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COMPARISON OF THE PRECISION OF GAIN AND OFFSET ESTIMATIONS OBTAINED WITH THE HISTOGRAM TEST OF ADCS

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Abstract – The Histogram Method of analogue to digital converter testing can be used to estimate their gain and offset error. There are two commonly used definitions for these parameters, namely "Terminal Based" and "Independently Based". In this paper the precision of those two different estimators is compared with the goal of assessing which one is more precise for the same number of samples and the same amount of additive noise.

Keywords: Analogue to Digital Converter; Histogram Method; Gain and Offset Estimation.

1. INTRODUCTION

The Histogram Test Method [1-3] can be used to determine the gain and offset error of an analogue to digital converter (ADC). These two parameters are important for ADC users since it directly affects how the digital output codes are converted to voltages at the ADC input.

There are two ways to define the gain and offset named "Terminal Based" and "Independently Based". According to the *Terminal Based Definition*, the offset error plus the product of the gain by the first and last *real* transition voltages, results in the first and last *ideal* transition voltages respectively. Hence the designation "Terminal Based" refers to the fact that the definition is based on the extremes of the transfer function, that is, on the value of the first (lowest) and last (highest) transition voltages. So that the gain (G) and offset error (O) satisfy the definition if they are computed with the following expressions [1]:

$$G = \frac{L_{ideal} - F_{ideal}}{L - F} \quad \text{and} \quad O = F_{ideal} - G \cdot F \, . \tag{1}$$

To simplify the notation, two variables were introduced: $F = T_1$ and $L = T_{2^{n_b}-1}$ for the first and last transition voltage respectively.

In the *Independently Based Definition*, the gain and offset error are defined as the two scalars that when used to multiply the estimated transition voltages, $\hat{T}_k$, (gain, G) and add to the result of the multiplication (offset error, O) lead to corrected transition voltages

$$\hat{T}_k^{corr} = G \cdot \hat{T}_k + O \tag{2}$$

that are as close as possible to the ideal transition voltages in a least square sense [2].

The way traditionally used to determine the independently based ADC gain and offset error is to use a linear regression procedure to fit the estimated transition voltages to the ideal ones:

$$T_k^{ideal} = \hat{G} \cdot \hat{T}_k + \hat{O} \, . \tag{3}$$

The estimated gain is the slope of the fitted straight line and the estimated offset error is the point of intersection of that straight line with the vertical axis (axis of the ideal transition voltages).

The presence of non-ideal effects like additive noise, phase noise, jitter and distortion, for example, lead to uncertain estimation of the ADC parameters. In particular, the ADC gain and offset error, which are determined from the transition voltages, will be affected by those non-ideal effects. In this paper the presence of additive noise is considered. This type of noise, usually of thermal origin, is stochastic. It can be described as a normally distributed random variable with null mean and standard deviation σ_{nv}. The consequence to the estimation of the ADC gain and offset is that each time the test is carried out on a given ADC the values obtained are different. They are also random and as such can be statistically characterized by a probability density function and statistical moments. Usually it is safe to consider that the gain and offset error are normally distributed since their determination is made from a large number of samples. The two statistical moments that can be used to characterize the distribution is the mean and the variance (or standard deviation). The knowledge of the mean is important to determine if an estimator is biased and eventually to correct the estimated values so that the estimation error, on average, is null. The knowledge of the standard deviation is important to access the precision of the results, namely, how concentrated are they in relation to the correct value. In practice the value of estimator standard deviation is used to compute a confidence interval which defines, with a certain degree of confidence, where the actual value of what is being estimated is [4]. Of course the lower the standard deviation the better since the smaller is the confidence interval.

This paper is about the precision of two different estimators used to estimate the same ADC parameters (gain and offset error). The goal is to determine which estimator is

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better in terms of having higher precision (lower standard deviation) when all other conditions are kept the same.

In the following the precision of the gain (section 2) and offset error (section 3) estimators will be compared. In section 4 the validity of the expressions presented will be verified using a Monte Carlo procedure and finally, in section 5, some conclusions will be drawn.

