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USING SINUSOIDAL INSTEAD OF TRIANGULAR STIMULUS SIGNALS ON THE IEEE 1057 STANDARD RANDOM NOISE TEST OF ADCS

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Abstract – The random noise test of analogue to digital converters (ADCs) recommended by the IEEE 1057 Standard for Digitizing Waveform Recorders recommends the use of a triangular signal to stimulate the ADC under test. Here it will be shown that a sinusoidal stimulus signal can be used instead. This gives greater flexibility when carrying out the test and allows sine fitting algorithms to be employed to determine the initial phase of the two data records that need to be acquired. The knowledge of this initial phase can be used to align the two records and reduce the errors introduced by additive noise in the instant of acquisition trigger.

Keywords: Analogue to Digital Converter; Random Noise Test; Sine Wave.

1. INTRODUCTION

The standard deviation of the random noise present in an analogue to digital converter (ADC) is an important parameter used to describe the performance of ADCs and to choose the proper ADC to use in a given application. The knowledge of the noise standard deviation in a test setup is also needed when performing other ADC tests, namely the Standard Static Test [1] and the Standard Histogram Test [1-7], for the determination of the error and precision of the ADC parameters estimated with them.

In [8] an analysis was presented regarding the precision of the estimates obtained with this test. An expression was also proposed for the minimum number of samples required to guarantee a certain bound on the uncertainty of the results. This is important in order to minimize the duration of the test because the right number of samples required can be calculated.

The test, as describe in [1], section 8.6.2, consists in acquiring synchronously two sets of a certain number of samples (M). The noise standard deviation (σ) is then estimated from the root mean square of the difference (*msd*) between the output codes of those two sets. If the noise standard deviation is high enough, a null input voltage is sufficient to perform the test, if not, a triangular stimulus signal should be used.

In this paper it is claimed that a sinusoidal stimulus signal can also be used. This, in itself, gives greater flexibility when doing the test. The main benefit, however, is that it allows traditional sine fitting algorithms to be used to determine the initial phase of the two records of data. Recall that two records need to be acquired so that the values of the samples can be subtracted from each other in order to eliminate the contributions of systematic errors like ADC non-linearity, gain and offset error and stimulus signal distortion. What remains are random effects like random additive noise. Note that other random errors, like amplitude or phase noise in the stimulus signal or jitter in the ADC will contribute to the end result of the test. For this reason the value estimated by this test is just an upper bound on the amount of additive random noise present in the ADC under test. In order for the systematic errors to cancel out, the two records should be perfectly aligned, that is, the acquisition should start at exactly the same instant relative to the stimulus signal period. In order words, the two records should have the same initial phase. To achieve this, the IEEE 1057 standard recommends that the start of the acquisition be triggered by the stimulus signal voltage. This triggering, however, is affected by the amplitude noise present since the instant of the first sample will depend not only on the ideal value of the stimulus signal, as it should, but also on the amount of random voltage noise that happens to be present.

Note that the curve fitting could also be done, in principle, on a triangular signal [9]. However triangular fitting procedures are not as straight forward as sine fitting ones.

In the following it will be demonstrated that a sinusoidal stimulus signal can be used and that the corresponding estimator is the same as the one used for the triangular stimulus signal. This is the goal of the paper. The details of exactly how the records can be aligned using the sine fitting information and the assessment of how much better is this procedure in comparison to the tradition one will be left for a future publication.

Section 2 deals with the variance of the ADC output codes considering three types of stimulus, namely, continuous (DC), triangular and sinusoidal. In section 3, the estimator for the sinusoidal case is derived and compared to those of the DC and triangular stimulus cases. Finally, in section 4, some conclusions are drawn.

2. VARIANCE OF THE ADC OUTPUT CODES

2.1. DC Stimulus Signal

Random additive noise in ADCs, as described in [1], is a non-deterministic fluctuation of the ADC output and is described by its frequency spectrum and statistical properties. It is usually considered that the noise present is white (flat frequency spectrum), presenting a stationary probability density function and that the noise is additive and independent of the stimulus signal.

Due to the presence of random noise at the ADC input, the output code (k) can be considered a discrete random variable which can assume any value between 0 and $2^{n_b}-1$ for a n_b-bit ADC.

