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ERROR MODELING OF STATIC ENERGY METERS

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Abstract – The scope here is to derive a model of the measurement error of static energy meters. The model proposed is based on three error parameters: gain, phase and bias error. The validity of the model is confirmed through the statistical analysis of the measurement results obtained during the calibration of a commercial energy meter, compliant with the Measuring Instruments Directive 2004/22/EC. The experiments also involved operating conditions of the meter beyond those required by the calibration procedure prescribed by the relevant standards. A further analysis is offered aimed at verifying the statistical significance of the parameters of the model.

Keywords: energy meters, measurement accuracy, Measuring Instruments Directive.

1. INTRODUCTION

The come into effect of the Measuring Instruments Directive (MID) 2004/22/EC and the corresponding harmonized technical standards promotes the investigation of appropriate test and calibration procedures capable to assess the compliance of the measuring instruments for legal metrological control with the needs of the modern life. It is in this context that we focus our attention on the active (static) electrical energy meters ([1], annex MI-003) and in particular on their accuracy performance.

The availability of a relatively simple model capable to predict the measurement error of an instrument as a function of its input quantities is essential for the design of the calibration plan and for the identification of possible critical aspects of the metrological confirmation process. It is indeed important to be aware that the adoption of different calibration plans might lead to different conclusions about the compliance of the same instrument with the error limits stated by the relevant standards.

Although the model proposed relies on basic physical considerations about the typical architecture of static energy meters the scope here is not to identify the various sources of error and/or to quantify them starting from an a-priori analysis of the physical structure of these devices (as described, for example, in [2]-[7]). The identification of each source of error is a difficult task, especially considering that different non-ideal effects may lead to the same error contribution. We will therefore follow a quasi-black box and empirical approach leading to a simple model having few (three) parameters which will be quantified making use of the statistical analysis of series of measurements.

Gain (multiplicative), phase and bias (additive) errors are considered. Gain and phase errors are associated to the gain and phase mismatch of the voltage and current channels of the energy meter, while the bias error is associated to a bias in both channels. Although measures are taken, in the energy meter architecture, in order to correct these errors the correction itself cannot be perfect and a residual error will unavoidably be present. All the parameters are assumed to be independent of load current. The plausibility of an additional contribution to gain error proportional to load current (originated from self-heating) is however tested through an appropriate statistical analysis.

In the following we will refer to power meters (PMs) instead of energy meters because the principle of operation of these instruments is that of a PM and the conversion from power to energy is obtained through a count operation, which is intrinsically error free.

2. ERROR MODEL

The error model of the PM is sketched in Fig. 1. It consists of three error contributions, namely the gain error α, the phase error φ_c and the bias error ε. The active power P delivered to the input of the PM is

$$P = VI\cos\varphi \tag{1}$$

(the meaning of the symbols is obvious).The reading of the PM, P_m, is

$$P_m = (1+\alpha)VI\cos(\varphi+\varphi_c)+\varepsilon \tag{2}$$

In absence of any error contribution, that is when $\alpha = 0$, $\varphi_c = 0$, $\varepsilon = 0$ we have $P_m = P$.

The error model, as expressed by (2) is non-linear. However, as it is verified in practice, errors are small and (2) can be linearized around the ideal zero error condition, that is

$$P_m \simeq VI\cos\varphi + \frac{\delta P_m}{\delta\alpha}\alpha + \frac{\delta P_m}{\delta\varphi_c}\varphi_c + \frac{\delta P_m}{\delta\varepsilon}\varepsilon \qquad (3)$$

where the partial derivatives in (3) are evaluated in $\alpha = 0$, $\varphi_c = 0$, $\varepsilon = 0$. If the errors and derivatives are known equation (3) can be used to correct the measured power P_m in order to predict the power $VI\cos\varphi$ actually delivered to the input of the PM. Thus the term $VI\cos\varphi$ in (3) corresponds to the power predicted by the model, P_p, and we have

$$P_p \simeq P_m - \left(\frac{\delta P_m}{\delta\alpha}\alpha + \frac{\delta P_m}{\delta\varphi_c}\varphi_c + \frac{\delta P_m}{\delta\varepsilon}\varepsilon \right) \qquad (4)$$

