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MATRIX METHOD FOR LCMM — CONNECTION BETWEEN SUBSPACES OF REFERENCE POINTS

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Abstract – The article describes the transplantation of Matrix Method as an assessment of accuracy of coordinate measurement for Large CMM. The method of the synthetic description of the accuracy is used successfully for small and medium-sized CMMs. It is implemented for large measurements as research project nr: PB 5T07/D03824. The issue involves many problems, especially from a technical point of view. This article presents experiments concerning connections between reference subspaces – the crucial elements for implementation of Matrix Method.

Keywords LCMM, calibration, uncertainty

1. INTRODUCTION

One of the most important problems of metrology today is the estimation of accuracy of coordinate measurement. Currently, used uncertainty model, based on the control length measurement, does not fully reflect universal aspects of coordinate measurement. This leads to a situation, where purchasing decisions about CMM and subsequent decisions about the type of measurements, are taken based on incomplete information.

This problem became the impulse of works on methods of accuracy assessment of coordinate measurement. Thanks to the works, the proposition of connection of various methods, appeared within the framework of the standard ISO 15530 [7].These works are the result of earlier research on accuracy assessment of CMM, done mainly by PTB, where one of the first virtual coordinate machine, giving the possibility of accuracy assessment by simulation, was elaborated [6].

This model was based on earlier identified geometric errors of CMM and errors from the probehaed. Similar model was elaborated in Cracow University of Technology [1].

Thanks to arbitrary acceptance of error model, these systems can not map the behavior of CMM. That's why the change of approach succeeded. As the base, not only the analytical error model of CMM was taken but also the synthetic model, based on the error of mapping of measuring point.

The method of recreation of measuring point by the machine was called the Matrix Method [3, 4].

Information from the error of recreation of measuring point, allows the later assessment of accuracy of any measuring task for any construction of coordinate machine with hybrid construction taken into account. It is caused by the fact, that measurement of point coordinates is the direct measurement (in coordinate metrology).

2. MATRIX METHOD

The base of Matrix Method is the acceptance of vector character of error of recreation of measuring point (Fig. 1). The error $\overline{e}$ is the difference between indication of point of contact $\overline{p}_m$ by the machine, treated as a leading vector extended from the beginning of coordinate system to this point, and real point of contact of the probehead with the measured space $\overline{p}_a$, treated here also as a leading vector.
It possible to present this by the following equation:

$$\overline{e} = \overline{p}_m - \overline{p}_a \tag{1}$$

In the research on errors of coordinate machines, it is possible to distinguish two errors from many of them, which depend on the same CMM. It is the error from contact system of CMM and error followed from geometric solution of the machine construction. Than the representation of error $\overline{e}$ from the equation (1) is possible:

$$\overline{e} = \overline{e}_{cmm} + \overline{e}_{prb} \tag{2}$$

where $\overline{e}_{cmm}$is the error of recreation of the position of CMM whereas $\overline{e}_{prb}$ is the component from contact system of the machine.

It was illustrated on Fig. 1.

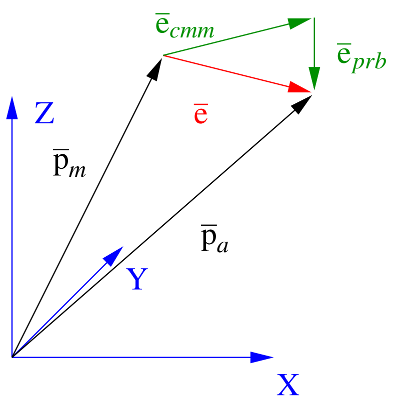

Fig. 1. Vector character of the error $\overline{e}$

The error $\overline{e}_{prb}$is dependent on the direction of invasion vector $[nx, ny, nz]$ and used configuration of gauge plunger. Whereas the error of position is dependent on XYZ position of the measuring point in measuring space of CMM.

