

DESIGN OF DSP WINDOWS USING WINDOW SPECTRUM ZEROS PLACEMENT

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Abstract – Windows are frequently used in digital signal processing, mainly for leakage reduction in non-coherent sampling (spectral analysis, detection of weak harmonic components close to a strong harmonic components, e.g. in ADC testing) or in FIR digital filter design. Many windows are described and compared in literature. This paper informs about a mathematical instrument and an algorithm allowing design of several classes of cosine windows with various prescribed properties. The design is based on iterative window spectrum zeros placing. Window coefficients are presented either as high-accuracy decimal numbers or in a more compact form of ratios. The proposed algorithm can be implemented using any computing instrument; we have selected Matlab environment. The designed Graphical User Interface allows easy inserting of required window spectrum properties and presentation of resulting window coefficients, window spectrum and window shape in time domain. The method allows design of all three window classes of Rife and Vincent windows (up to the window order 10) but allows also design of windows with different properties, corresponding to some newly defined window classes. Both symmetrical and periodic (DFT) windows can be designed. The principle of the method is described and definitions of the new window classes and examples of designed windows are presented.

Keywords: digital signal processing, data windows, windows design

1. INTRODUCTION

Multiplication of the signal part to be processed by a function of the same length N , usually symmetrical around axis in its centre and smoothly approaching zero values at its ends (sometimes with the left-out last sample) and called “window” is common practice in digital signal processing. The aim of windowing is reducing energy leakage in frequency domain in spectrum analysis by DFT (discrete Fourier Transform) in case of non-coherently sampled signal [1], [2] or reducing frequency response ripple by designing FIR filters [3]. Reference [2] presents overview, definitions of basic parameters, coefficient values and comparison of spectra and properties of a large number of windows. Some corrections to windows described in [2] and some additional

windows are presented in [4] (design is based on Fourier Transform of continuous data windows, cosine windows with order up to 3 are designed for minimum side-lobes level or rapid side-lobe decay combined with minimal side-lobes level and compared with Kaiser-Bessel windows). Generalized cosine windows are used by far most frequently because of the ease of their mathematical handling. Coefficients of these windows for the window orders up to 10 found for minimum side-lobe magnitudes are presented as decimal numbers of the length of 16 in [5]. A design procedure based on window spectrum zeros position is also presented there, but the mathematical handling in [5] differs from the one presented here and the in [5] described algorithm is applied only to design of windows with minimum relative side-lobe magnitude.

Signal windowing influences values of the estimated signal parameters as e.g. RMS value [6], [7], [8], active power [9], power spectral density [10], phase or THD, so appropriate numerical corrections of the bias produced by windowing have to be used. Windowing is used also in interpolated DFT [11] and its applications (e.g. [12]). A window design method based on energy parameters allows high accuracy estimation of signal harmonic component parameters using small number of signal samples and is therefore suitable for real-time parameter estimation [13]. Besides bias reduction of the measured signal parameter caused by windowing, also influence of additive noise on windowed signal parameter estimation should be taken into account. It is determined by window equivalent noise bandwidth [2] and for RMS value estimation it is investigated for flattop cosine windows in [7] and for Rife-Vincent windows of the classes 1 to 3 in [12]. Design procedure based on iterative minimization of maximum ripple of window spectrum within basic window magnitude spectrum frequency bin, numerical parameters of several flattop windows and their comparison to classical windows by RMS values estimation without using DFT interpolation are presented in [7].

