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ASSESSMENT OF THE APPLICABILITY OF THE WEIGHT VECTOR THEORY FOR CORIOLIS FLOWMETERS

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Abstract − The weight vector theory for Coriolis flow meters has been the subject of research presented by Hemp and co-workers in various articles. The underlying theory may not be easily understood. This paper explains the application of the weight vector theory for Coriolis flowmeters. The theory is applied to simple theoretical meter configurations consisting of a single straight pipe. The application of the weight vector approach is of relevance when investigating velocity profile effects, e.g., in Coriolis flow meters. Promising results have been found in recent literature showing the vulnerability of straight pipe Coriolis flowmeter configurations to velocity profile effects. The application of the weight vector theory is shown to be either limited to the investigation of few parameters or employs unrealistic boundary conditions and lacks comparative studies, making a more comprehensive study desirable. The usefulness of the weight vector theory to predict velocity profile effects for bended tube is not apparent from today's state-of-the-art literature, but of great interest for flowmeter manufacturers since bended tubes designs are frequently used in today's Coriolis flowmeters.

Keywords: weight vector, velocity profile effects, Coriolis flow meter

1. INTRODUCTION

The concept of the weight vector has been developed by J.A. Shercliff [1] and M.K. Bevir [2]. For electromagnetic flowmeters, this approach is extensively applied and experimentally validated. The weight vector theory for Coriolis flow meters has been developed with the purpose of predicting velocity distribution effects [3].

The extensive work done by J. Hemp has evidently had an impact on the development of the weight-vector approach for electromagnetic and Coriolis flowmeters. He initially worked with the theory for electromagnetic flowmeters, see for example [4,5], and has in the recent years put his focus on the application of the same theory for Coriolis flowmeters.

Generally speaking, the weight vector theory for Coriolis flowmeters provides means to express the phase difference between sensing signals as a function of the steady flow field in the tube and a weight-vector field, which depends on vibrational flows in the appropriately vibrating tube without the steady flow.

The basic weight vector theory for Coriolis flowmeters has been described in [6]. A technique is presented for developing an analytical expression for the weight vector. A first application of the same technique for Coriolis flowmeters is presented in [3] and [7]. The former shows the derivation of the weight vector theory for Coriolis flowmeters, whereas the latter presents the calculation of the Coriolis flowmeter sensitivity if the effect of fluid viscosity is to be taken into account. More recent studies are published in [8], [9] and [10]. A review of the state-of-the-art findings and open questions regarding velocity profile effects in Coriolis mass flowmeters is presented in [11]. The presented study related to the weight vector theory considers straight tube configurations and employs results from [3,6].

The weight vector theory for Coriolis flowmeter may not be understood from the related literature. To remedy this and encourage further application and testing, the theory is briefly revised, the necessary equations are determined and their application is illustrated for single straight tube configurations. The aim of this paper is to discuss the applicability of the weight vector theory, e.g. to predict velocity profile effects of Coriolis flowmeters, and point out its vulnerability.

2. WEIGHT VECTOR THEORY FOR CORIOLIS FLOWMETERS

The measuring tube is assumed to be a straight circular cylindrical shell with its geometry being defined by its length L, wall thickness h and inner tube radius R_i, Fig. 1. According to the weight vector theory for Coriolis flowmeters [10], the flow induced phase difference $\Delta\phi$ between sensor signals can be determined assuming linearity between the measured signals and the flow field

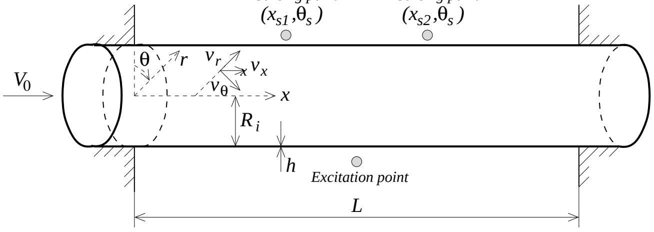

Fig. 1. Model of straight, clamped, fluid-conveying pipe.