After simple calculations we obtain

$$\frac{\delta P_m}{\delta\alpha} = VI\cos\varphi \qquad (5)$$

$$\frac{\delta P_m}{\delta\varphi_c} = -VI\sin\varphi \qquad (6)$$

$$\frac{\delta P_m}{\delta\varepsilon} = 1 \qquad (7)$$

Note that the sensitivity to the gain and phase errors is large when the apparent power VI is large. Also, the sensitivity to gain error is higher for higher values of the power factor while the reverse is true for phase error. The sensitivity to bias error does not depend on the load. The bias contribution dominates the PM error when the apparent power is low.

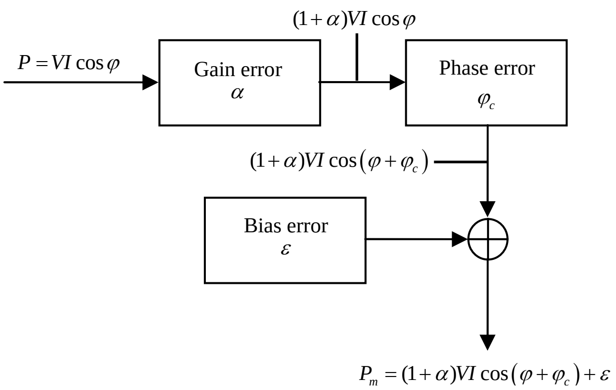


Fig. 1. Schematic diagram showing the various error contributions and their effect on the power measured by the PM.

3. ERRORS ESTIMATION

The errors are here evaluated through the least-squares statistical estimation method. The deviation $\Delta_{i,j}$ between the power predicted by the model $(P_p)_{i,j}$ (see (4)), and the power $P_{i,j}$ delivered to the PM by a reference source is calculated for various values I_i of the load current ($i = 1,2,...N$) and of the power factor $(\cos\varphi)_j$ ($j = 1,2,...M$). We have

$$\Delta_{i,j} = (P_p)_{i,j} - P_{i,j} \qquad (8)$$

The values of the errors α, φ_c and ε are those minimizing the relative sum-of-the-squares SS given by

$$SS = \sum_{i=1}^{N}\sum_{j=1}^{M}\left(\frac{\Delta_{i,j}}{VI_i}\right)^2 \qquad (9)$$

The preference for the *relative* rather than the *absolute* deviation is based on the fact that the inaccuracy of the reference source is typically proportional to VI, the constant of proportionality being nearly independent of load current, voltage and power factor.

4. EXPERIMENTAL RESULTS

The model was tested against measurements performed on two commercial PMs. Similar results were obtained, leading to the same favourable conclusion about the validity of the model. Here we limit our attention to the set of results corresponding to one of the two PMs calibrated. The measurement plan was designed according to [1]. In particular load current ranged from 500 mA to 32 A and power factor was set to 1, 0.5 inductive and 0.8 capacitive. The investigation included additional power factors, namely 0.1 and 0.3 inductive and 0.1 and 0.3 capacitive, in order to verify the validity of the model outside the limits set by the standard calibration procedure. The voltage was set to 230 V. The number of combinations of load currents and power factors was therefore 112 ($N = 16$ current values times $M = 7$ power factor values) and for each combination 5 readings of the PM were taken totally collecting 560 measurements. The mean value of the five readings corresponding to each current-power factor combination, $(P_m)_{i,j}$, was calculated together with the standard deviation of the mean $(u_m)_{i,j}$. The power delivered to the input of the power meter, $P_{i,j}$, was generated by a reference source (a commercial calibrator) capable to independently adjust voltage, current and phase angle through a wide range of values. The inaccuracy of the reference generator is quantified by an expanded uncertainty $U_{i,j}$, stated at the 95 % confidence level. The deviation $\delta_{i,j}$ between the PM reading and the reference power was calculated as