The measure of the accuracy of recreation of measuring point by the machine in its measuring space is the sum of the longest length of the vector $\overline{e}_{cmm}$ and maximum length of vector $\overline{e}_{prb}$ what can be written by the equation:

$$U_{MM} = \max|\overline{e}_{prb}| + \max|\overline{e}_{cmm}| \tag{3}$$

2.1. *Technical realization of the determination of U_{MM}*

The determination of the error $\overline{e}_{prb}$ is connected with the measurement of spherical artifact or artifact ring. In the second case in publication [4]there was the proposition to measure this artifact 64 times. The diameter of this artifact should not exceed 30 [mm]. Obtained information allows to identify the error $\overline{e}_{prb}$in perpendicular direction. It is sufficient in measurements of mechanical parts where the direction of invasion on the measuring point is usually perpendicular (measurements of the holes) or parallel (measurements of surfaces) to the tip axis.

However, taking into account the growing number of measuring tasks connected with the control of elements with curved spaces of freedom like the measurements of body of the car elements or geometry of car windows and also turbine blades, the error $\overline{e}_{prb}$ need to be identified on the basis of the measurement of spherical artifact, where the research, leading to determination of the error as the function dependent on all three direction of the invasion, is possible to be done.

In the case of the determination of the error $\overline{e}_{cmm}$the elaboration of the network of referee points in measuring space of the machine is necessary. In the uniformly spaced points in network, determination of error of recreation of position by CMM is possible.
The technical realization of the network is based on plate artifact. The realization consists in measurement of the artifact in various positions and three orientations parallel to the main surfaces of the measuring space. Thanks to that, in this space, the network of referee points is created and these points are the centres of the referee elements of the artifact.
On the Fig. 2 this method is presented.

In this case the artifact is measured in parallel to the three main surfaces of the measuring space. From the obtained local deviation dx and dy, in particular surfaces, there is the possibility to determine the spatial error $\overline{p}_p$ in accordance with the equation:

$$\overline{e}_{cmm} = \frac{1}{2}\begin{bmatrix} dx_{XY} + dx_{XZ} \\ dy_{XY} + dx_{YZ} \\ dy_{XZ} + dy_{YZ} \end{bmatrix} \tag{4}$$

The devation dx_{XY} means the deviation of the plate, in direction of local axis x, settled in measuring space of the machine in parallel to surface XY. Particular coordinates of the vector $\overline{p}_p$ are, in this case, determined as the diameter of the deviations in the same direction from the two mutually perpendicular positions of the artifact.

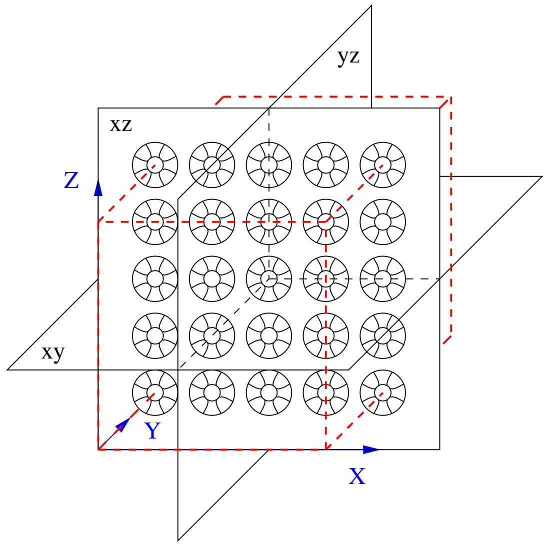

Fig. 2. The positions of the plate artifact to built the network of reference points

3. MATRIX METHOD FOR LCMM

Within the framework of research project PB:5/T07/D03824 it was decided to transpose Matrix Method on LCMM.
Lots of problems ware found connected with technical realization of the network of referee points in measuring spaces of LCMM. It is mainly connected with lack of the artifacts, which could be the substitute used in small and medium-sized machines.

he solution is the elaboration of networks of points in subspaces — where the measurements are most often made. Than subspaces can be built with the help of plate artifacts, nowadays available on the market for small and medium-sized machines. The build method is the same what in matrix method for the machines of smaller measuring spaces(Fig. 2).