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$$\Delta\phi = \int_{-\infty}^{\infty}\int_{0}^{2\pi}\int_{0}^{R_i} \mathbf{V}_0 \cdot \mathbf{W}_\phi \, dr\, d\theta\, dx \tag{1}$$

where $\mathbf{V}_0$ is the steady fluid velocity vector in absence of pipe vibrations and $\mathbf{W}_\phi$ the weight vector for the phase difference. The weight vector depends on certain vibrational flow fields in the absence of steady flow. The integral is taken over the entire volume of the fluid. The weight vector for the phase difference $\mathbf{W}_\phi$ is defined as [10]

$$\mathbf{W}_\phi = \mathrm{Im}\left(\frac{\mathbf{W}}{v_{r1}^{(1)}(x_{s1},\theta_s)}\right) \tag{2}$$

where $\mathbf{W}$ represents the weight vector and $v_{r1}^{(1)}(x_{s1},\theta_s) = i\omega u_r^{(1)}(x_{s1},\theta_s)$ the radial tube velocity of the working mode at the sensing point (x_{s1},θ_s) with ω being the angular operation frequency and $u_r^{(1)}(x_{s1},\theta_s)$ the radial displacement of the working mode. The weight vector $\mathbf{W}$ can be defined using [3,6,10]

$$\mathbf{W} = -\rho\left[\left(\mathbf{v}^{(2)}\cdot\nabla\right)\mathbf{v}^{(1)} - \left(\mathbf{v}^{(1)}\cdot\nabla\right)\mathbf{v}^{(2)}\right] \tag{3}$$

where ρ is the density of the fluid and $\mathbf{v}^{(1)}$ and $\mathbf{v}^{(2)}$ are fluid vibrational velocity fields. The field $\mathbf{v}^{(1)}$ is a result of the tube vibration without steady flow in the symmetric working mode, i.e. driven by a central force. The field $\mathbf{v}^{(2)}$ results from the antisymmetric tube vibration, without the presence of a flowing fluid, driven by equal and opposite unit forces applied at the sensing points (x_{s1},θ_s) and (x_{s2},θ_s).

3. APPLICATION EXAMPLES

To exemplify the application of the theory presented in section 2, two examples from the state-of-the-art literature will be summarized. Major results will be presented and a reflection will be given on the value of these examples with respect to a future application on other Coriolis flowmeter designs.

3.1. Determination of weight vector and phase difference for straight Coriolis flowmeter with non-supported ends

Hemp [8] investigated a straight Coriolis flowmeter, consisting of a single tube defined by the tube length L and the inner tube radius R_i, with non-supported free ends infinitely close to but unattached to adjacent piping. Neglecting viscosity and compressibility of the fluid, the equations for $\mathbf{v}^{(1)}$ and $\mathbf{v}^{(2)}$ are

$$i\omega\rho\,\mathbf{v} = -\nabla p \tag{4}$$

$$\nabla\cdot\mathbf{v} = 0 \tag{5}$$

with p being the pressure on the fluid. Equation (4) and (5) correspond to the momentum and continuity equation derived by employing mass conservation and Newton's second law on a fluid element.

Equations (4) and (5) have the approximate locally rigid tube solutions

$$\begin{aligned} &v_r = V\cos\theta;\, v_\theta = -V\sin\theta;\, v_x = \Omega\, r\cos\theta; \\ &p = -i\omega\rho V r\cos\theta \end{aligned} \tag{6}$$

where V is the local linear velocity of the tube and Ω the local angular velocity of the tube. Inserting (6) into (3) results in

$$\mathbf{W} = \rho\left(V^{(1)}\Omega^{(2)} - V^{(2)}\Omega^{(1)}\right)\mathbf{k} \tag{7}$$

indicating that the weight vector away from the tube ends is independent of r and θ.

