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BIAS IN ADC TERMINAL BASED GAIN AND OFFSET ESTIMATION USING THE HISTOGRAM METHOD

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Abstract – It is demonstrated that, when using the Histogram Test Method to test an analogue to digital converter, the presence of additive noise in the test setup or in the converter itself causes a bias in the terminal based estimation of the gain but not in the estimation of the offset. This will be demonstrated here by analytically determining the estimation error as a function of the sinusoidal stimulus signal amplitude and the noise standard deviation. A closed form approximate expression will be proposed for the computation of the bias of terminal based gain. The results presented are numerically validated using a Monte Carlo procedure.

Keywords – Analogue to Digital Converter; Histogram Test Method; Estimator Bias.

1. INTRODUCTION

The Histogram Method is a method widely used in analogue to digital converter (ADC) testing [1-2]. It allows the estimation of different ADC parameters, namely the transition voltages, code bin widths, integral non-linearity, differential non-linearity, gain and offset error. The estimators for these parameters are affected by non-ideal effects in the test setup or in the ADC itself, like additive noise, phase noise, jitter, stimulus signal distortion and frequency error, among others.

In this paper we address the influence of additive noise on the estimation error of the ADC gain and offset error. This is important not only to properly characterize statistically the results obtained when testing an ADC but also to allow the correction of the estimations made which is possible when the amount of additive noise present is known or can be estimated.

In section 2 an introduction to the terminal based definition of gain and offset error is made. In section 3 the error of the estimators are derived and in section 4 the results presented are numerically validated using a Monte Carlo procedure. Finally in section 5 some conclusions are drawn.

2. TERMINAL BASED GAIN AND OFFSET ERROR

The purpose of an ADC is to convert the values of a current or voltage present at the input, which is a continuous variable, into a digital word that should represent that input. That relationship, between the input variable and output digital words (or codes) is known as the ADC transfer function and is determined by the ADC manufacturer [3]. In the rest of the text we will consider that the input variable is a voltage. There are different types of transfer functions. One of them, used with bipolar ADCs, is the mid-riser (also known as “with no true zero”) which is represented in Fig. 1. Variable n_b represents the ADC number of bits and FS the full scale voltage.

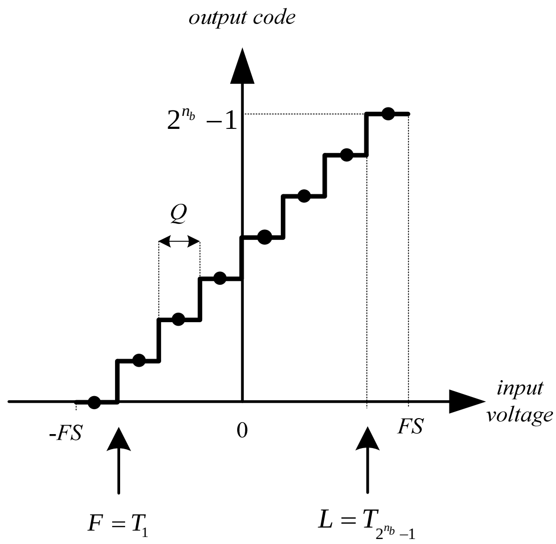

Fig. 1. Illustration of the transfer function of a bipolar ADC. This type of transfer function is known as mid-riser or “with no true zero”.

Each output code corresponds to a range of input voltage values (horizontal lines). Given an output code one cannot determine exactly which was the input voltage at the time of the sampling and posterior analogue to digital conversion. It is conventional to adopt the middle point of the ranges mentioned as the value of the input voltage for a given output code (black circles).

The transition voltages, T_k, define the ADC transfer function, that is, the relation between input voltage and output code, k. For an ideal ADC, the transition voltages of the transfer function, defined as in Fig. 1, are

$$T_k^{ideal} = -FS + k \cdot Q . \quad (1)$$

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They are equally spaced by an amount Q given, from the definition of the transfer function, by

$$Q = \frac{2 \cdot FS}{2^{n_b}} . \qquad (2)$$

In an actual ADC, the real transition voltages will be different from the ideal ones. To express those differences several parameters are used. Two of those are the ADC gain and offset error. They can be defined in different ways. Two of the most used are the *Terminal Based Definition* and the *Independently Based Definition* [1]. In this paper we will focus our attention on the first one. According to the *Terminal Based Definition*, the offset error plus the product of the gain by the first and last *real* transition voltages, results in the first and last *ideal* transition voltages respectively. Hence the designation "Terminal Based" refers to the fact that the definition is based on the extremes of the transfer function, that is, on the value of the first (lowest) and last (highest) transition voltages. So that the gain (G) and offset error (O) satisfy the definition if they are computed with the following expressions [4]:

$$G = \frac{L_{ideal} - F_{ideal}}{L - F} \quad \text{and} \quad O = F_{ideal} - G \cdot F . \qquad (3)$$

To simplify the notation, we introduced the variables $F = T_1$ and $L = T_{2^{n_b}-1}$.