Even if by the most applications some of classical windows, the coefficients of which are generally known, are used, sometimes another window with its spectrum fitted to a given application would be more useful. The ideal window would have minimum main lobe width, minimum side-lobes level, maximum side-lobes fall-off and minimum equivalent noise bandwidth, but these requirements cannot be fulfilled

simultaneously. There is in fact no ideal signal window, any given application requires its own optimum window parameters. These optimum parameters depend on frequency content of the measured signal and on the estimated signal parameter. For example, minimum main lobe width is required if there are some strong frequency components close to each other in the processed signal. The low side-lobes level is required if weak frequency components close to a strong component in a multi-frequency signal have to be detected. If a high broadband additive noise is present, a window with low equivalent noise bandwidth should be used. The used window presents usually a compromise of several conflicting requirements. This paper presents a general design method of cosine windows and some new window classes with selectable additional properties as compared to the known cosine window classes. Classical Rife-Vincent windows of all three classes can be designed by this procedure, also for window orders higher than those presented in [1], [12], and other publications known to authors. Window design is based on iterative placing of window spectra zeros on frequency axis so that the desired magnitude window spectrum properties are fulfilled. We have used Matlab environment for all calculations. A graphical user interface allowing easy definition of desired window spectrum properties, calculation and presenting of window spectra zeros and window coefficients (allowing their presentation also as ratios or decimal numbers with selected numerical accuracy) is prepared. This GUI also shows designed window spectrum, and window shape in time domain. Examples of windows designed by the described method are included in parts 3.2 and 3.3.

2. PRINCIPLE OF THE WINDOWS DESIGN BY WINDOW SPECTRUM ZEROS PLACEMENT

2.1. Cosine window spectrum, its zeros and their context with window coefficients.

Cosine window of the order L , defined for samples $n = 0, 1, \dots, N_{win} - 1$, where N_{win} is window length in samples and D_r are window coefficients (numerical values of which are in vast majority of in practice used windows changing signs regularly with increasing r , starting from the positive value of D_0) can be expressed as

$$w(n) = \sum_{r=0}^L D_r \cos\left(2\pi r \frac{n}{N_{win}}\right) \quad (1)$$

Its magnitude spectrum is composed of sum of $2L+1$ spectra of rectangular window with magnitudes determined by window coefficients and shifted by 0 to $\pm L$ frequency bins. The complex DFT spectrum of a sampled rectangular window (normalized to maximum value of 1 and of the length of N samples) can be expressed as the so-called Dirichlet kernel

$$W_r(\theta) = e^{-j\pi\theta \frac{N-1}{N}} \frac{\sin(\pi\theta)}{N \sin\left(\frac{\pi\theta}{N}\right)} \quad (2)$$

where θ is frequency expressed in frequency bins.

The corresponding continuous-time magnitude window spectrum can be found from (2) as a limit for $N \rightarrow \infty$ and simultaneously $T_s \rightarrow 0$ (T_s being sampling period) so, that the window length in seconds $T_r = N \cdot T_s$ is preserved. It is

$$|W_r(\theta)| = \frac{\sin(\pi\theta)}{\pi\theta} \quad (3)$$

The normalized magnitude spectrum of a continuous-time general cosine window can be expressed as

$$|W(\theta)| = \frac{\sin \pi\theta}{\pi} \theta \sum_{r=0}^L \frac{D_r}{(\theta^2 - r^2)} \quad (4)$$

Equation (4) can be expressed decomposed into two multiplicative components

$$|W(\theta)| = \left(\frac{\theta \sin(\pi\theta)}{\pi \prod_{r=0}^L (\theta^2 - r^2)} \right) \cdot \left(\sum_{r=0}^L D_r \prod_{\substack{k=0 \\ k \neq r}}^L (\theta^2 - k^2) \right) \quad (5)$$

The first component does not depend on window coefficients and this component's zeros (we shall call them *fixed window spectrum zeros*) define the shape and properties of the initial window. The initial window spectrum can be modified by the second component. The desired modification is achieved by a suitable placement of the zero values of this component. These zeros depend on window coefficients. We shall call them *movable window spectrum zeros*. The first component zeros lie on the integer frequencies expressed in frequency bins of $\pm(L+1)$ and outside this interval. Without the influence of the second component of (5) the first zero value corresponds to the half of the window spectrum main-lobe width (and it is, similarly like all fixed zeros, a real value zero).