Assume a fully developed flow velocity profile, i.e. the phase shift $\Delta\phi$ can be determined using an axisymmetric velocity profile $\mathbf{v} = v(r)\,\mathbf{k}$. Equation (1) turns out to be [8]

$$\Delta\phi = \int_0^{R_i} 2\pi r\, v(r) W_\phi(r)\, dr \tag{8}$$

where the axisymmetric weight function for phase shift $W_\phi(r)$ is defined using (2)

$$W_\phi(r) = \mathrm{Im}\left(\frac{W(r)}{(v_x)_P}\right) \tag{9}$$

with the axisymmetric weight function $W(r)$ being

$$W(r) = W_0(r) + W'(r) \tag{10}$$

with

$$W_0(r) = \rho\int_0^L \left(V^{(1)}\Omega^{(2)} - V^{(2)}\Omega^{(1)}\right) dz \tag{11}$$

and

$$W'(r) = 2\rho\left(V^{(1)}\Omega^{(2)} - V^{(2)}\Omega^{(1)}\right)\Big|_{z=0} b\tilde{W}(r) \tag{12}$$

where $\tilde{W}(r)$ is the end-effect axisymmetric weight function with

$$\tilde{W}(r) = -0.941 + 0.816\left(\tfrac{r}{b}\right)^2 - 0.214\left(\tfrac{r}{b}\right)^4 + 0.551\left(\tfrac{r}{b}\right)^6 \tag{13}$$

As a final result [8] states, that the phase difference $\Delta\phi$ neglecting end effects can be determined using

$$\Delta\phi = \dot{M}\,\frac{L^3\omega}{EI}\,F\left(\tfrac{\zeta}{L}\right) \tag{13}$$

where $\dot{M}$ is the mass flow rate, EI the tube rigidity, ζ the sensor position and $F(\zeta/L)$ the non-dimensional sensor position function, cf. [8].

The study presented in [8] illustrates, in a straight forward way, the application of the weight vector theory to determine an expression for the phase difference between sensor signals $\Delta\phi$ of a straight tube Coriolis flowmeter.

The chosen unrealistic boundary conditions, i.e. unsupported pipe ends unattached to adjacent piping, are a major handicap of the study. A more realistic investigation would incorporate clamped or at least hinged pipe ends, since these are better representations of actual Coriolis flowmeter designs. The reason for choosing the investigated boundary condition is not stated in [8]. The cause for this could for example be, that alternative boundary conditions, e.g. simply-supported pipe ends, would just complicate the results without changing the conclusions, or that the theory presented in [8] simply does not hold for other boundary conditions.

In addition, the developed formulas have not been illustrated by numerical calculations. The results from [8] have neither been compared to nor confirmed by analytical or numerical results obtained with alternative solution procedures. On the basis of the information given in [8], it cannot be concluded whether the presented formulas are beneficial or not.

3.2. Weight vector study of velocity profile effects in straight tube Coriolis flowmeter

A straight Coriolis flowmeter with clamped ends is considered in [10], Fig. 1. The measuring tube has the following dimensions inner tube radius R_i = 10 mm, wall thickness h = 0.5 mm and length L = 200 to 600 mm and material properties density ρ = 4510 kg/m^3, Young's modulus E = 102.7 GN/m^2 and Poisson's ratio $\nu = 0.34$. The results are presented for the working mode, with the distance between the sensing points being $s = L/2$ and in terms of variations of the flowmeter's mass flowrate sensitivity [10]

$$K = \frac{\Delta\phi}{q_m} \tag{14}$$

corresponding to the ratio between the phase difference $\Delta\phi$, determined using (1), and the mass flowrate q_m

$$q_m = \pi \rho V_0 R_i^2 \tag{15}$$

where V_0 is the mean flow velocity. In [10] the velocity profile effect is presented as variations in the ratio between the mass flowrate sensitivities for chosen velocity profiles K and for the flat, plug-flow profile K_0,

$$\frac{K}{K_0} = \frac{\int_0^{R_i} r V_{x0}(r)\, \bar{W}_x(r)\, dr}{V_0 \int_0^{R_i} r\, \bar{W}_x(r)\, dr} \tag{16}$$

where $\bar{W}(r)$ is the axisymmetric weight function, see [10]. Equation (16) is illustrated by Fig. 2, which shows variations of the mass flowrate sensitivities with aspect ratio L/R_i for laminar and turbulent flows, respectively K_{lam} and K_{turb}, relative to the sensitivity for the flat velocity profile K_0, assuming a circumferential mode where the tube cross-section is not deformed during the tube vibration. From Fig. 2 it can be seen that the ratio between the turbulent flow and flat velocity profile sensitivity is almost 1 for long tubes. This means that there is no difference between assuming a simple plug flow rather than a more realistic turbulent flow to determine the flowmeter sensitivity for long tubes using weight vector theory. This does not apply for short tubes as it can be seen in Fig 2. Similar conclusions are drawn when laminar flow is assumed. For short tubes, the sensitivity pre-

