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CALIBRATION OF CAPACITANCE STANDARDS WITH A QUADRATURE BRIDGE

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Abstract – National Metrology Institutes employ the quadrature bridge in the traceability chain to derive the farad from the ohm (represented with the quantum Hall effect). The bridge calibrates the capacitance product of the two capacitors being measured; a separate measurement with a ratio bridge is usually required to estimate each capacitor independently.

We describe here a method which, by measuring three capacitance standards with the quadrature bridge only, permits to derive the value of each capacitor without resorting to other measurement systems. The method has been checked with an automated quadrature bridge at the level of 1 nF, and verified by measurement on a ratio bridge.

Keywords : Capacitance measurement, bridge circuits, digital systems.

1. INTRODUCTION

The quadrature bridge [1, 2] is a product bridge which compares four standards together, two resistors $R_{1,2}$ of equal nominal value R, and two capacitors $C_{1,2}$ of nominal value C, at angular frequency $\omega = 2\pi f = (R{\cdot}C)^{-1}$. The model equation of the bridge is $\omega^2 R_1 R_2 C_1 C_2 = 1 + \delta$, where $\delta << 1$ is a quantity derived from bridge settings at equilibrium.

The quadrature bridge is a key step in the complex traceability chain which links electrical capacitance and resistance. A number of national metrology institutes (NMIs) realize the farad from the representation of the ohm given by the dc quantum Hall effect; the quadrature bridge is employed to calibrate the product $C_1{\cdot}C_2$ given the estimate of $R_1{\cdot}R_2$. To obtain the estimates of C_1 and C_2, a separate measurement with a resistance ratio bridge is usually performed [2].

We propose here a simple method to obtain, with the quadrature bridge, not only the estimate of the capacitance product, but a separate estimate for each capacitance employed; this without relying on separate ratio bridge.

At INRIM we are developing a new traceability chain to link ohm and farad; two new bridges (a resistance ratio bridge, and a quadrature bridge [3, 4]), both semi-automated and based on the same digital polyphase digital sinewave generator, have been developed.

An older capacitance ratio bridge, manually operated, is still employed to perform scaling down to maintained national standards, to lower capacitance values.

The implementation of the method proposed in this paper, benefits of the automation given by the new quadrature bridge: results obtained are compared with those given by the capacitance ratio bridge.

2. THE METHOD

The method relies on the availability of *three* capacitance ratio standards $C_{A,B,C}$ to be employed in the quadrature bridge instead of the usual two needed.

Three measurements are performed in the sequence with the three possible capacitance couplings (C_A; C_B), (C_B; C_C) and (C_C; C_A), against the same two resistance (R_1; R_2) at angular frequency ω; three bridge readings δ_{AB}, δ_{BC} and δ_{CA} are thus obtained. If we set the parameter $K = (\omega R_1 R_2)^{-1/2}$ the three bridge measurement equations can be written:

$$\begin{aligned} C_A C_B &= K^2(1+\delta_{AB}), \\ C_B C_C &= K^2(1+\delta_{BC}), \\ C_C C_A &= K^2(1+\delta_{CA}). \end{aligned} \tag{1}$$

The system of equations (1) is easily solved to give

$$\begin{aligned} C_A &= K^{-1}\left[\frac{(1+\delta_{AB})\cdot(1+\delta_{CA})}{(1+\delta_{BC})}\right]^{\frac{1}{2}}, \\ C_B &= K^{-1}\left[\frac{(1+\delta_{AB})\cdot(1+\delta_{BC})}{(1+\delta_{CA})}\right]^{\frac{1}{2}}, \\ C_C &= K^{-1}\left[\frac{(1+\delta_{BC})\cdot(1+\delta_{CA})}{(1+\delta_{AB})}\right]^{\frac{1}{2}}. \end{aligned} \tag{2}$$

Keeping only the first-order terms, equations (2) can be approximated by

$$C_{\mathrm{A}} = K^{-1}\left[1+\frac{1}{2}\left(\delta_{\mathrm{AB}}+\delta_{\mathrm{CA}}-\delta_{\mathrm{BC}}\right)\right],$$

$$C_{\mathrm{B}} = K^{-1}\left[1+\frac{1}{2}\left(\delta_{\mathrm{AB}}+\delta_{\mathrm{BC}}-\delta_{\mathrm{CA}}\right)\right], \qquad (3)$$

$$C_{\mathrm{C}} = K^{-1}\left[1+\frac{1}{2}\left(\delta_{\mathrm{BC}}+\delta_{\mathrm{CA}}-\delta_{\mathrm{AB}}\right)\right].$$

3. IMPLEMENTATION

The method described in Sec. 2 has been implemented with three capacitance standards $C_{\mathrm{A,B,C}}$ of equal nominal value $C_{\mathrm{N}} = 1$ nF, and the new quadrature bridge [3], at the frequency $f = 1541.4339$ Hz. The resistance standards employed in the bridge have a nominal value of $R_{1,2} = 103.250$ kΩ; K is given by a traceability chain involving a resistance ratio bridge, a calculable resistor and dc resistance measurements traceable to the representation of the ohm given by dc quantum Hall effect [4].