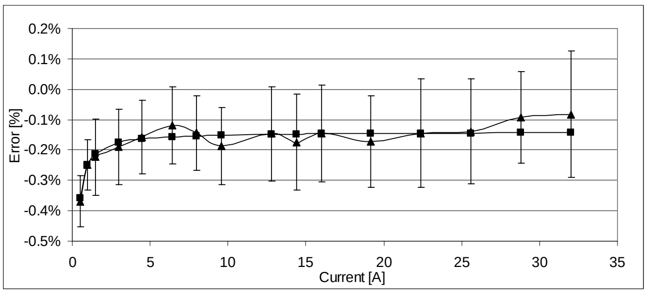
$$\delta_{i,j} = (P_m)_{i,j} - P_{i,j} \qquad (10)$$

and its expanded uncertainty, or calibration uncertainty $(U_\delta)_{i,j}$, is

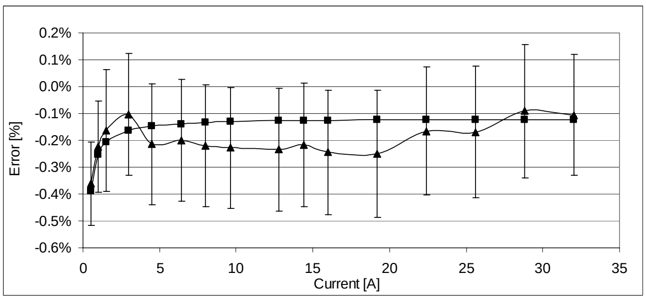
$$\left(U_{\delta}\right)_{i,j} = \sqrt{U_{i,j}^{2} + \left[\left(U_{m}\right)_{i,j}\right]^{2}} \tag{11}$$

where $\left(U_{m}\right)_{i,j} = 2.78\left(u_{m}\right)_{i,j}$ is the expanded uncertainty due to the non-repeatability of the PM and corresponding to 95 % confidence level (2.78 is the critical value of the Student's t distribution with four degrees of freedom and corresponding to 95 % confidence interval).

The error parameters α, φ_c and ε appearing in the error model (4) were computed according to the least-squares estimation procedure described in section 3. Finally the deviation $\Delta_{i,j}$ between the power predicted by the model $\left(P_p\right)_{i,j}$ and the reference power $P_{i,j}$ (see (8)) was calculated. The comparison between the relative residuals $\delta_{i,j} / P_{i,j}$ and $\Delta_{i,j} / P_{i,j}$ as a function of current is shown in figures 2(a) through 2(g). Each figure corresponds to a different value of the power factor (see the caption of Fig. 2). The amplitude of the (symmetric) uncertainty bar around $\delta_{i,j} / P_{i,j}$ is $2\left(U_{\delta}\right)_{i,j} / \left(P\right)_{i,j}$.



(a)



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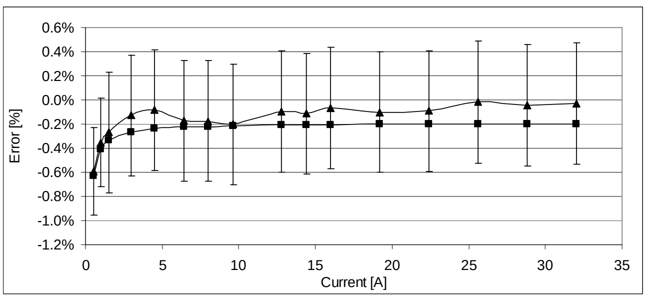
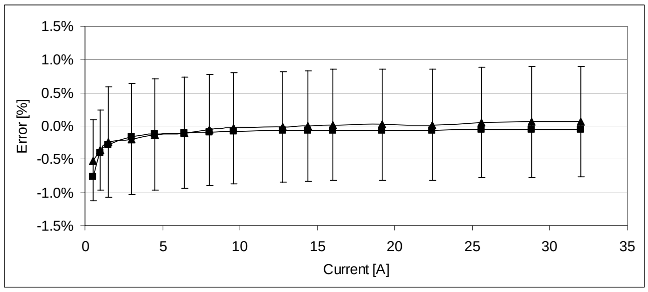


(c)