The *movable* spectrum zeros can be found by solving the equation

$$\sum_{r=0}^L D_r \prod_{\substack{k=0 \\ k \neq r}}^L (\theta^2 - k^2) = 0 \quad (6)$$

and the magnitude spectrum of the window can be then expressed (Z_r are the movable spectrum zeros) as

$$|W(\theta)| = \frac{\sin \pi\theta}{\pi\theta} \cdot \prod_{r=1}^L \frac{\theta^2 / Z_r^2 - 1}{\theta^2 / r^2 - 1} \quad (7)$$

Number of the positive real part value movable zeros corresponds to the order of window L and it represents also number of degrees of freedom by window design. A direct analytical movable zero finding from (6) is simple for windows of the order 1 and 2, but difficult for window order 3 and practically impossible for higher order windows. Calculation of movable zeros from window coefficients and vice versa is possible if the used mathematical tool offers implementation of procedures suitable for finding zeros of a given function, for building a polynomial from its zeros and for basic matrix operations. One of the frequently used

instruments offering all these capabilities is Matlab, offering besides matrix operations also functions *roots* and *poly*. For any window order L a matrix A of the dimensions $(L+1, L+1)$ can be found (based on (6)) for which there is

$$Z = \pm \sqrt{\text{roots}(A \cdot D)} \quad (8)$$

D being vector of window coefficients (size $L+1$) and Z being vector of movable zeros of the window (size L for positive real-part window zeros and size $2L$ for all window zeros, with regard to the spectrum symmetry).

The window coefficients corresponding to the movable zeros can be found as (in Matlab notation)

$$D = A \setminus \text{poly}(Z^2) \quad (9)$$

The research of windows design by placing the window spectrum zeros is motivated by the comparatively low sensitivity of the window magnitude frequency spectrum to the change in window spectrum zeros values compared to the sensitivity of this spectrum shape to changing window coefficient values. The real-value spectrum zeros correspond to frequencies on which the window magnitude spectrum crosses the zero level. Window coefficients are bound with these crossing values by the functions *roots* and *poly* (see (8) and (9)). A small change in coefficient values can lead to essential changes of the movable zeros and consequently also of window shape and corresponding properties. The searched space of possible values of movable zeros is more smooth than a space of values of the coefficients, and that is why a simple search algorithm can be used to find the zeros values.

2.2. Window spectrum zeros and their influence on window shape and properties

Based on (5), it is possible to take into account separately a component depending solely on window order (defining “initial” window shape, determined by window spectrum fixed zeros), and another component of window spectrum zeros vector, composed of movable zeros. These zeros allow to change the designed window shape and to optimize the window properties. The design of classical hamming window shown in Fig.1 can serve as a typical example. Placing its only one movable zero on the frequency of approximately 2.598 frequency bins results in

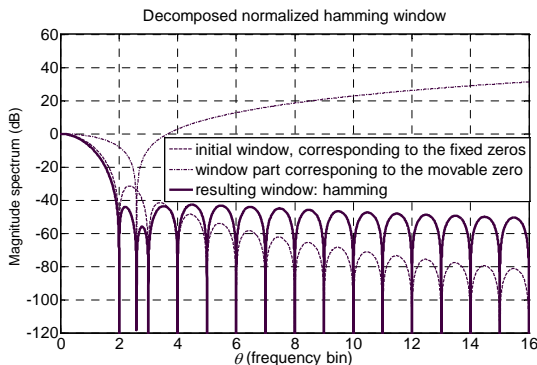


Fig. 1. Magnitude frequency spectra of hamming window and its two components, corresponding to fixed and to movable window spectrum zeros (there is only one movable zero here).

substantial decrease of the window spectrum first side-lobe magnitude (but paid for it by slower side-lobe fall-off).

Rectangular window is a zero-order cosine window and it offers therefore no movable zero. Higher-order windows offer several movable zeros (their number in each frequency half-plane equals to window order L , and their values are symmetrical for positive and negative frequencies). Magnitude window spectrum of higher-order windows can therefore be more distinctly changed and more complex requirements on its properties can be fulfilled. The difficulty of movable zeros optimal placement increases with window order, since also number of dimensions of possible solutions increases.

