ERROR IN THE IEEE 1057 STANDARD RANDOM NOISE TEST OF ADCS

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Abstract − The random noise test of analogue to digital converters recommended by the IEEE 1057-2007 Standard for Digitizing Waveform Recorders is studied highlighting the estimation error and the influence of stimulus signal offset and amplitude on the estimation.

Keywords: Analogue to Digital Converter Testing, Estimation Error, Random Noise.

1. INTRODUCTION

The amount of random noise present in an analog to digital converter (ADC) affects directly its performance by reducing the effective number of bits. When a problem with a given ADC is suspected, for instance, one of the first things to consider is the amount of random noise present. To estimate the standard deviation of the random noise, the IEEE 1057 Standard for Digitizing Waveform Recorders suggest a specific test procedure [1]. It consists of shorting the ADC input and acquiring two sets of samples with the same length (*M*) and subtracting the output codes obtained sample by sample. This eliminates systematic contributions of the ADC or the stimulus signal but keeps the random noise. The mean square difference is then computed and used in an estimator that returns a value for the random noise standard deviation.

When there is too little random noise present, the output codes may not vary from sample to sample and from set to set. If this is the case the estimator used does not work. It is suggested that a small amplitude triangular signal be used to stimulate the ADC causing the output codes to vary. The estimator to use in this situation is different from the one used for a short-circuited ADC. It is an heuristically derived estimator that is biased. It is this procedure that is the focus of the paper. The goal is to determine how large the bias of the estimator is.

Note that the knowledge of the amount of random noise present in the ADC or in the test setup is also useful for the computation of the uncertainty of the results obtained with other ADC test methods like the Standard Static Test [1] or the Standard Histogram Test [1-7].

In the following we will analyze the expression used in [1] to estimate the standard deviation from the mean square error. We then present a study about the influence of the stimulus signal amplitude and offset on the estimation of the random noise standard deviation. Finally we determine the maximum estimation error incurred when estimating the random noise using the expression presented in [1].

The stimulus signal amplitude and offset affect the ADC random noise estimation because the heuristically derived estimator is based in asymptotic cases of noise standard deviation which are not dependent on stimulus signal amplitude or offset when in fact the ADC output codes do depend on the stimulus signal amplitude and offset.

2. RANDOM NOISE ESTIMATOR

Due to the presence of random noise at the ADC input, the output code (k) can be considered a discrete random variable which can assume any value between 0 and $2^{n_b}-1$ for a n_b-bit ADC. The variance of the output codes of the ADC when its input has a constant value of y is (adapted from eq. D.7 of [1])

$$\sigma_{k|y}^2 = \frac{1}{4} - \frac{1}{4} \cdot erf^2\left(\frac{U[k]-y}{\sqrt{2}\cdot\sigma_r}\right) \quad , \quad \sigma_r \ll 1, \quad (1)$$

where σ_r is the random noise standard deviation in LSB and $U[k]$ is the kth normalized transition voltage of the ADC (transition voltage divided by the ideal code bin width Q). The variance of the output codes for a triangular stimulus signal can be calculated from the amplitude distribution, f_y, of the stimulus signal [8],

$$\sigma_k^2 = \int_{-\infty}^{\infty} \sigma_{k|y}^2(y) \cdot f_y(y) dy . \quad (2)$$

For a triangular stimulus signal, with an amplitude A and an offset C (in volt), normalized by the ideal code bin width ($A_Q=A/Q$ and $C_Q=C/Q$) the amplitude distribution is

$$f_y(y) = \begin{cases} \dfrac{1}{2A_Q} & , \quad |y-C_Q| < A_Q \\ 0 & , \quad \text{otherwise} \end{cases} . \quad (3)$$

The output codes variance is thus

$$\sigma_k^2 = \frac{1}{2A_Q} \int_{C_Q-A_Q}^{C_Q+A_Q} \sigma_{k|y}^2(y) dy . \quad (4)$$

The corresponding standard deviation is represented in Fig. 1 by a solid line.