A thorough verification of the results of the method would require an international intercomparison; at the moment, we performed a check by comparing the differences (C_{A} - C_{B}), ($C_{\mathrm{B}} - C_{\mathrm{C}}$) and ($C_{\mathrm{C}} - C_{\mathrm{A}}$) with those obtained with the capacitance ratio bridge by substitution.

3.1. Capacitance standards

The capacitance standards $C_{\mathrm{A,\,B,\,C}}$ have been constructed at INRIM [5], [6] starting from General Radio 1404-A sealed gas-dielectric capacitors. The standards are thermostated at 23 °C with 1 mK stability by battery-operated Peltier elements, and configured as four terminal-pair impedance standards; in the present experiment they are employed in with a two terminal-pair configuration (defining planes at the end of the coaxial cables employed for connection). A photo of one of the standards is shown in Fig.1.

Fig. 1. A photo of one 1 nF capacitance standard.

3.2. The quadrature bridge

The quadrature bridge [3] is fully coaxial; all standards R_1, R_2, C_1, C_2 are defined as two terminal-pair standards at the end of the connection cables. The bridge is based on a multiphase digital sinewave generator, $G_{\mathrm{A,B,C,D,Mag,}}$ which provide adjustable main, quadrature and balancing voltages and an a semi-automated procedure to achieve bridge equilibrium.

The equilibrium procedure achieves residual voltages on the null detector, D, below 20 nV; the corresponding uncertainty contribution is lower than 1×10^{-7} on the product ($C_1 \cdot C_2$) being measured.

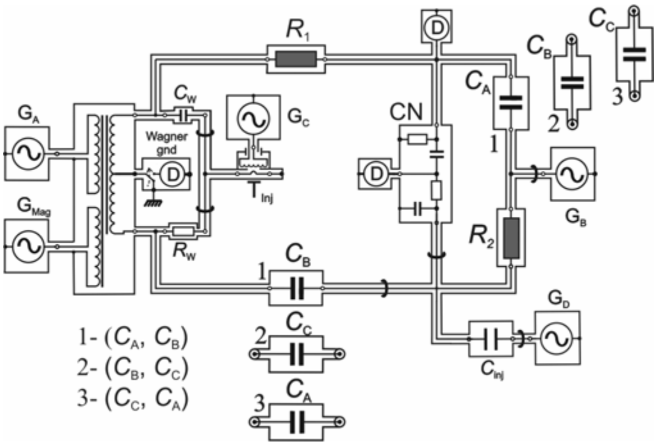


Fig. 2. Simple schematic of the quadrature bridge and three capacitance standards $C_{\mathrm{A,B,C}}$ used in this experiment.

Fig. 2 shows the arrangement of the quadrature bridge and how the capacitance standards are connected with respect to the main arms of the bridge. The numbers 1, 2 and 3 indicates three consecutive sequences:

- 1 - (C_{A}, C_{B}) pair is calibrated against the pair of resistance standards (R_1, R_2) and the capacitor C_{C} is out off the quadrature bridge;
- 2 - (C_{B}, C_{C}) pair is calibrated against the same pair of resistance standards and the capacitor C_{A} is out off the bridge;
- 3 - (C_{C}, C_{A}) is the last pair combination calibrated against the same pair of resistance standards and the capacitor C_{B} is out off the quadrature bridge.

3.3. Capacitance ratio bridge

The capacitance ratio bridge is a two terminal-pair coaxial transformer bridge which permits 1:1 and 10:1 comparisons on 10, 100 and 1000 pF nominal capacitance values at the working frequency of $f = 1592$ Hz. It is normally employed for 10:1 capacitance scaling; the ratio is not directly calibrated, a step-up procedure involving three capacitors is usually employed. Adjustments toward equilibrium are performed by acting on multi-decade manually-operated inductive voltage dividers.