When the additive noise standard deviation is higher than the ADC ideal code bin width (Q) the suggested method in [1] is to short circuit the ADC input and acquire two sets of samples (ka_j and kb_j) and subtract the codes obtained. This eliminates fixed errors of the ADC but preserves the random nature of the output codes.

Here normalized voltages, expressed in LSB (least significant bits) units, will be used, by dividing the voltages with the ideal ADC code bin width Q. The normalized stimulus signal voltage is represented by y and the normalized random noise voltage is represented by r. The normalized sampled voltage at instant t_j is thus given by

$$u(t_j)=y(t_j)+r(t_j). \tag{1}$$

Considering that the normalized additive noise has a null mean and a standard deviation represented by σ_r (σ/Q), the sampled voltage, which is also a random variable, has

$$\mu_u = y \quad \text{and} \quad \sigma_u=\sigma_r. \tag{2}$$

Being the additive noise normally distributed, the sampled voltage probability density function is [10]

$$f_u(u\,|\,y)=\frac{1}{\sqrt{2\pi}\sigma_r}\cdot e^{-\frac{(u-y)^2}{2\sigma_r^2}} \tag{3}$$

and its distribution function is [10]

$$F_u(U\,|\,y)=\int_{-\infty}^{U} f_u(u)\cdot du=\frac{1}{2}+\frac{1}{2}\cdot erf\left(\frac{U-y}{\sqrt{2}\cdot\sigma_r}\right). \tag{4}$$

Due to the subtraction of the codes obtained with the two sample sets, it is appropriate to consider the ADC as having an ideal behaviour since any fixed errors were eliminated by the subtraction and random errors can be considered as being present in the stimulus signal input.

The probability p_k of a sample having output code k is equal to the probability of the sampled voltage being equal to or lower than the transition voltage T[k+1] and equal to or greater than transition voltage $T[k]$ (for the middle codes):

$$p_k=P\{U[k]\le u\le U[k+1]\} \quad , \quad k=1,\ldots,2^{n_b}-2, \tag{5}$$

where the normalized transition voltage $U[k]=T[k]/Q$ was used. The probability p_k can thus be expressed with the help of the sampled voltage distribution function:

$$p_k(y)=\begin{cases}F_u(U[1]\,|\,y) & , \quad k=0\\ F_u(U[k+1]\,|\,y)-F_u(U[k]\,|\,y) & , \quad k=1,2,\ldots,2^{n_b}-2\\ 1-F_u(U[2^{n_b}-1]\,|\,y) & , \quad k=2^{n_b}-1\end{cases} \tag{6}$$

The mean, second moment and variance of the output codes are, by definition [10],

$$\mu_{k|y}=\sum_{k=0}^{2^{n_b}-1}k\cdot p_k(y),\; m_{2k|y}=\sum_{k=0}^{2^{n_b}-1}k^2\cdot p_k(y)\;,\; \sigma_{k|y}^2=m_{2k|y}-\mu_{k|y}^2\,. \tag{7}$$

2.2. Triangular Stimulus Signal

When the amount of random noise present is small, in comparison with the ADC ideal code bin width, the IEEE 1057 standard [1] suggests the use of a triangular stimulus signal that spans several ADC codes (about 10). The variance of the output codes can be calculated from the amplitude distribution, f_y, of the triangular stimulus signal [10] using

$$\sigma_k^2=\int_{-\infty}^{\infty}\sigma_{k|y}^2(y)\cdot f_y(y)dy\,. \tag{8}$$

For a triangular stimulus signal, with an amplitude A and an offset C, normalized by the ideal code bin width ($A_Q=A/Q$ and $C_Q=C/Q$) the amplitude distribution is

$$f_y(y)=\begin{cases}\dfrac{1}{2A_Q} & , \quad |y-C_Q|<A_Q\\ 0 & , \quad \text{otherwise}\end{cases}\,. \tag{9}$$

The output codes variance is thus

$$\sigma_k^2=\frac{1}{2A_Q}\int_{C_Q-A_Q}^{C_Q+A_Q}\sigma_{k|y}^2(y)dy\,. \tag{10}$$