For small additive noise standard deviation the variance σ_k^2 is obtained by inserting (1) into (4) leading, after some manipulation, to

$$\sigma_k^2 = \frac{\sqrt{2} \cdot \sigma_r}{8 A_Q} \int_{C_Q - A_Q}^{C_Q + A_Q} 1 - erf^2(x) dx \quad , \quad \sigma_r \ll 1 . \qquad (5)$$
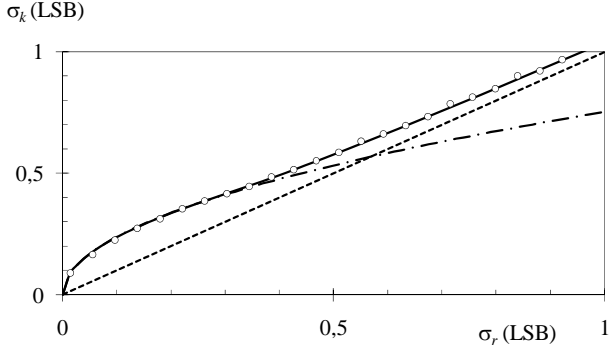

Fig. 1. Representation of the code standard deviation as a function of the additive noise standard deviation (solid line) for a triangular stimulus signal with amplitude $Q/2$. The dashed and the dot-dashed lines represent the approximation for small and large additive noise standard deviation given by (6) and (7) respectively. The circles represent the uncertainty of the codes standard deviation obtained experimentally.

In order to understand the dependence of σ_k on the noise standard deviation the asymptotic cases of small and large noise standard deviation are going to be considered. To compute the value of this integral when the noise standard deviation goes to 0 the stimulus signal amplitude can be considered infinite, leading to

$$\sigma_k^2 \big|_{\sigma_r \to 0} = \frac{1}{\sqrt{\pi}} \sigma_r . \qquad (6)$$

This approximation is represented in Fig. 1 by the dash-dotted line. It can be seen that in fact it approximates the solid line for small values of σ_r.

In the case of a noise standard deviation much higher than the ideal code bin width $(\sigma_r \to \infty)$ the codes standard deviation approximates the noise standard deviation:

$$\sigma_k \big|_{\sigma_r \to \infty} = \sigma_r \qquad (7)$$

This situation is represented by the dotted line in Fig. 1.

3. EXPERIMENTAL VALIDATION

A triangular stimulus signal, with amplitude 39.216 mV and frequency 10 Hz, was applied to a 12-bit data acquisition board (Keithley DAS 1601) in the ±10V range and two sets of 10000 samples were acquired at a sampling frequency of 100 kHz. The value of amplitude and frequency choosen does not have any particular meaning and were measured with a high precision multimeter. The purity of the triangular stimulus signal is terms of harmonic distortion in not critical since the subtraction performed eliminates deterministic non-idealities in the ADC or the stimulus signal.

Only the 8 most significant bits were used so that the ADC would present an almost ideal behavior. This was validated by testing the data acquisition board (using all 12

bits) with the Standard Histogram Test and obtaining an integral non-linearity (INL) and differential non-linearity (DNL) lower than 0.5 LSB.

A Stanford DS360 function generator was used to generate the additive noise and a Wavetek 9100 calibrator was used to generate the triangular wave. Each instrument was connected to a differential input of the data acquisition board to add the Gaussian noise to the triangular wave.

The results are represented with circles in Fig. 1 where a good agreement with the theoretical curve, represented by the solid line, is observed.

4. STIMULUS SIGNAL AMPLITUDE

As observed in equation (4), the code standard deviation depends on the stimulus signal amplitude. This can be verified in Fig. 2 where two cases are plotted, one for $A_Q = 0.5$ (thin line) and another for $A_Q = 0.75$ (thick line). The difference is highest for values of noise standard deviation smaller than 0.5LSB.
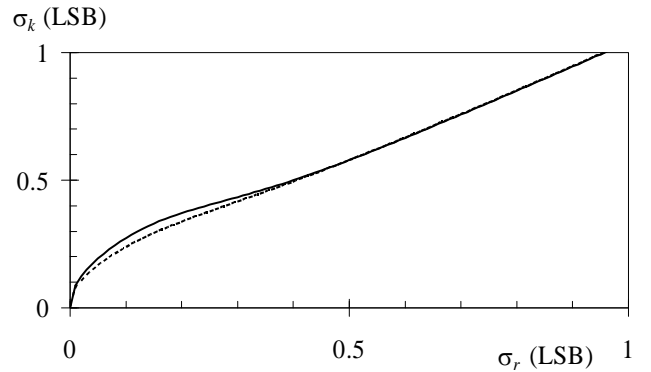

Fig. 2. Representation of the code standard deviation as a function of the additive noise standard deviation for a triangular stimulus signal with amplitude $Q/2$ (dotted line) and an amplitude 0.75Q (solid line).

That dependence can be also observed in Fig. 3 for two different values of triangular wave offset. The case of null amplitude ($A_Q = 0$) corresponds to the DC input case. It can be seen that the higher the triangular wave amplitude the less influence it has on the code standard deviation. Whenever the amplitude A is a multiple of $Q/2$, that is, $2A_Q$ is integer, the code standard deviation has the same value.