Important window properties can be influenced by movable window placement, including e.g. main lobe width, side-lobes fall-off or effective noise bandwidth of the window (*ENWB*).

3. SEARCH ALGORITHM, WINDOW CLASSES AND DESIGNED WINDOWS EXAMPLES

3.1. Selection and design of the search algorithm

The window design task is to find optimal placement of movable spectral zeros to fulfill prescribed spectrum properties. Movable zero positions form an L -dimensional space of real positive values. This space is searched through by a suitable search algorithm. *Greedy search algorithm* finding *gradient* in a point surrounding was selected and for increasing search speed and security against finishing search on a local extreme a stochastic search component was included to it. It means that in every step a random point in close surrounding of the actual point is selected in the searched L -dimension movable zeros space, and the range of proximity (simultaneously in every dimension) is step by step diminished so that the algorithm be faster at the beginning and more precise close to the end of search.

The magnitude spectrum is computed for each new tested state (zeros value set), its basic parameters are evaluated and the state is evaluated according to the selected optimum criterion (window class). We have experimentally found that the algorithm is not much sensitive to the initial placement of the movable zeros.

The search is finished after a selected number of consequent unsuccessful tests of the point surrounding is passed and when the selected accuracy of the movable zeros positions was achieved. It was experimentally found that the accuracy of movable zeros corresponding to three decimal places is sufficient for the good agreement with the commonly used window coefficients accuracy.

3.2. Cosine window classes (known and newly defined)

The three basic classes of cosine windows were defined by D. C. Rife and G. A. Vincent [1]. They are denoted here as classes RV1, RV2 and RV3. Windows of the class RV1 are designed for maximization of window spectrum side-lobes fall-off. There is an analytical prescription for the design, based on number of time domain window derivatives zeros on window ends [4].

Windows of the class RV2 (based on Dolph-Tchebyshev windows) are designed for minimization of the window spectrum main-lobe width for a given maximum

level of side-lobes relative magnitude R . There is no exact analytical prescription for the Dolph-Tchebyshev windows, but an approximation (Taylor approximation) with analytical formula providing good results for low enough R is used.

Windows of the class RV3 are designed as a compromise between windows RV1 and RV2 – i.e. between window side-lobes fall-off and the window main-lobe width by given relative amplitude of the-window side-lobes.

A comparison of these windows for window order $L = 5$ is presented in Fig.2.

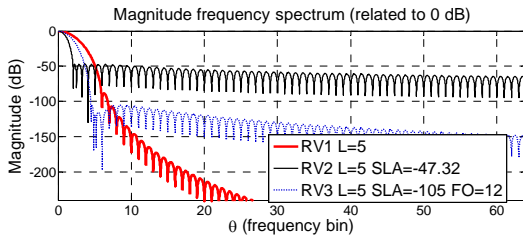


Fig. 2. Magnitude frequency spectra of windows RV1, RV2 and RV3, window order 5, parameters of windows RV2 and RV3 see the legend (SLA is the highest side-lobe level in dB, FO is side-lobes fall-off in dB/oct).

Some new additional classes are proposed in this paper.

Windows of the *class 4* (C4) minimize relative side-lobe level by a prescribed side-lobes fall-off. A comparison of window RV3 and C4 for identical window order L and side-lobe fall-off FO is presented in Fig.3.

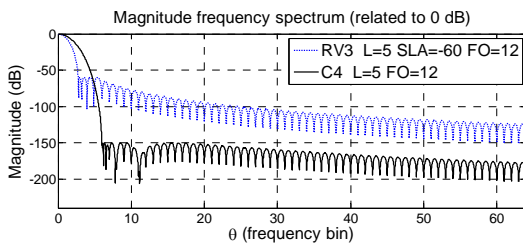


Fig. 3. Magnitude frequency spectra of windows RV3 and C4, window order 5, parameters of windows see the legend (SLA is highest side-lobe level in dB, FO is side-lobes fall-off in dB/oct).