2.3. Sinusoidal Stimulus Signal

In the case of a sinusoidal stimulus signal, the variance of the output codes can be calculated the same way as in the previous paragraph using now [10]

$$f_y(y)=\begin{cases}\dfrac{1}{\pi\sqrt{A_Q^2-(y-C_Q)^2}} & , \quad |y-C_Q|<A_Q\\ 0 & , \quad \text{otherwise}\end{cases}\,. \tag{11}$$

Inserting this into (8) leads to

$$\sigma_k^2=\int_{C_Q-A_Q}^{C_Q+A_Q}\sigma_{k|y}^2(y)\frac{1}{\pi\sqrt{A_Q^2-(y-C_Q)^2}}dy\,. \tag{12}$$

3. RANDOM NOISE ESTIMATORS

The random noise estimator is computed from the mean square difference

$$msd=\frac{1}{M}\sum_{j=0}^{M-1}(ka_j-kb_j)^2\,. \tag{13}$$

The mean square difference obtained has twice the variance of each set since they are independent of each other. According to [10] the expected value of the mean square difference determined by (13) is twice the variance of the output codes:

$$E\{msd\} = \sigma_{ka}^2 + \sigma_{kb}^2 = 2\sigma_k^2 \,. \tag{14}$$

Taken this into account, and considering that the variance of the output codes is equal to that of the additive noise, the estimated variance of the latter is just

$$\widehat{\sigma_r} = \sqrt{\frac{msd}{2}} \,. \tag{15}$$

The expected value of the estimated noise standard deviation can be approximated, using (15), by

$$\mathrm{E}\left\{\widehat{\sigma_r}\right\} \approx \sqrt{\frac{\mathrm{E}\{msd\}}{2}} \,. \tag{16}$$

Inserting (14) leads to

$$\mathrm{E}\left\{\widehat{\sigma_r}\right\} \approx \sigma_k \,, \tag{17}$$

where σ_k is given by (7).

As seen in Fig. 1, for small values of random noise standard deviation, the expected value of the estimator is much different than the actual value of noise standard deviation (dashed and dotted curves).

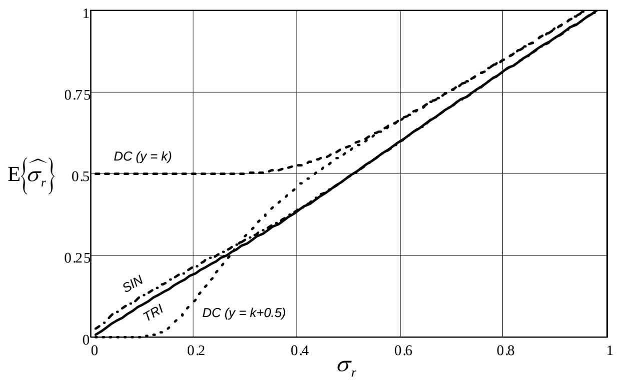

Fig. 1. Expected value of the estimated random noise as a function of the actual standard deviation of the random noise for a DC stimulus (dashed and dotted), a triangular stimulus (solid) and a sinusoidal stimulus (dash-dotted). Two cases of DC stimulus are represented: value equal to one of the ADC transition voltages (dashed) and exactly in between two consecutive ADC transition voltages (dotted). The triangular and sinusoidal stimulus signals have an amplitude of 5 LSB.

In the case of a triangular stimulus signal, the estimator recommended in the IEEE 1057 standard is

$$\widehat{\sigma_r} = \left[\left(\sqrt{\frac{msd}{2}}\right)^{-4} + \left(\frac{\sqrt{\pi}}{2} msd\right)^{-4}\right]^{\frac{-1}{4}} . \tag{18}$$

This expression was heuristically obtained from the two extreme cases of a large amount of random noise (eq. (15)) and a small amount of random noise. This latter case is determined from (6) and (7) but considering that the stimulus signal only spans two ADC output codes. In that case one has

$$\sigma_k^2 = \frac{1}{2A_Q} \int_{C_Q - A_Q}^{C_Q + A_Q} \left[\frac{1}{4} - \frac{1}{4} \cdot erf^2\left(\frac{U[k] - y}{\sqrt{2} \cdot \sigma_r}\right)\right] dy \quad , \quad \sigma_r \ll 1 \,. \tag{19}$$