However there is the problem with connection of the subspaces. There is the possibility to use the overlap method and try to cover all the measuring space of LCMM but this is long process and it can lead to considerable errors caused by movements of artifact [6].
The connection is in order to connect two subspaces in one. Thanks to that, there is the possibility to find out what is the error of position of machine in a case when the beginning of the coordinate system is in one subspace and the measuring point in other subspace. In this case the connection is the vector, which allows to create one subspace instead of two. The problem is illustrated on Fig. 3

The vector $\overline{v}_{I-II}$ is the vector which connect the subspace $P^I$with the subspace P^{II}.Assuming that the beginning of the system is in subspace P^I we want all the research points to be connected with this subspace. That's why in the case when the measuring point is in subspace P^{II} the error $\overline{e}^{(I-II)}_{cmm}$ is:

$$\overline{e}^{(I)}_{cmm} = \overline{v}_{I-II} + \overline{e}^{(II)}_{cmm} \tag{5}$$

The aim of the research is determination of the vector $\overline{v}_{I-II}$.

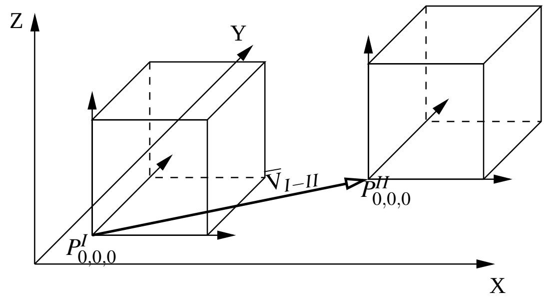

Fig. 3. The connection of subspaces

One of the possibilities was the movements of plate artifact with the help of guides and control of its position and orientation with the help of laser interferometer. The conception of this construction is presented in [5]. Farther work on this solution did not bring satisfactory results. That's why it was decided to use the analytical model of geometric errors as the connection between the particular subspaces.

This model describes the error of measurement by the equation [6]:

$$\overline{E} = \overline{T} + \mathbf{A} \cdot \overline{X} + \mathbf{A}_p \cdot \overline{X}_p \tag{6}$$

where $\overline{T}$ is the vector summing the errors of translation , $\mathbf{A}$ — the matrix accumulating the errors of rotation having an influence on the measurement by referee tip , $\overline{X}$ — Coordinates of referee point , $\mathbf{A}_p$ —matrix accumulating the errors of rotation having an influence on indication of CMM in case of using no-referee tip in measurement $\overline{X}_p$ — vector extended between referee tip and tip used to the measurement.

To create the connection between subspaces, the information about errors of position xtx, yty, ztz and error of translation, being in the vector $\overline{T}$, is not enough. The information about the errors in matrix $\mathbf{A}$ is also needed.
It can be written that the error of position $\overline{E}_p$ is:

$$\overline{E}_p = \overline{T} + \mathbf{A} \cdot X \tag{7}$$

where [6]:

$$\overline{T} = \begin{bmatrix} xtx + xty + xtz \\ ytx + yty + ytz \\ ztx + zty + ztz \end{bmatrix} \tag{8a}$$

$$\mathbf{A} = \begin{bmatrix} 0 & -ywz - xrz & xwz + xry + yry \\ 0 & 0 & -ywz - xrx - yrx \\ 0 & xrx & 0 \end{bmatrix} \tag{8b}$$

$$\overline{X} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \tag{8c}$$

Determination of these errors is possible with the help of laser interferometer or artifacts. In the case of LCMM the possibility of use of two-dimensional artifacts is limited. That's why the procedure concerning determination of geometric errors LCMM using Laser Step Gauge was elaborated [2].