When performing the test to estimate the random noise standard deviation, a value of triangular wave amplitude has to be chosen. This choice will influence the estimated value, however, observing Fig. 3, there are some values that seem more advantageous, namely odd multiples of $Q/2$ ($A_Q = 0.5$, 1.5, 2.5, …) because the slope of the curve in those points is smaller making the estimate less sensible to errors in the triangular wave amplitude that in practical conditions are sure to exist. Also the output code variance becomes practically independent of the stimulus signal offset. The higher the triangular wave amplitude, the higher will be the additive noise introduced by the function generator (because of its different ranges with higher amplification) making very high values of amplitude not advisable.
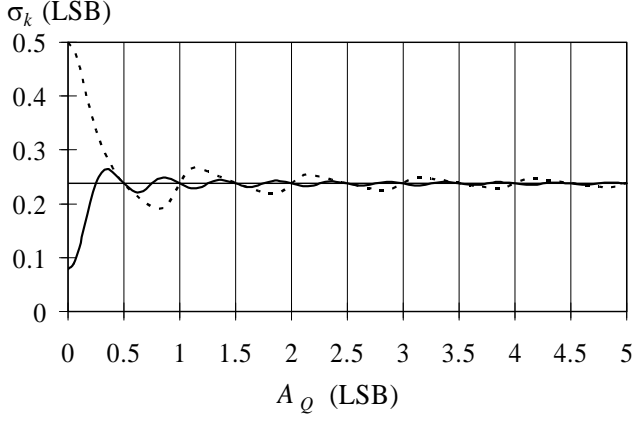
Fig. 3. Representation of the code standard deviation as a function of the triangular wave amplitude divided by the ideal code bin width. The solid line represents is for an offset equal to an ADC transition voltage and the dotted line a situation where the offset value is exactly between two consecutive transition voltages. The dotted line represents the value taken by the code standard deviation when the triangular amplitude is $Q/2$. A value of noise standard deviation of 0.1 LSB was used.

The IEEE standard [1] suggests the use of $A_Q = 5$ without explaining the reason. We hope the work presented here sheds some light on the subject. As seen here, a marginally better choice would be, for instance, $A_Q = 5.5$.

5. ESTIMATION ERROR

The IEEE standard [1] presents a heuristically derived expression,

$$\widehat{\sigma_r} = \left[\left(\sqrt{\frac{msd}{2}} \right)^{-R} + \left(\frac{\sqrt{\pi}}{2} msd \right)^{-R} \right]^{-\frac{1}{R}} \quad , \quad R = 4 \,, \quad (8)$$

where

$$msd = \frac{1}{M} \sum_{j=0}^{M-1} \left(ka_j - kb_j \right)^2 . \quad (9)$$

Note that the variables σ and mse used in [1] are expressed in volt and squared volt respectively while the variables σ_r and msd used here do not have dimensions.

The expected value of the mean square difference is [8]

$$E\{msd\} = \sigma_{ka}^2 + \sigma_{kb}^2 = 2\sigma_k^2 . \quad (10)$$

The expected value of the estimated random noise standard deviation, given by (8) can be approximated by

$$E\{\widehat{\sigma_r}\} \approx \left[\left(\sqrt{\frac{E\{msd\}}{2}} \right)^{-R} + \left(\frac{\sqrt{\pi}}{2} E\{msd\} \right)^{-R} \right]^{-\frac{1}{R}} \quad , \quad R = 4 . \quad (11)$$

Inserting (10) into (11) leads to

$$E\{\widehat{\sigma_r}\} \approx \left[(\sigma_k)^{-R} + \left(\sqrt{\pi} \sigma_k^2 \right)^{-R} \right]^{-\frac{1}{R}} \quad , \quad R = 4 , \quad (12)$$

where σ_k is given by (5).

The error of the additive noise standard deviation estimation is given by

$$e_{\widehat{\sigma_r}} \triangleq E\{\widehat{\sigma_r}\} - \sigma_r \quad (13)$$

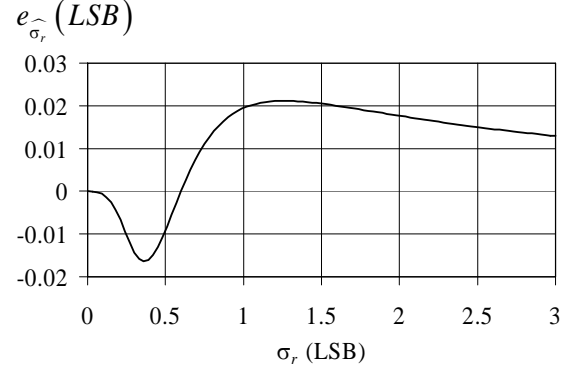and is represented in Fig. 4.