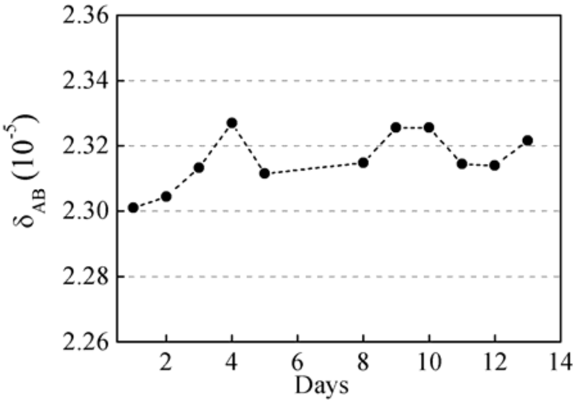
In the present application, the bridge is employed to measure the capacitance differences among the three 1 nF capacitors, by substitution (using the third capacitor as a pivot). The sensitivity of the difference measurement is lower than 3×10^{-8}.

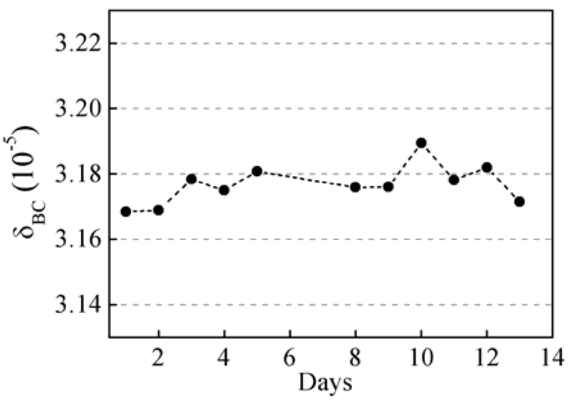
4. RESULTS

In a first stage, several measurements have been made with the quadrature bridge; each measurement is represented by a reading δ_{AB}, δ_{BC} and δ_{CA} shown in Fig. 2. With the corresponding value of K given by the traceability chain, and by solving the system of equations (3), C_A, C_B and C_C capacitance values can be computed.

After the completion of the measurements with the quadrature bridge, the capacitors have been moved to the capacitance ratio bridge.

Several measurements by substitution have been performed, by cycling the roles of C_A, C_B, and C_C (two capacitors measured by substitution, and one employed as pivot); hence, the values $(C_A - C_B)$, $(C_B - C_C)$ and $(C_C - C_A)$ have been estimated.





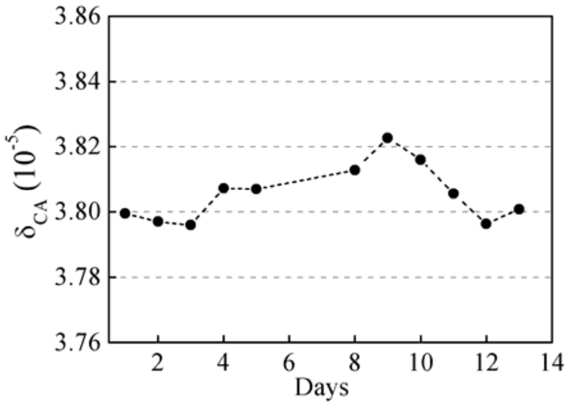


Fig. 2. Quadrature bridge readings δ_{AB}, δ_{BC} and δ_{CA} of the products $(C_A \cdot C_B)$, $(C_B \cdot C_C)$ and $(C_C \cdot C_A)$ during a period of two weeks.

Table 1 reports a comparison of the difference values (expressed in relative terms) given by the new method with the quadrature bridge, and those obtained with the substitution measurement.

Table 1. Relative difference between capacitors standard evaluated by means of the quadrature bridge and the capacitance ratio bridge.

Capacitance difference (relative to nominal value)	From quadrature bridge (ppm)	From capacitance ratio bridge (ppm)
$(C_A - C_B)/C_N$	-6.28	-6.13
$(C_B - C_C)/C_N$	14.89	14.73
$(C_C - C_A)/C_N$	-8.61	-8.51

4. CONCLUSIONS AND PERSPECTIVES

An evaluation of the uncertainty of the differences reported in Tab. 1 for the two measurements is still in course: however, known contributions sum up to less than 1×10^{-7} for both methods. Therefore, the two columns of Tab. 1 give at the moment slightly incompatible results, caused by a small systematic error of unknown origin.

A possible explanation could be in the discovery of an incomplete shielding (causing electric flux leak) in one of the capacitor. Adjustments to the capacitor and further investigations will be performed before the Conference.

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