Application of position error $\overline{E}_p$leads to determination of the error which would be done by the machine in beginning point of subspace P^{II}if the coordinate system would be in subspace P^{I}. So the conclusion that vector $\overline{v}_{I-II}$ is:

$$\overline{v}_{I-II} = \overline{E}_p(P_0^{II}) - \overline{E}_p(P_0^{I}) \tag{9}$$

where $\overline{E}_p(P_0^{II})$ is the error of position determined in accordance with equation (7) for the beginning of subspace P^{II}.

4. RESEARCH

Within the framework of research, above method, concerning determination of accuracy, was verified on the basis of the assumptions of matrix method using subspaces. Research done on GLOBA IMAGE machine was divided in two parts:

1. Determination of subspaces in order to determination of the error e_{cmm} in subspaces.

2. Determination of geometrical errors CMM in order to determination of connection between particular subspaces.

4.1. Determination of the error e_{cmm} in control subspaces

Within the framework of research the spherical plate in scale 1:2 was made in comparison to common spherical plates (Fig. 4. Thanks to it, the simulation of the conditions of LCMM was possible using medium-sized machines. We could also compare obtained results from the method based on determination of subspaces with the method described in [4]. Within the framework of research four control spaces were determined in the way, that two of them overlapped on last two. In Fig. 5the particular positions are presented in diagram.

The building of the subspace was carried out in accordance with the diagram showed in Fig. 2.Each position of the plate was measured few times in order to determinate the variation of the error $\overline{e}_{cmm}$ of the machine.

Obtained results for subspace $P^{I}$are presented in Fig. 6

4.2. Determination of position error CMM on the basis of geometrical errors

Determination of geometrical errors CMM was the next step in research. Their identification allows to determine the position errors $\overline{E}_p$ for the beginnings of systems of subspace.

In order to determinate the position errors, the laser interferometer ML GOLD Renishaw with the set to measurements of rectilinearity, rotation and also mutual perpendicularity errors was used. In accordance with the equations (7-8b) the following geometrical errors: $xtx, xty, xtz, xrx, xry, xrz, yty, ytx, ytz, yry, yrx, yrz, ztz, ztx, zty$ along specified axes and also mutual perpendicularity axes errors were determined. Fig. 7 shows the determination of error ztx using Wollastone V-block and the results.

