-t_1}\int_{t_1}^{t_2} f_2^2(t)\,dt = \frac{1}{N}\frac{\Delta_1^2}{4}F_2^2 \qquad (4)$$

where F_2 is the L_2 norm of applied orthonormed function set. In the case of Fourier orthonormed set, where

$$a_i = \frac{2}{T}\int_0^T f_1(t)\cos iwt\,dt \qquad (5)$$

$$b_i = \frac{2}{T}\int_0^T f_1(t)\sin iwt\,dt\ ,\ \left(i = 0, 1, 2, ..., M\right) \qquad (6)$$

using Weierstrass approximation theorem, function $y_1 = f_1(t)$ can be approximated using trigonometric polynomial:

$$f_1(t) \approx \frac{a_0}{2}+\sum_{i=1}^{M}\frac{a_i\cos iwt + b_i\sin iwt}{2} \qquad (7)$$

For Fourier orthonormed set, $F_2 = \frac{1}{\sqrt{2}}$ stands for all coefficients, except for a_0, where $F_2 = 1$. Consequently, estimated measurement uncertainty u_i has the same value for every coefficient (except a_0):

$$u_i = \frac{1}{\sqrt{N}}\frac{\Delta_1}{2}\frac{1}{\sqrt{2}} \ge s_{\bar{e}} \qquad (8)$$

For a_0, it is $u_0 = \sqrt{2}\,u_i$.

If the sampling frequency f_s is 250 kHz in each channel, then N = 5000 samples are taken in every period of the fundamental 50 Hz frequency. If the A/D converter range is ±2.5 V and its resolution is 8 bits, then the ratio of u_i and range (2.5 V) falls between:

$$\frac{1}{2^{14}} > \frac{u_i}{2{,}5\text{ V}} > \frac{1}{2^{15}} \qquad (9)$$

For a correct representation of a decomposed input signal, a large number of coefficients need to be obtained simultaneously i.e. the order of trigonometric polynomial, *M*, should be as high as possible. This is achieved by using 2*M*+1 elementary instrument structures (two flash A/D converters, a multiplier and an accumulator) working in parallel. The beneficial feature is that only two dither generators are sufficient – one applied to channel 1 and the other to channel 2 of every parallel elementary structure.

If a large number of elementary structures were implemented in full, the hardware of the resulting multi-component instrument can become very complex and cumbersome. However, there is a neat simplification because the signals in channels 2 are known basic functions. Therefore the signals, the dither and the A/D converter outputs can be calculated in advance and stored in a memory. In this way, the A/D converter and the dither generator in channel 2 of every elementary instrument are physically eliminated from the hardware. The output from the memory is directly connected to the multiplier, as shown in Fig. 3 [7, 8]. This elementary structure has been named “stochastic digital processor of orthogonal transforms”.

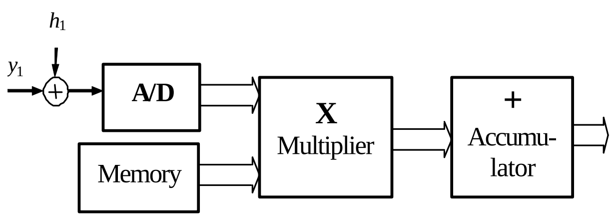

Fig 3. Schematic of the stochastic instrument
for measurement of one orthonormed component.

3.5. A practical implementation

The above instrument structure lends itself for and additional way of parallelising – several different input signals can be simultaneously decomposed. The prototype instrument of [8], shown in Fig.4, is implemented on a single chip and can perform harmonic analyses for the DC component and up to 49 harmonics (both cosine and sine components) in seven different input channels - three phase voltages and four (three line and neutral) currents. The measurement period (i.e. the spectrum refresh rate) can be set to an arbitrary number of fundamental cycles. If needed, a 20 ms measurement period can be chosen and it will give refresh rate of 50 times a second, but the accuracy will be limited. A longer measurement period will increase the total number of samples and hence significantly reduce the measurement uncertainty due to both A/D conversion process and signal noise. The resulting speeds of measuring orthonormed coefficients can be higher than using the FFT method and a standard DSP.

Fig 4. The prototype instrument for Fourier coefficients measurements, with the Fourier spectrum on the LCD display [8].

It has been shown in [7] that the resolution of the memorised dithered base functions should be at least two bits better than the resolution of the applied flash A/D converter. In such a case, for measuring orthogonal coefficients, the upper limit of the standard deviation is decoupled from the signal shape, the coefficient order and is identical for every coefficient, as shown by (4). The upper limit of the standard deviation depends only on the quantum size (resolution) of the applied flash A/D converter and on the norm of the transformation [7]. The result is that coefficients of orthonormed functions can be obtained accurately even without floating point arithmetic. As the short-word integer arithmetic is sufficient, a radical simplification of the necessary processing hardware is possible. Prototypes for measuring Fourier coefficients utilise a 6x8-bit and a 8x10-bit integer multiplying accumulators [7,8].

3.6. A view on mathematical tools

The mathematics that governs the operation all types of the above presented instruments is relatively simple but very interesting. Rather than discussing the details of mathematical modelling, here we want to point out an element of philosophy. The average value on an interval is actually the integral of the measured function. As the digital integration is a process of adding, the commutative principle applies. Therefore the elements of the sum can be added in a deterministic fashion (as they are sampled in time), in an arbitrary chosen order, or in an absolutely random sequence – it is completely unimportant. Consequently, from the average value point of view, the time within the interval can be treated as a stochastic variable with a uniform distribution. In this way, the problem of measurement over the interval can be classified in the Probability theory and the area of Statistic theory of samples.

For explanation of both simulation and experimental results, we use the Central Limit Theorem in a generalised form. We haven't found this in the available literature, but thousands of results confirm its application. The calibration equipment of a top-class accuracy [12] has confirmed the accuracy of prototypes and therefore the validity of the derived formulae.

4. CONCLUSION

The paper compares strategies of digital measurements in a point and measurements over an interval. It is highlighted that measurement in a point puts high demands on A/D converters, which cannot provide high sampling rate and high precision simultaneously.

The measurement over an interval strategy and its methods bring advantages for accurate measurements – high accuracies can be achieved even with a simple hardware. The simple hardware also enables easy paralleling of the elementary instrument structures and this provides possibilities for simultaneous measurements of several variables. Application of measurement over an interval methods to measure average signal values, signal products, root-mean-square values and signals in transform domains are presented. It is shown that very low measurement uncertainty can be obtained even with a coarse 8-bit A/D converter. A realized instrument for Fourier transform is also presented.

ACKNOWLEDGMENTS

The authors are sincerely grateful to colleagues, Ph.D. and M.Sc. candidates and technicians who have over the years contributed to the development of measurement over an interval theory, applications and realized instruments.

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