2. PRECISION OF THE GAIN ESTIMATORS

In [1] and [2], the precision of the terminal based and independently based gain (and offset error), respectively, was determined.

The result obtained in [1] for the standard deviation of the terminal based (TB) gain was

$$\sigma_{\widehat{G_{TB}}} \approx \frac{\pi\sqrt{A^2 - T_0^2}}{\sqrt{2}\left(FS - Q\right)M}\sqrt{\max\left(\frac{1}{4}, \frac{M}{\pi\sqrt{\pi}}\frac{\sigma_{nv}}{\sqrt{A^2 - T_0^2}}\right)}, \quad (4)$$

where FS is the ADC full scale, Q is the ideal code bin width, A is the stimulus signal amplitude, M is the number of acquired samples, σ_{nv} is the additive noise standard deviation and T_0 is the lowest transition voltage.

The result obtained in [2] for the standard deviation of independently based (IB) gain was

$$\sigma_{\widehat{G_{IB}}} \approx 1.65\frac{A}{\left(FS - Q\right)^2}\frac{\sigma_{nv}}{\sqrt{M}}. \quad (5)$$

In order to make it easier to compare the two expressions and to shed some light on the quantities that influences the standard deviation of the gain estimation, some considerations will be made. First it is going to be assumed that the ADC under test has a mid-riser type of transfer function [5]. This does not invalidate the results obtained for other types of transfer functions but eases the interpretation of the expressions. The mid-riser transfer function has symmetrical transition voltages and the lowest transition voltage is given by

$$T_0 = -FS + Q. \quad (6)$$

Furthermore, it can be said that the ideal code bin width (Q) is typically much lower than the ADC full scale (FS). Taking these two considerations into account, expression (4) can be written as

$$\sigma_{\widehat{G_{TB}}} \approx \frac{\pi\sqrt{\alpha^2 - 1}}{\sqrt{2}M}\sqrt{\max\left(\frac{1}{4}, \frac{M}{\pi\sqrt{\pi}}\frac{\sigma_{nf}}{\sqrt{\alpha^2 - 1}}\right)}, \quad (7)$$

where two new variables have been introduced, namely the normalized additive noise standard deviation

$$\sigma_{nf} = \frac{\sigma_{nv}}{FS}, \quad (8)$$

and the overdrive amount

$$\alpha = \frac{A}{FS}. \quad (9)$$

Usually the ADC is overdriven, that is, the stimulus signal amplitude is made greater that the ADC full scale, which would be the value necessary to stimulate all the ADC codes so as to guarantee that even if there are errors in the sine wave amplitude, a non null sine wave offset or a ADC gain higher than 1, all the ADC code still get stimulated. Overdrive is also used to limit the error introduced by additive noise in the estimation of the ADC transition voltages near the edges of the ADC range. The amount of overdrive, α, is thus always greater than 1 (typically 5 % to 20 % greater).

A finally consideration is to assume that the additive noise standard deviation is not too low as to make the 2nd argument in the "max" function in (7) lower than the 1st one. This corresponds to having

$$\sigma_{nf} > \frac{\pi\sqrt{\pi}}{4M}\sqrt{\alpha^2 - 1}. \quad (10)$$

This allows (7) to be written as

$$\sigma_{\widehat{G_{TB}}} \approx 0.941\left(\alpha^2 - 1\right)^{\frac{1}{4}}\sqrt{\frac{\sigma_{nf}}{M}}, \quad (11)$$

Making the same consideration regarding (5) it is possible to write

$$\sigma_{\widehat{G_{IB}}} \approx 1.65\alpha\frac{\sigma_{nf}}{\sqrt{M}}. \quad (12)$$

By comparing expressions (11) and (12) several conclusions can be made. The first one is that the terminal based gain depends on the square root of the noise standard deviation while the independently based gain depends linearly on the noise standard deviation, as can be observed in the two curves depicted in Fig. 1.