Inserting (19) into (10) leads to

$$\lim_{\substack{\sigma_r \to 0 \\ A_Q \to \infty}} \sigma_k^2 = \frac{1}{\sqrt{\pi}} \sigma_r \,. \tag{20}$$

Considering (14) and (20), a possible estimator for the case of low amount of random noise would be

$$\widehat{\sigma_r} = \frac{\sqrt{\pi}}{2} msd \quad , \quad \sigma_r << 1 \,. \tag{21}$$

It is (15) and (21) that leads to (18). The expected value of (18) can be approximated by

$$\mathrm{E}\left\{\widehat{\sigma_r}\right\} \approx \left[\left(\sqrt{\frac{\mathrm{E}\{msd\}}{2}}\right)^{-4} + \left(\frac{\sqrt{\pi}}{2}\mathrm{E}\{msd\}\right)^{-4}\right]^{\frac{-1}{4}} . \tag{22}$$

Inserting (14) leads to

$$\mathrm{E}\left\{\widehat{\sigma_r}\right\} \approx \left[(\sigma_k)^{-4} + \left(\sqrt{\pi}\sigma_k^2\right)^{-4}\right]^{\frac{-1}{4}} , \tag{23}$$

where σ_k is given by (7). This expected value is depicted in Fig. 1 (solid curve) which shows that it is a good estimator of σ_r.

To use a sinusoidal stimulus signal one can derive an estimator using the same reasoning as in the case of the triangular signal. Instead of using (9) and (10) one would use (11) and (12). The result obtained for the extreme case of small random noise standard deviation and large stimulus signal amplitude is the same as in the case of the triangular stimulus signal, namely (20). The heuristically derived estimator to use in the case of a sinusoidal stimulus is thus the same as used for the triangular stimulus, that is, (18). The expected value of the estimator in the sinusoidal case is also depicted in Fig. 1 (dash-dotted curve). It can be seen to be very similar to the triangular one (solid curve).

In Fig. 2 the error of the estimators, defined as

$$e_{\widehat{\sigma_r}} \triangleq \mathrm{E}\left\{\widehat{\sigma_r}\right\} - \sigma_r \,, \tag{24}$$

is represented. For small amounts of random noise the triangular and sinusoidal estimators have a much lower error than the DC estimator. For large amounts of random noise, all estimators are equally good.

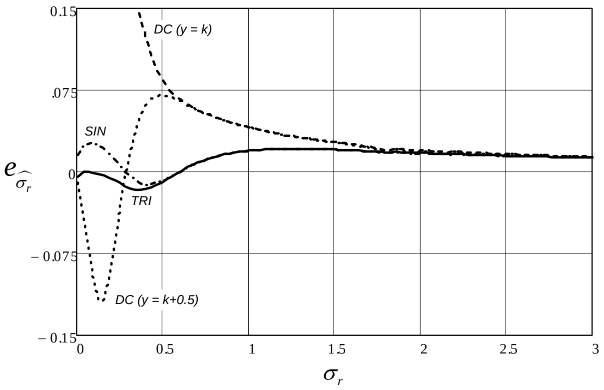

Fig. 2. Error of the estimators used obtain the random noise standard deviation as a function of the actual standard deviation of the noise for a DC stimulus (dashed and dotted), a triangular stimulus (solid) and a sinusoidal stimulus (dash-dotted). Two cases of DC stimulus are represented: value equal to one of the ADC transition voltages (dashed) and exactly in between two consecutive ADC transition voltages. The triangular and sinusoidal stimulus signals have an amplitude of 5 LSB.

4. CONCLUSIONS

In this paper we proposed the use of a sinusoidal stimulus signal to estimate an ADC random noise standard deviation. The actual estimator expression proposed is the same as suggested by the IEEE 1057 standard. This allows more flexibility in performing the test and also opens the way to consider using sine fitting and record alignment to eliminate the record triggering necessary with the triangular stimulus signal. This would, in principle, eliminate that source of uncertainty.

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