Fig. 4. Ball-plate to verification of the method

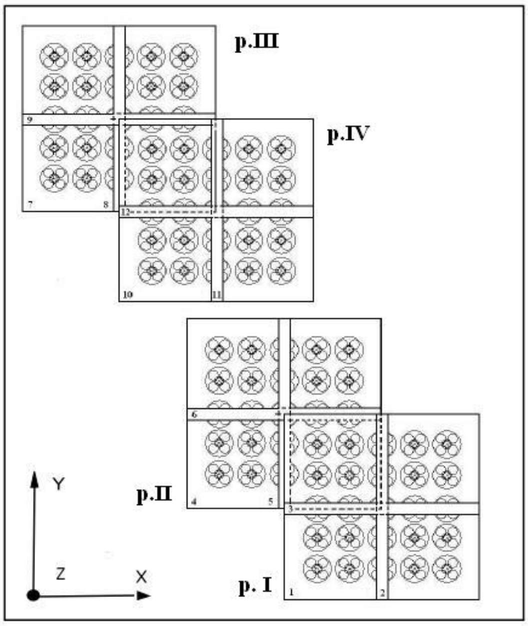

Fig. 5. Particular positions of control subspaces in measuring space CMM

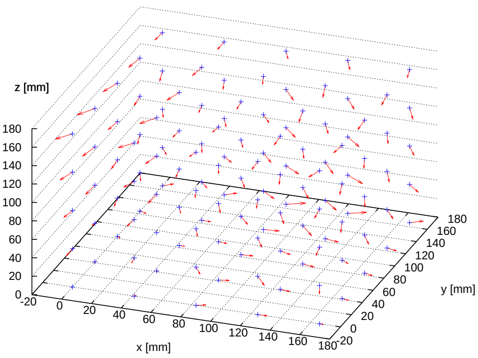

Fig. 6. Determined subspace P^I

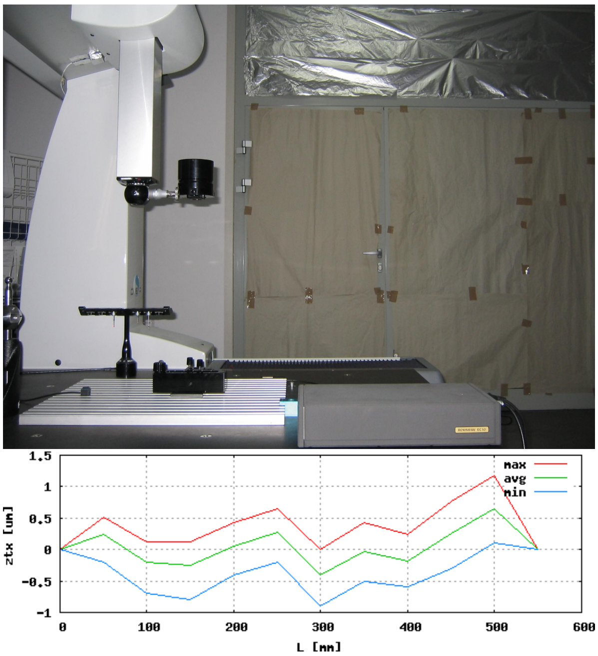

Fig. 7. Determination of geometrical error ztx and obtained results

Tab. 1. Error $\overline{\mathrm{E}}_p$ for particular beginnings of subspaces

nr	subspace			vector $\overline{\mathrm{E}}_p$		
	X	Y	Z	X	Y	Z
I	386.0	146.0	-647.1	-0.0035	-0.0015	-0.0003
II	51.0	709.0	-647.1	-0.0010	-0.0025	-0.0008
III	175.5	584.5	-647.1	-0.0045	-0.0039	-0.0013
IV	258.5	270.5	-647.1	-0.0041	-0.0029	-0.0008

Determined errors $\overline{\mathrm{E}}_p$ of all subspaces in Fig. 5 are presented at in Tab. 1

It allows to determine the vector of connection $\overline{\mathrm{v}}$ in accordance with (9) in further stage.

4.3. Determination of error $\overline{\mathrm{e}}_{prb}$

In further part of research the error $\overline{\mathrm{e}}_{prb}$ was determined. It was done by the measurement of spherical artifact in 16th intersections form the equator to the pole. On the equator the sphere was measured in 64 points. The number of points was decreased with the intersection so there was only one measurement on the pole.

4.4. Determination of the uncertainty of positioning

Determination of error $\overline{E}_p$ for particular subspaces allows to describe the error $\overline{e}_{cmm}$ for the research space. In accordance with (3) this error is the maximum length of the vector determined in accordance with equation (5).

Obtained maximum values of error $\overline{e}_{cmm}$ for particular subspaces are in Tab. 2.

Tab. 2. Values u_p for each subspace

Subspace	u_p
P^{I}	0.0014[mm]
P^{II}	0.0026[mm]
P^{III}	0.0031[mm]
P^{IV}	0.0023[mm]

The vectors of connections between particular subspaces are presented in Tab. 3

Tab. 3. Vectors of connection between particular subspaces

subspace		P^{I}	P^{II}	P^{III}	P^{IV}
P^{I}	X		-0.0045	0.0025	0.0016
	Y		-0.0025	0.0039	0.0019
	Z		0.0005	0.0010	0.0005
P^{II}	X	0.0045		0.0035	0.0031
	Y	-0.0025		0.0014	0.0006
	Z	-0.0005		0.0005	0.0000
P^{III}	X	-0.0025	-0.0035		-0.0009
	Y	-0.0039	-0.0014		-0.0020
	Z	0.0010	-0.0005		-0.0005
P^{IV}	X	-0.0016	-0.0031	0.0009	
	Y	-0.0019	0.0006	0.0000	
	Z	0.0009	0.0020	0.0005	

Using the results from table 2 and 1, determination of error u_{cmm} is possible as a maximum error of position in research space. For the research machine the final value u_{cmm} is 0.0045 [mm].