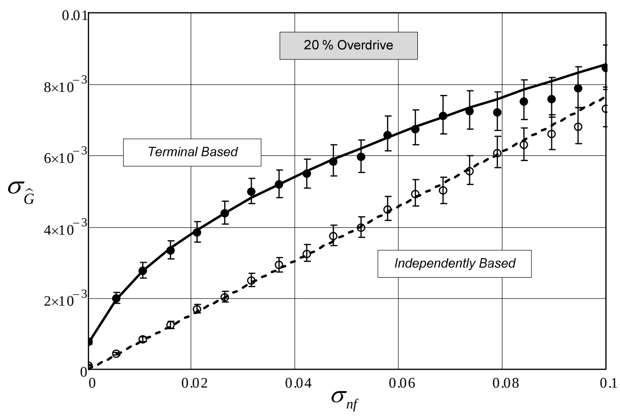

Fig. 1. Standard deviation of the estimated ADC gain as a function of the normalized additive noise standard deviation for a stimulus signal amplitude of 1.2 V and an 8-bit ADC with 1 V full scale. The circles represent the values obtained numerically. The black circles correspond to the terminal based gain and the white circles correspond to the independently based gain. The vertical bars represent the confidence intervals for a 99.9% confidence level. The lines are the representation of the analytical expressions.

The second conclusion is that both gain definitions are inversely proportional to the square root of the number of acquired samples.

By comparing the two curves in Fig. 1 it can be seen that for noise standard deviations smaller that 10 % the ADC full scale ($\sigma_{nf} < 0.1$), which is a typical situation encountered in practice, the independently based gain has lower standard deviation (better precision). In Fig. 2, which is for a 5 % overdrive instead of 20 %, the situation is similar.

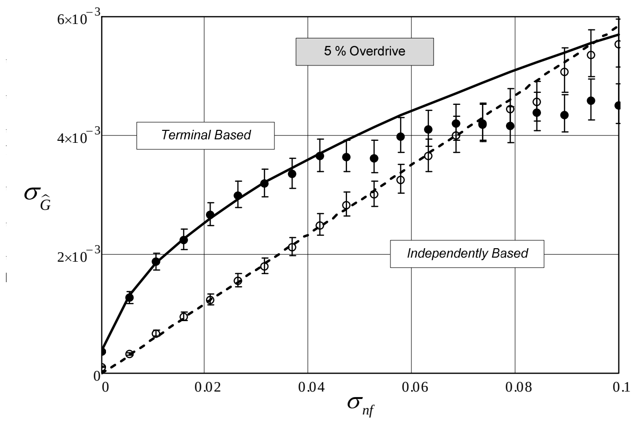

Fig. 2. Standard deviation of the estimated ADC gain as a function of the normalized additive noise standard deviation for a stimulus signal amplitude of 1.05 V and an 8-bit ADC with 1 V full scale. The circles represent the values obtained numerically. The black circles correspond to the terminal based gain and the white circles correspond to the independently based gain. The vertical bars represent the confidence intervals for a 99.9% confidence level. The lines are the representation of the analytical expressions.

The intersection point between the two curves can be determined by equating (11) to (12) which leads to

$$\sigma_{nf} = 0.325 \frac{\sqrt{\alpha^2 - 1}}{\alpha^2} . \tag{13}$$

For the case of 20% overdrive, depicted in Fig. 1, the intersection occurs at $\sigma_{nf} = 0.15$, while for the case of 5%, depicted in Fig. 2 it occurs at $\sigma_{nf} = 0.094$.

3. PRECISION OF THE OFFSET ERROR ESTIMATORS

In [1] it was determined that the standard deviation of the terminal based offset estimation is

$$\sigma_{\widehat{O_{TB}}} = \left(FS - Q\right)\sigma_{\widehat{G_{TB}}} , \tag{14}$$

which can be written as (using $FS >> Q$)

$$\sigma_{\widehat{O_{TB}}} \approx 0.941 FS \left(\alpha^2 - 1\right)^{\frac{1}{4}} \sqrt{\frac{\sigma_{nf}}{M}} , \tag{15}$$

while in [2] it was determined that the standard deviation of the independently based offset estimation is

$$\sigma_{\widehat{O_{IB}}} \approx 1.1 \frac{A}{FS - Q} \frac{\sigma_{nv}}{\sqrt{M}} , \tag{16}$$

which can be written as

$$\sigma_{\widehat{O_{IB}}} \approx 1.1 FS \alpha \frac{\sigma_{nf}}{\sqrt{M}} , \tag{17}$$

These two equations, (15) and (17) are similar to (11) and (12) respectively. The difference is the multiplication by *FS* and the numeric factor in the independently based definition which was 1.65 for the gain and is 1.1 for the offset.