When testing an ADC with the Standard Histogram Test we obtain an estimate of the transition voltages (not the real transition voltages). From those estimates we can compute the estimated ADC gain and offset error using

$$\hat{G} = \frac{L_{ideal} - F_{ideal}}{\hat{L} - \hat{F}} \quad \text{and} \quad \hat{O} = F_{ideal} - \hat{G} \cdot \hat{F} . \qquad (4)$$

The "hat" symbol over the variables means that they are an estimative and not the actual values for the ADC under test.

3. DERIVATION OF THE ESTIMATION BIAS

The presence of additive noise, which is a stochastic perturbation of the voltage sampled by the ADC, causes all the quantities estimated with the Histogram Method to be also stochastic variables. This is so for the estimated ADC gain and offset error given by (4) since they are determined from the estimated values of the first (F) and last (L) transition voltages.

The expected value of the estimates given by (4) can be computed from the statistical properties of $\hat{F}$ and $\hat{L}$. This can be done using

$$\mathrm{E}\{g\} \simeq g + \frac{1}{2}\left(\frac{\partial^2 g}{\partial x^2}\sigma_x^2 + \frac{\partial^2 g}{\partial x \partial y} r \sigma_x \sigma_y + \frac{\partial^2 g}{\partial y^2}\sigma_y^2 \right), \qquad (5)$$

where g is a function of two random variables x and y with correlation coefficient r [5, p. 156]. The function and its derivatives are evaluated at $x = \mathrm{E}\{x\}$ and $y = \mathrm{E}\{y\}$. In this paper, however only the first term in (5) is going to be used. This leads to a poorer approximation, however, since the analytical expressions for expected values of x and y ($\hat{F}$ and $\hat{L}$ in the context of this paper) that are going to be used are themselves coarse approximation for their exact value, it would be unnecessary to consider the second term in (5). This choice will be numerically validated latter using Monte Carlo simulations.

The expected value of the function will thus be considered equal to the function of the expected value. Using (4) leads to

$$\mathrm{E}\{\hat{G}\} = \frac{L_{ideal} - F_{ideal}}{\mathrm{E}\{\hat{L}\} - \mathrm{E}\{\hat{F}\}} , \; \mathrm{E}\{\hat{O}\} = F_{ideal} - \mathrm{E}\{\hat{G}\} \cdot \mathrm{E}\{\hat{F}\} . \; (6)$$

In [6], the amount of overdrive to use in order to minimize the error in the estimation of the transition voltages due to additive noise was studied. There, an expression for the computation of the expected value of the transition voltages estimated with the Histogram Method has been derived,

$$\mathrm{E}\{\widehat{U_k}\} \approx -\cos\left\{ \int_0^{\pi} \left[\frac{1}{2} + \frac{1}{2}\mathrm{erf}\left(\frac{U_k + \cos(\varphi)}{\sqrt{2}\sigma_n} \right) \right] \mathrm{d}\varphi \right\}, \qquad (7)$$

where U_k and σ_n are the normalized values of transition voltage and additive noise standard deviation. They are obtained from the real values by subtracting the stimulus signal offset C (just in the case of the transition voltages) and dividing by its amplitude, A:

$$U_K = \frac{T_k - C}{A} \quad \text{and} \quad \sigma_n = \frac{\sigma_{nv}}{A} , \qquad (8)$$

where σ_{nv} is the additive noise standard deviation in volt.

Note that the expected value of the estimated transition voltage, $\widehat{U_k}$,given by (7) depends on the actual transition voltage U_k. It is, however, safe to use the ideal value of the transition voltage in place of the actual value which is unknown, since typically they are similar.

In order to simplify the presentation, it is assumed we are dealing with an ADC that has a mid-riser transfer function like the one depicted in Fig. 1. In this type of transfer function the transition voltages are symmetric which eases the analytical derivations. One has, thus,

$$L_{ideal} = -F_{ideal} = FS - Q , \qquad (9)$$

where (1) and (2) were used. Using this it is possible to write (6) as

$$\begin{aligned} \mathrm{E}\{\hat{G}\} &= \frac{2(FS - Q)}{\mathrm{E}\{\hat{L}\} - \mathrm{E}\{\hat{F}\}} \\ \mathrm{E}\{\hat{O}\} &= -FS + Q - \mathrm{E}\{\hat{G}\} \cdot \mathrm{E}\{\hat{F}\} \end{aligned} . \qquad (10)$$

Also, expression (7) is odd in relation to U_k. Leading to

$$\mathrm{E}\{\hat{L}\} = -\mathrm{E}\{\hat{F}\} . \qquad (11)$$

Using (11) it is possible to write (10) as,

$$\mathrm{E}\left\{\hat{G}\right\} = \frac{FS - Q}{\mathrm{E}\left\{\hat{L}\right\}} \quad \text{and} \quad \mathrm{E}\left\{\hat{O}\right\} = 0\,. \qquad (12)$$

This shows immediately that the offset error estimator is unbiased since its expected value is equal to its ideal value, which is zero.