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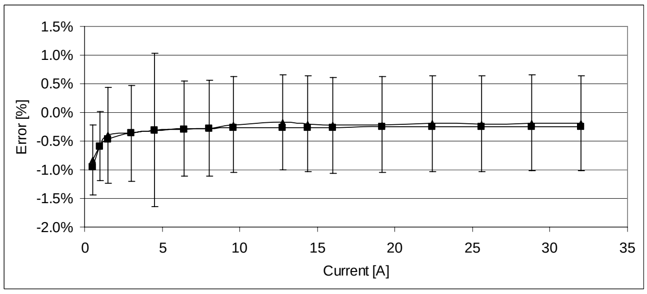
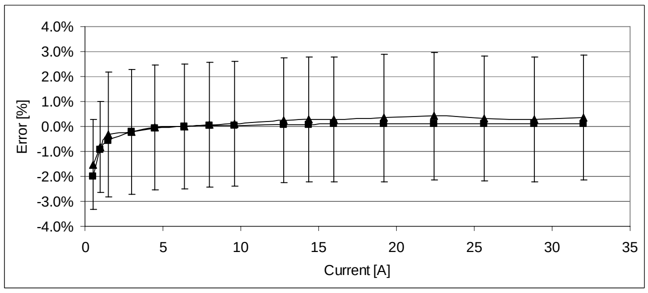
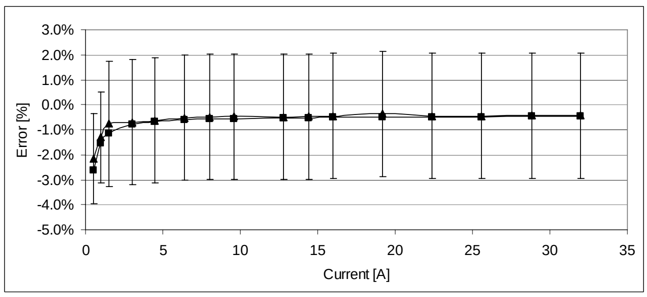


(e)



(f)



(g)

Fig. 2. Relative deviation between the measured power and the reference power (continuous line with triangles) compared to the corresponding deviation predicted by the model (continuous line with squares): (a) $\cos\varphi = 1$, (b) $\cos\varphi = 0.8$ capacitive, (c) $\cos\varphi = 0.5$ inductive, (d) $\cos\varphi = 0.3$ capacitive, (e) $\cos\varphi = 0.3$ inductive, (f) $\cos\varphi = 0.1$ capacitive, (g) $\cos\varphi = 0.1$ inductive.

The relative calibration uncertainty is of the order of a few tenths of percent when the power factor is relatively high ($\cos\varphi = 1$, 0.8 capacitive, 0.5 inductive, i.e. the calibration values set in [8]) while reaches few percents when the power factor is low ($\cos\varphi = 0.1$ inductive and capacitive representing the worst cases). It is important to observe that the dominant contribution to the calibration uncertainty is due to the uncertainty of the calibrator, which ranges from 0.08 % ($\cos\varphi = 1$, load current less than 1 A) to 2.5 % ($\cos\varphi = 0.1$, inductive or capacitive, load-current greater than 10 A) while the non-repeatability of the PM ranges from 0.02 % to 0.2 %.

The limit of maximum PM error, as specified in [8] is 1.0 % or 1.5 % depending on the value of apparent power and power factor. The magnitude of the measured errors is lower than these limits (see Fig. 2(a), (b) and (c)).

Note that the discrepancy between the error predicted by the model and the measured error is generally much lower than the calibration uncertainty. It is thus impossible to ascertain whether such discrepancy is due to model imperfections or it is intrinsic to measurement results. Anyway this demonstrates the ability of the model to predict the PM errors within the accuracy ordinarily achievable through these calibrations. Also, the good repeatability of the PM allows for a substantial reduction of calibration uncertainty by using a higher quality calibrator.

5. MODEL VALIDATION

The model is here tested in order to verify the statistical significance of its parameters. The significance test is performed according to the analysis of variance technique [9]. In essence the technique consists in verifying if the introduction of an additional parameter into the model produces a substantial reduction of the discrepancy between measurements and predictions. The test-statistic is the Fisher variable F. The following possible cases are considered and listed below in order of increasing complexity of the model.