Windows of the *class 5* (C5) minimize relative side-lobe level by a prescribed number of movable zeros in infinity (NZ). Using remaining movable zeros for side-lobe minimization leads to the best relation between side-lobe attenuation and side-lobe fall off (an example of two magnitude spectra of class 5 windows see Fig. 4).

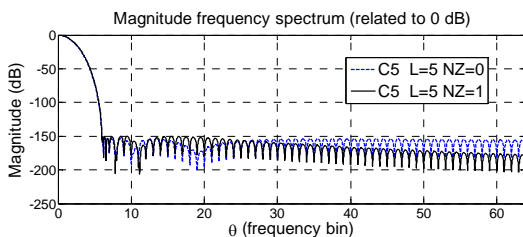


Fig. 4. Magnitude frequency spectra of windows of class C5, window order 5, number of zeros in infinity (NZ) see the legend.

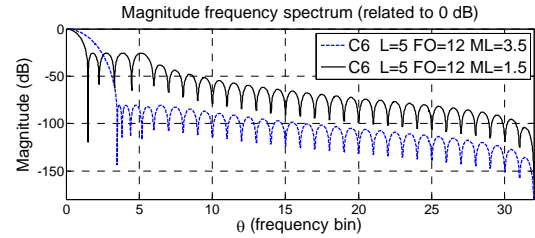


Fig. 5. Magnitude frequency spectra two windows of class C6, window order 5, FO is side-lobe fall-off in dB/oct., ML is a half of the main-lobe width in frequency bins.

Windows of the *class 6* (C6) minimize the relative side-lobes level by a prescribed main-lobe width (in frequency bins) and prescribed minimal side-lobe fall off (Fig. 5). The main-lobe width can be selected for a concrete application so that the main-lobes placed on closest neighboring signal components do not overlap.

Windows of the *class 7* (C7) minimize relative side-lobe level starting from a prescribed (“critical”) frequency (Fig. 6). The design is similar to design used in class 8 windows but it allows designing windows with lowest side-lobe level from a prescribed frequency.

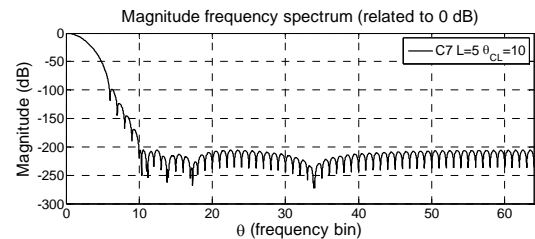


Fig. 6. Magnitude frequency spectrum of a window C7, window order 5, θ_{CL} is critical frequency in frequency bins - see the legend (the achieved relative side-lobe level is -205.95 dB).

Windows of the *class 8* (C8) minimize the frequency above which the spectrum side-lobes level does not surpass a given level (Fig. 7).

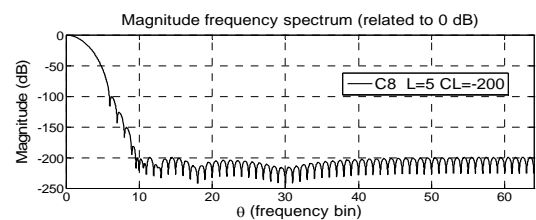


Fig. 7. Magnitude frequency spectrum of a window C8 and window order 5, CL is side-lobes level in dB (corresponding minimum frequency θ_{CL} that is here 9.49 frequency bins).