Inserting (7) into (12) and using the ideal value for the last transition voltage in place of the actual value in (7), leads to

$$\boxed{\mathrm{E}\left\{\hat{G}\right\} \approx \frac{FS - Q}{-\cos\left\{\int_0^{\pi}\left[\frac{1}{2}+\frac{1}{2}\mathrm{erf}\left(\frac{\frac{FS-Q}{A}+\cos(\varphi)}{\sqrt{2}\sigma_n}\right)\right]\mathrm{d}\varphi\right\}}}. \qquad (13)$$

In [6] a careful analysis of the dependence of (7) on the values of the transition voltage and the additive noise standard deviation was carried out. It was possible to determine the maximum value of the estimation error of the transition voltages for a given additive noise standard deviation. The expression obtained was

$$e_{\hat{U}\max} \approx \frac{\sigma_n}{5} \quad \text{for} \quad \sigma_n < 0.1\,, \qquad (14)$$

where

$$e_{\hat{U}} \triangleq \mu_{\hat{U}} - U\,. \qquad (15)$$

This is valid for values of normalized noise standard deviation lower than 0.1 which is equivalent to having a noise standard deviation lower than 10 % the stimulus signal amplitude, a situation generally encountered in practice. Using this, an approximate expression for the upper bound of the expected value of the normalized transition voltage is

$$\mathrm{E}\left\{\hat{U}\right\} < U + \frac{\sigma_n}{5} \quad \text{for} \quad \sigma_n < 0.1\,. \qquad (16)$$

Using (16) in (12) leads to

$$\mathrm{E}\left\{\hat{G}\right\} < \frac{FS - Q}{FS - Q + \frac{\sigma_{nv}}{5}}\,. \qquad (17)$$

Considering that the ideal code bin width (Q) is much smaller than the ADC full scale (FS), it is possible to write (17) as

$$\mathrm{E}\left\{\hat{G}\right\} < \frac{5}{5 + \frac{\sigma_{nv}}{FS}}\,. \qquad (18)$$

The relative error of the estimator, defined by

$$\varepsilon_{\hat{G}} \triangleq \left|\frac{\mathrm{E}\left\{\hat{G}\right\} - G_{ideal}}{G_{ideal}}\right| = \left|\mathrm{E}\left\{\hat{G}\right\} - 1\right|, \qquad (19)$$

is thus

$$\boxed{\varepsilon_{\hat{G}} < \frac{1}{5}\frac{\sigma_{nv}}{FS}}. \qquad (20)$$

This expression allows the determination of an upper bound on the relative error in the estimation of the ADC gain using the Histogram Method. It can be seen that this upper bound is proportional to the standard deviation of the additive noise (σ_{nv}) relative to the ADC full scale (FS).

4. NUMERICAL VALIDATION

In order to validate the derivations presented here and the approximations made, namely the substitution of the expected value of a function by the function of the expected value of it argument in (6), the use of expression (7), derived in [6] and which is in itself an approximate expression and the upper bound on the estimation error of the transition voltages made in (14), a Monte Carlo procedure was used. It consists in repeatedly simulating a sinusoidal stimulus signal corrupted by additive noise and using the Histogram Test to estimate the terminal based gain and offset of a simulated ADC.

In Fig. 2 the expected value of the estimated ADC gain is depicted as a function of the normalized additive noise standard deviation (black circles). The vertical bars represent the confidence interval for a 99.9 % confidence level obtained with 1000 repetitions of the Histogram Test. It can be seen that the results conform to expression (13) which was derived here and which can be used to analytically determine the estimation error. The dashed line is the representation of expression (16) which gives a bound for the estimation error and which is much easier to use than (13).

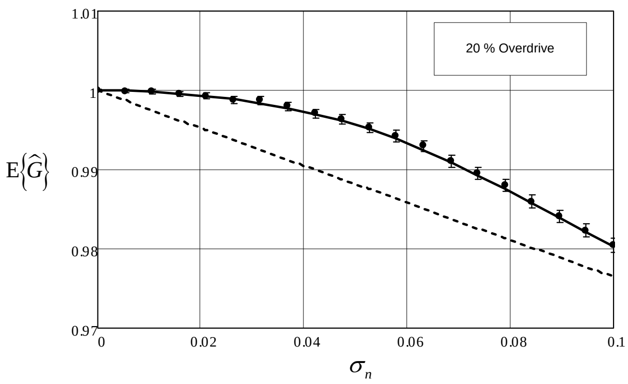

Fig. 2. Expected value of the estimated ADC gain as a function of the normalized additive noise standard deviation for a stimulus signal amplitude of 1.2 V. The circles represent the values obtained numerically. The vertical bars represent the confidence intervals for a 99.9% confidence level. The solid line is the representation of expression (13) and the dashed line is the representation of (18).