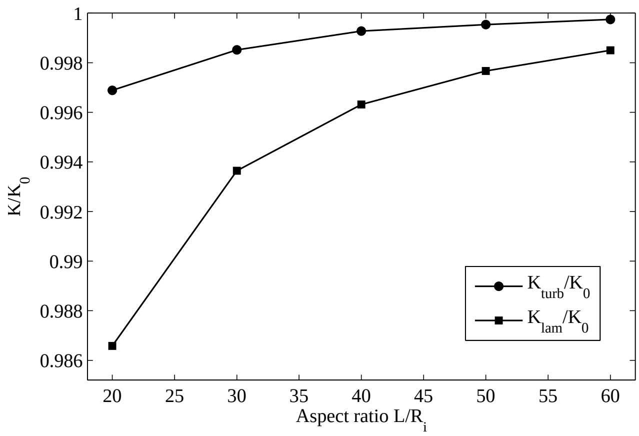

Fig. 2. Variations of mass flowrate sensitivities versus aspect ratio L/R_i for laminar and turbulent flow, respectively K_{lam} and K_{turb}, (relative to sensitivity for plug flow K_0) in case of circumferential mode with non-deformed cross-section during tube vibration, adapted from [10].

dicted using a plug flow assumption is larger than the sensitivity predicted using the laminar flow assumption. This also applies for long tubes, however the difference not as pronounced as for short tubes.

Table 1 shows mass flowrate sensitivities calculated for the two lowest circumferential modes, i.e. assuming the tube cross-section to be, respectively, non-deformed and deformed during the tube vibration, and for different tube aspect ratios using a direct and the weight vector solution procedure. This corresponds to a quantitative test of the theory for a few parameters. The direct solution procedure is described in [10]. It can be seen in Table 2 that the results, with a few exemptions, are in agreement. This illustrates the applicability of the weight vector theory as it is presented in [10].

The applicability of the weight vector theory to determine the sensitivity of a straight Coriolis flowmeter has been shown in [10]. A quantitative test employing a few parameters shows agreement between the results obtained by the weight vector theory and a direct solution procedure. Indications are found that the simple plug flow assumption can be used to estimate the sensitivity in case of high aspect ratios, e.g. long tubes. Furthermore it is shown that short tubes are more vulnerable to velocity profile effects than long tubes. The weight vector theory predicts a higher sensi-

Table 1. Comparison of the mass flowrate sensitivities from the direct and the weight vector solution procedure, adapted from [10]. Flat velocity profile with ρ = 1000 kg/m^3 and V_0 = 1 m/s.

Aspect ratio α	Mass flowrate sensitivity K_0 (rad/(kg/s))			
	Non deformed tube cross-section		Deformed tube cross-section	
	Direct solution	Weight vector	Direct solution	Weight vector
20	$4.349 \cdot 10^{-3}$	$4.349 \cdot 10^{-3}$	$2.996 \cdot 10^{-2}$	$2.996 \cdot 10^{-3}$
40	$7.348 \cdot 10^{-3}$	$7.348 \cdot 10^{-3}$	$1.705 \cdot 10^{-1}$	$1.705 \cdot 10^{-1}$
60	$1.062 \cdot 10^{-2}$	$1.062 \cdot 10^{-2}$	$4.259 \cdot 10^{-1}$	$4.262 \cdot 10^{-1}$

tivity when shifting from laminar to turbulent flow, which agrees with practical experiences from a Coriolis flowmeter manufacturer. Compared to [3,8], [10] offers indications that the weight vector theory in fact can be used to evaluate velocity profile effects for Coriolis flowmeters. It is apparent from the results presented in [10], that velocity profile effects cannot generally be neglected for Coriolis flowmeters, especially when a short tube design is employed. This is valuable information, since velocity profile effects often are ignored when designing Coriolis flowmeters. The work [10] leaves open questions, since it does not investigate the influence of other parameters, e.g. alternative boundary conditions or curved measuring tube shapes, on velocity profile effects.