5. ASSESSMENT OF MEASURING POINT ACCURACY

Research with small and medium-sized machines showed that there is the possibility to use matrix method to accuracy assessment of coordinate measurement [3].

In information which are obtained from determination of error $\overline{e}_{prb}$ and $\overline{e}_{cmm}$ allows to determine the uncertainty of recreation of mesuring point by research CMM. This action is showed in diagram (8)

In order to approximate the function of errors $e_{prb}(nx,ny,nz)$ and $e_{cmm}(X,Y,Z)$ the neuron networks were used. The other network was used to simulate the position error and other to approximate the errors of contact system. Teaching files were prepared on the basis of files

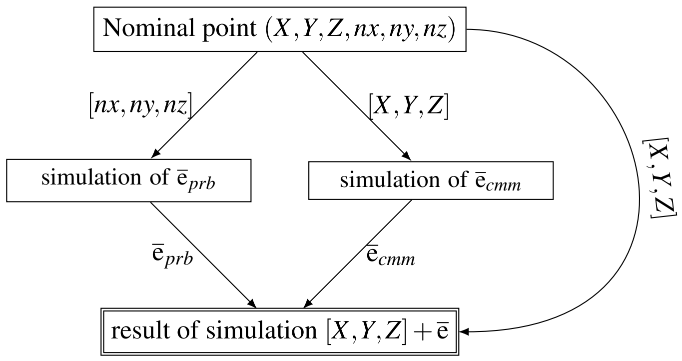

Fig. 8. Virtual measuring machine — determination of uncertainty of measuring task

obtained from the determination of uncertainty of measuring point of CMM. The working method of the simulator was presented on the diagram Fig. 9.

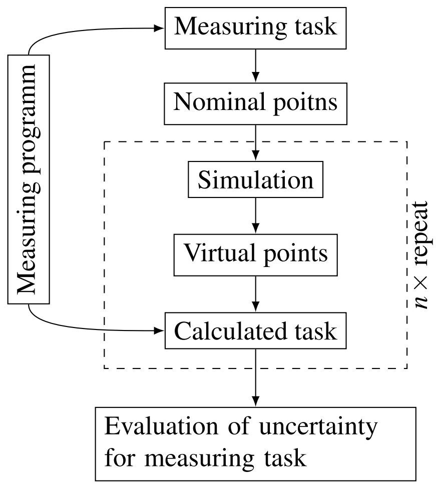

Fig. 9. Virtual measuring machine — determination of uncertainty of measuring task

For the given measuring task, data about nominal points (coordinates and invasion vector) from the computer program are taken. They are simulated appropriate number of times in a virtual measuring machine. Obtained measured virtual points are used to determination of measuring task again also n-times. Than from the obtained multiple result, the determination of uncertainty of measurement is possible.

In the case of this method it is very important that except averaging data, teaching files need to obtain the information about the variation of specified errors. It can be the standard deviation in particular points of network of referee points or in case of error of head $\overline{e}_{prb}$ standard deviation dependant on invasion angles.

Taught neuron networks can, except averaging error $\overline{e}$, give appropriate standard deviation what can be used in simulator to simulate the normal distribution with the help of appropriate function. In this way simulated error $\overline{e}$ can be also saddled with error component.

6. SUMMARY

In the article the technical realization of connection of subspaces of referee points on the basis of analytical model of geometrical error LCMM was presented.

Then there is the possibility to elaborate the method of determination of measuring errors of coordinate machine and assessment of accuracy of this measurement.

Nowadays this problem is especially essential because of the possibilities accuracy assessment of real coordinate measurements with the help of the method based on simulation of coordinate measurements.

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