3.3. An example selection of designed windows and their basic parameters)

A few examples of designed windows (their zero positions, window coefficients and basic window parameters) are presented in table 1. Majority of the presented windows corresponds to figures in the part 3.2. Following abbreviations and symbols are used in this table: D - window coefficients normalized to maximum window time-domain value, i.e. maximum level of the window in time domain being 1, coefficients are given for windows

defined for positive time, Z - movable spectral zeros; ML - half of main-lobe width (in frequency bins) - note that it does not mean the complete window main-lobe, SL - first side-lobe level (in dB), SLA - highest side-lobe level (in dB), BW (-3dB) 3 dB main-lobe bandwidth, FO - side-lobes fall-off (in dB/octave), $ENBW$ - equivalent noise bandwidth BW (in frequency bins), L - window order, NZ - prescribed number of zeros in infinity, CL - (critical level, in dB) maximum side-lobe level behind selected frequency θ_{CL} , θ_{CL} - (critical frequency in frequency bins) frequency reached for given CL .

Table 1. Examples of selected windows basic parameters, windows spectrum zeros Z and windows coefficients D .

Window	movable zeros R, window coefficients D						
	ML (fr.bin)	SL (dB)	SLA (dB)	BW (-3dB) (fr.bin)	FO (dB/oct.)	$ENBW$ (fr.bin)	additional
RV1 L=5	6,00	-87,94	-87,94	2,75	72,39	2,91	
	R=[Inf Inf Inf Inf Inf] D=[63/256 -105/256 15/64 -45/512 5/256 -1/512]						
RV2 L=5 SLA=-47.32	2,07	-47,32	-47,32	1,33	5,69	1,39	
	R=[2.07096 2.52934 3.25873 4.11879 5.04723] D=[8665/16446 -1655/3557 49/9634 -125/32111 -37/20152 -8/11403]						
RV3 L=5 SLA=-105 FO=12	4,50	-105,00	-105,00	2,02	17,31	2,13	
	R=[4.49857 4.89090 5.20852 6.02999 Inf] D=[87865/260803 -163088/340445 487432/2996403 -53756/2565221 572/1343281 -1/1436838]						
C4 L=5 FO=12	6,00	-150,27	-150,27	2,32	16,94	2,46	
	R=[6.18579 6.56122 7.84510 11.15333 Inf] D=[10278995/35334188 -11383814/25293643 8869544/43553097 -1010395/20299417 257578/47321941 -1464/9195433]						
C5 L=5 NZ=0	6,00	-153,57	-153,57	2,30	5,30	2,43	
	R=[6.17587 6.51849 7.76470 10.28907 19.05314] D=[1872717/6379399 -6502663/14388489 3612425/17935034 -512357/10690410 20451/4068748 -2060/14974699]						
C5 L=5 NZ=1	6,00	-150,27	-150,27	2,32	14,71	2,46	
	R=[6.18579 6.56123 7.84509 11.15333 Inf] D=[7094377/24387021 -123035759/273372586 6971137/34231138 -3225252/64797149 196116/36030181 -1802/11318395]						
C6 L=5 FO=12 ML=3.5	3,50	-80,70	-80,70	1,80	17,59	1,90	
	R=[3.50000 3.82589 4.40949 5.18228 Inf] D=[54441/143207 -35591/71888 10153/84762 -487/98298 29/471144 14/318861]						
C6 L=5 FO=12 ML=1.5	1,50	-25,66	-25,66	1,17	17,11	1,22	
	R=[1.50000 2.25740 3.30254 4.48199 Inf] D=[379/622 -1737/4438 -299/6074 -174/3605 -1134/18869 -573/9496]						
C7 L=5 $\theta_{CL}=10$	6,00	-98,73	-98,73	2,57	9,91	2,72	$CL=-205,95$ dB
	R=[10.31773 11.17172 13.78587 17.32376 33.84168] D=[6567959/25013889 -50589330/118760947 42844267/191072178 -6954194/95222661 221466/16781899 -149108/150161583]						
C8 L=5 CL=-200	6,00	-100,84	-100,84	2,54	9,40	2,69	$\theta_{CL}=9,5$ fr.bin
	R=[9.55481 10.43171 12.38790 17.03281 32.27557] D=[14640889/55220399 -86468074/201877425 6432467/28906250 -6461703/91264951 1624676/131703081 -117185/133356736]						

3.4. Complex values of movable window spectra zeros and flattop windows.

As we have mentioned in part 3.1, the movable zeros used by us for window design are real numbers. Some movable zeros of cosine window spectra can nevertheless be complex numbers. They are not suitable for the above mentioned window classes design, since they do not lead to so good window properties as in the case of window design

based on all zeros being real. They also do not correspond to (real) frequency values for which the magnitude window spectrum crosses the zero level.