4. DISCUSSION AND CONCLUSION

The early literature regarding the weight vector theory for Coriolis flowmeters does not provide sufficient information to enable a straight-forward application of the theory on Coriolis flowmeters. The usefulness of the weight vector theory to evaluate velocity profile effects for Coriolis flowmeters is not shown.

In the recent literature, promising results have been published showing that especially short tube designs are vulnerable to velocity profile effects, so that velocity profile effects cannot generally be neglected for Coriolis flowmeters. However the quantitative test, to compare the results from weight vector theory calculations to results obtained by direct solution procedures, is limited to a few parameters, so a more comprehensive study is necessary.

The major lack of the state-of-the-art weight vector theory for Coriolis flowmeters is that it is limited to straight pipes with more or less realistic boundary conditions.

Even though it would be of great interest for manufacturers of Coriolis flowmeters with bended tube designs, the usefulness of the weight vector theory to predict velocity profile effects of bended tube configurations is not apparent from the today's literature. A first step in this direction is made by J. Hemp, who has determined the weight vector for the three straight sections of rigid u-tube flowmeter, however, without determining the weight vector in the curved corners of the meter, cf. [3,6], which would lead to a more complicated expression for the weight vector. The three obtained constant expressions for the weight vectors are parallel to the tube axis and pointing in the flow direction in each straight section. This indicates that the influence of bended tubes cannot be neglected when determining the weight vector for bended tube designs. If it could be shown, that the weight vector theory for Coriolis flowmeters is also useful to study velocity profile effects in bended tube configurations, this would provide a powerful tool, e.g. for flowmeter manufacturers. Its major advantage will be that it can validate and possibly replace the time-consuming and computational demanding simulations which are used today, i.e. numerical fluid-structure-interaction simulations.

The steady flow assumption indicates that the presented theory is only valid when the flow in the flowmeter is laminar. However, in real Coriolis flow meter applications the flow is usually turbulent. J. Hemp argues in [3], that the weight vector theory probably still is valid, since filtering of the sensor signals should remove the effect of turbulence related velocity fluctuations. An experimental validation of this statement using real Coriolis flowmeters is not apparent from the literature. Others have used mathematical expressions for describing turbulent velocity profiles, cf. [10], and used these as input in the presented theory. This approach gives promising results. However this approach does not replace the necessity of further investigations to confirm the applicability of the weight vector theory in case of turbulent flow. Numerical methods and/or experiments with real Coriolis flowmeters can, in this context, be tools to employ.

Computational fluid dynamics can be used to clarify whether fluid properties, e.g. viscosity and compressibility, and pipe characteristics which influence the fluid flowing in the vibrating pipe, e.g. internal pipe wall roughness, can be neglected or how they might change the weight vector theory for Coriolis flowmeters. Under certain assumptions, e.g. neglecting the effect of viscosity, J. Hemp argues based on ultrasonic flowmeter theory, that the expression for determining the weight vector is the same for compressible and incompressible flow [3]. This indicates that neglecting the effect of viscosity has an influence on the applicability weight vector theory for Coriolis flowmeters. This indication should be checked to test it and its extend, e.g. by using numerical methods, since it has not been studied further according to the know literature.

To sum up, it has been shown that the weight vector theory is applicable for predicting velocity profile effects in Coriolis flowmeters. The theory is however vulnerable, since comprehensive studies are missing and realistic tube designs and boundary conditions have not been investigated. This leads to the still open question: Can the weight vector theory for Coriolis flowmeters be easily applied to real Coriolis flowmeter designs, or not? The theory seems to hold a significant potential but also does not seem straight-forward in practical application involving real Coriolis flowmeters. The required fluid vibrational velocity fields may, e.g., not be readily set up, and may even involve computational demanding and time-consuming numerical simulations.

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