Fig. 4. Representation of the estimation error of the additive noise standard deviation, obtained with (8), as a function of the actual standard deviation.

It can be seen that the error is always smaller, in absolute value, than 0.022 LSB. When $\sigma_r \to \infty$ the error goes to 0.

The factor 4 used in (8) is very close to the value of 3.8203 which leads to the optimum approximation for an expression of the kind of (8) and corresponds to the position of the minimum in Fig. 5.
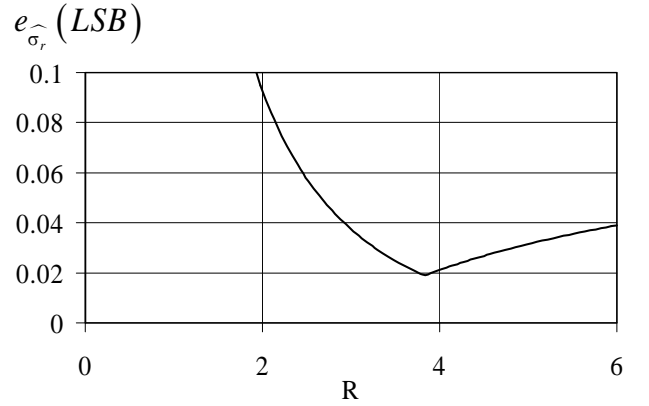

Fig. 5. Representation of the estimation error of the additive noise standard deviation, obtained with (8), as a function of the parameter R.

In Fig. 6, the maximum estimation error for different values of triangular stimulus signal amplitude and offset is represented. It can be seen that for small values of amplitude the maximum error depends strongly on the triangular wave offset. For values of amplitude higher than twice the ideal code bin width the maximum estimation error is practically independent of both amplitude and offset and is lower than 0.022 LSB.
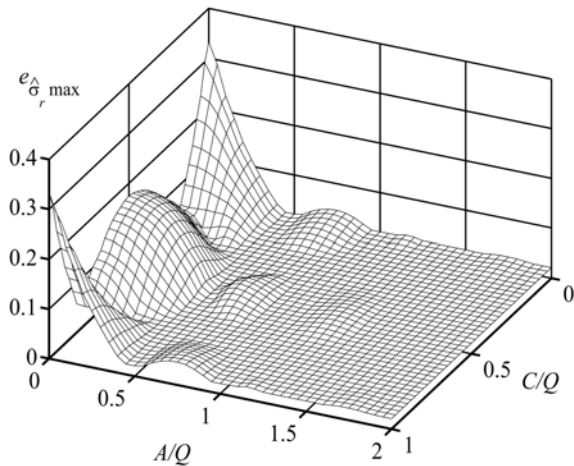
Fig. 6. Representation of the maximum estimation error of the additive noise standard deviation, obtained with (8), as a function of the triangular wave amplitude and offset.

6. CONCLUSIONS

In this paper, the test method proposed by the IEEE 1057-2007 standard [1] to estimate the random noise standard deviation was analyzed in detail. The expression presented there to calculate the additive noise standard deviation from the ADC output codes mean square difference is explained here from a different perspective. The influence of both triangular wave amplitude and offset on the estimation error was studied. The actual error on estimating the noise standard deviation when using that expression presented in [1] is calculated and it was concluded that it was smaller than 2.2% of the ideal ADC code bin width (0.022 LSB). The choice of triangular stimulus amplitude is studied and a justification for the value proposed in [1] is given. This study of the estimator bias complements the study of the estimator precision published in [9].

We suggest here that in the case were a triangular wave generator is not available the test be carried out with a DC stimulus signal, with any value inside the ADC range, and that the noise standard deviation be estimated as being equal to the code root mean square error, divided by 2. If the value obtained is smaller than Q/2 the value used as the noise standard deviation estimative should be Q/2. Of course the standard deviation could be smaller but, as shown here, it could not be higher. In summary, we propose, just for the case where a triangular generator cannot be used, the following expression for the estimation:

$$\widehat{\sigma}_r = \min\left(\frac{1}{2}, \sqrt{\frac{mse}{2}}\right). \qquad (14)$$

Experimental results were presented that validate the approach taken.

ACKNOWLEDGMENTS

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