Nevertheless, also these zeros can be used in window design. The so-called *flattop windows*, i.e. window designed for maximum flatness of their main lobe in the frequency range $\pm 0,5$ frequency bin, can be designed with their use. The goal of flattop window design is to design a window with such a spectrum, that it will be without any additional processing as much as possible insensitive to the non-coherency of sampling. These windows are useful in multi-frequency signal frequency analysis, since when using them for processing individual components of these signals, there is no necessity using interpolated DFT.

After selecting a suitable cost function for the search algorithm, the above described method can be adapted also for design of flattop windows of various orders. Their design is more complicated because of the additional dimension of window zero values.

Magnitude frequency spectra of two flattop windows (one of the 3rd order, the other one of the 4th order) designed by means of zeros placement, both with one imaginary movable zero, and magnitude frequency spectrum of the window implemented in Matlab as "flattop window" are shown in Fig. 8 and Fig. 9. Note especially the better flatness of the new windows in the interval ± 0.5 frequency bin clearly shown in the zoomed spectra in the first frequency bin in Fig.9.

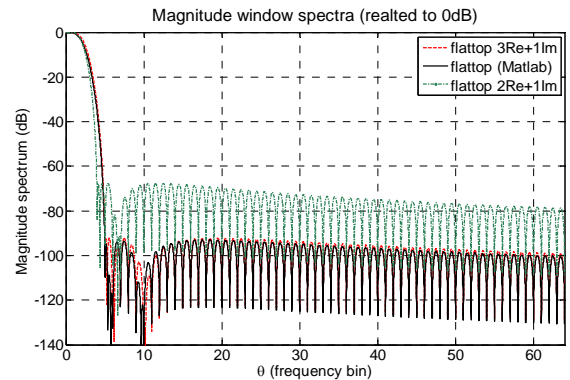


Fig. 8. Magnitude frequency spectrum of three flattop windows

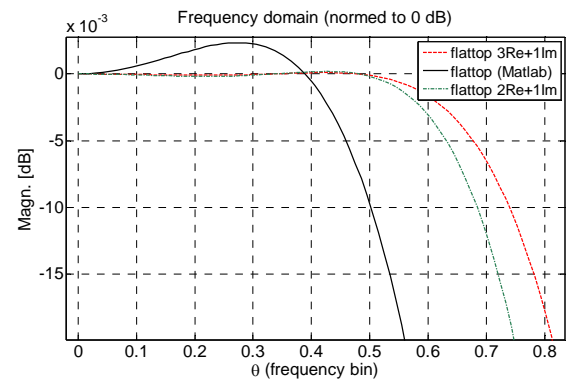


Fig. 9. Magnitude frequency spectrum of three flattop windows shown for frequency around 0.5 frequency bin

4. CONCLUSIONS

An effective method of iterative design of cosine windows of nine different classes including the three known Rife-Vincent window classes, the flattop window class and five newly defined window classes was presented and examples of window magnitude spectra, window coefficients and basic window properties were overviewed for selected windows representing all the newly defined classes. We have simplified the presented window design by developing a specialized graphical user interface in Matlab environment, where also all computations were performed. The newly defined window classes allow some additional window spectrum properties prescription compared to windows described in [1] and [5], but design of windows from [1] and [5] is also covered by the presented method and the used algorithm. The described method can be used also for flattop-type windows design. Some window zeros may in this case be complex numbers and can optimize the main-lobe window magnitude spectrum flatness.

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