In Fig. 3 the same information is depicted but now using 5% overdrive. Again the numerical results obtained are in conformance with the analytical expressions presented here.

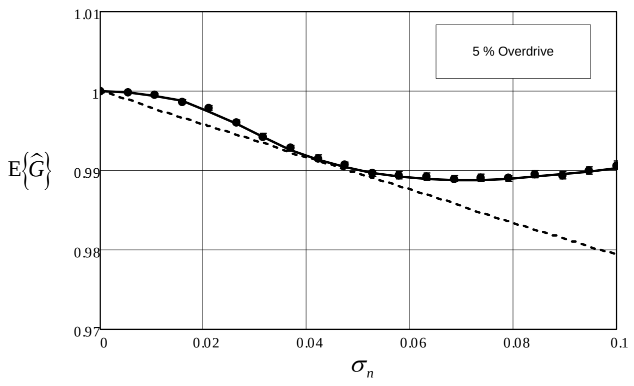

Fig. 3. Expected value of the estimated ADC gain as a function of the normalized additive noise standard deviation for a stimulus signal amplitude of 1.05 V. The circles represent the values obtained numerically. The vertical bars represent the confidence intervals for a 99.9% confidence level. The solid line is the representation of expression (13) and the dashed line is the representation of (18).

Table 1 list the values of the parameters of the test setup used in the numerical simulation.

Table 1. List of test setup parameters used in the numerical validation.

Test Parameter	Value
Number of Bits of the ADC (n_b)	8
ADC Full Scale (*FS*)	1 V
Sinusoidal Stimulus Amplitude (*A*)	1.05 V and 1.2 V
Sinusoidal Stimulus Offset (*C*)	0
Number of Samples (*M*)	1000
Additive Noise Standard Deviation (σ_{nv})	0 to 0.1×A
Number of Repetitions	1000
Confidence Level	99.9 %

In Fig. 4 it can be seen that the estimation error of the ADC offset is in fact null as demonstrated earlier.

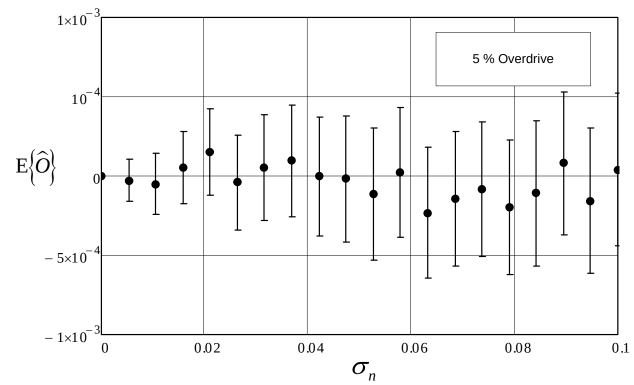

Fig. 4. Expected value of the estimated ADC offset error as a function of the normalized additive noise standard deviation for a stimulus signal amplitude of 1.05 V (5 % overdrive). The circles represent the values obtained numerically. The vertical bars represent the confidence intervals for a 99.9% confidence level.

5. CONCLUSIONS

In this paper it was shown that the presence of additive noise causes a bias in the estimation of the terminal based gain of an ADC using the Histogram Method but does not cause a bias in the offset error estimation.

Two analytical expressions were derived to compute the estimation error. The first one, expression (13), allows the calculation of the expected value of the estimated gain with an high degree of accuracy of the typical situation of additive noise lower than 10 % of the stimulus signal amplitude. It is however not a closed form expression and thus it is more difficult use, although of value when accuracy is necessary.

The second one, expression (20), can be used to obtain an upper bound on the relative error of the estimation. It is much easier to use but it only gives a bound on the error. As a rule of thumb it can be said that the relative error of the estimation is lower than one fifth of the noise standard deviation relative to the ADC full scale.

It was also demonstrated here that the gain estimator is not consistent, that is, its bias does not go to 0 when the number of samples goes to infinite. In fact, it was shown that the estimation error does not depend on the number of samples acquired but only on the amount of additive noise present in the test setup or in the ADC itself.

In the future further studies should be carried out to account for other non ideal effects like phase noise, jitter and stimulus signal harmonic distortion.

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