The comparison between the two definitions is thus similar to that done for the gain as seen in Fig. 3. The independently based definition has even more of an advantage in relation to the terminal based one, when considering the ADC offset errors.

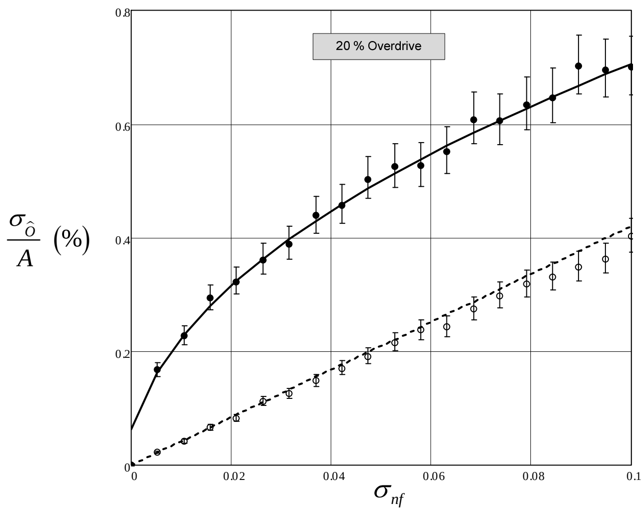

Fig. 3. Standard deviation of the estimated ADC offset error as a function of the normalized additive noise standard deviation for a stimulus signal amplitude of 1.2 V and an 8-bit ADC with 1 V full scale. The circles represent the values obtained numerically. The black circles correspond to the terminal based offset and the white circles correspond to the independently based offset. The vertical bars represent the confidence intervals for a 99.9% confidence level. The lines are the representation of the analytical expressions.

In the case of the offset, the intersection point is given by

$$\sigma_{nf} = 0.732 \frac{\sqrt{\alpha^2 - 1}}{\alpha^2} , \tag{18}$$

which is considerable higher than the case of the gain given by (13).

4. NUMERICAL VALIDATION

To validate the derivations presented and the approximations made, a numerical simulation of the Histogram Method, using a Monte Carlo procedure was carried out. 1000 repetitions were carried out and the confidence intervals for the standard deviation of the gain and offset error was determined for a 99.9 % confidence level.

Table 1 list the values of the parameters of the test setup used in the numerical simulation.

Table 1. List of test setup parameters used in the numerical validation.

Test Parameter	Value
Number of Bits of the ADC (n_b)	8
ADC Full Scale (*FS*)	1 V
Sinusoidal Stimulus Amplitude (*A*)	1.05 V and 1.2 V
Sinusoidal Stimulus Offset (*C*)	0
Number of Samples (*M*)	1000
Additive Noise Standard Deviation (σ_{nv})	0 to 0.1×A
Number of Repetitions	1000
Confidence Level	99.9 %

The confidence intervals are represented by vertical bars in the previous figures for comparison with the analytical expression. The results are all in agreement except those related to the terminal based gain in Fig. 2. That is because the approximations made in [1] for the standard deviation of the transition voltages is not very good when the transition voltages are close to the ADC range edges. The analytical expression is however an upper bound for the actual values of gain standard deviation.

5. CONCLUSIONS

In this paper a comparison was made between the precision of the two estimators normally used for the ADC gain and offset error. It was concluded that the independently based definition of gain and offset error has a better precision, that is, a lower standard deviation, than the terminal based definition, considering the same amount of overdrive and number of acquired samples. It is thus advisable to use the independently based definition of ADC gain and offset error when possible. Note that this definition is, however more computationally intensive than the terminal based one.

ACKNOWLEDGMENTS

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