Model I (two parameters: α and ε)

$$P_m^I = (1+\alpha)VI\cos\varphi + \varepsilon \tag{12}$$

Model IIa (three parameters: α , φ_c and ε)

$$P_m^{IIa} = (1+\alpha)VI\cos(\varphi+\varphi_c) + \varepsilon \tag{13}$$

Model IIb (three parameters: α , β and ε)

$$P_m^{IIb} = (1+\alpha+\beta I)VI\cos\varphi + \varepsilon \tag{14}$$

Model III (four parameters α , β , φ_c and ε)

$$P_m^{III} = (1+\alpha+\beta I)VI\cos(\varphi+\varphi_c) + \varepsilon \tag{15}$$

The model adopted in the previous sections is model IIa, where the phase error (φ_c) is introduced in addition to the basic gain (α) and bias (ε) error parameters in Model I. A linear dependence of gain error on the load current is included in models IIb and III through the error parameter β .

Table 1. Analysis of variance: model IIa with respect to model I

	No. of parameters	Degrees of freedom	Sum of squares of residuals	Mean square
Model I	2	110	$5.38\cdot10^{-5}$	$M_I = 4.89\cdot10^{-7}$
Model IIa	3	109	$4.88\cdot10^{-5}$	$M_{IIa} = 4.48\cdot10^{-7}$
Difference	1	1	$4.96\cdot10^{-6}$	$M_{IIa-I} = 4.96\cdot10^{-6}$
$F = \frac{M_{IIa-I}}{M_{IIa}} = 11.07 > 6.87 = F_c$				

Table 2. Analysis of variance: model IIb with respect to model I

	No. of parameters	Degrees of freedom	Sum of squares of residuals	Mean square
Model I	2	110	$5.38\cdot10^{-5}$	$M_I = 4.89\cdot10^{-7}$
Model IIb	3	109	$5.14\cdot10^{-5}$	$M_{IIb} = 4.71\cdot10^{-7}$
Difference	1	1	$2.39\cdot10^{-6}$	$M_{IIb-I} = 2.39\cdot10^{-6}$
$F = \frac{M_{IIb-I}}{M_{IIb}} = 5.07 < 6.87 = F_c$				

Table 2. Analysis of variance: model III with respect to model IIa

	No. of parameters	Degrees of freedom	Sum of squares of residuals	Mean square
Model IIa	3	109	$4.88\cdot10^{-5}$	$M_{IIa} = 4.48\cdot10^{-7}$
Model III	4	108	$4.63\cdot10^{-5}$	$M_{III} = 4.29\cdot10^{-7}$
Difference	1	1	$2.51\cdot10^{-6}$	$M_{III-IIa} = 2.51\cdot10^{-6}$
$F = \frac{M_{III-IIa}}{M_{III}} = 5.86 < 6.88 = F_c$				

The results of the analysis of variance are schematically reported in Tables 1, 2 and 3. The models to be compared appear in the first column (second and third rows), in order of increasing complexity. The number of parameters to be estimated is reported in the second column. In the third column we have the number of degrees of freedom, which corresponds to the difference between the total number of load current-power factor combinations ($M\cdot N = 112$) and the number of parameters to be estimated. The sum of the square of residuals is in column four. Each residual corresponds to the relative difference between the prediction of the model and the mean indication of the reference generator. The mean square, see the fifth column, is the ratio between the sum of the square of residuals and the number of degrees of freedom. In the row named "Difference" we have the difference between the values in the two rows above, with the exception of the figure in the last column which corresponds to the ratio between the value in the fourth column and that in the third one. The effect of the introduction of a new parameter in the model is considered significant if the ratio between the mean square of residuals of the difference and that of the model with increased complexity is greater than F_c, where F_c is the critical value of the Fisher variable at the 1 % level of significance. The number of degrees of freedom of F_c are those of the difference at the numerator (always equal to 1 in our case) and those of the more complex model at the denominator.

We see that the introduction of the phase error parameter φ_c is significant (Table 1), while the additional parameter β in the gain error does not appreciably reduce the sum of the square residuals (see Tables 2 and 3). The introduction of β is not significant at the 1 % level and it is slightly significant at the 5 % level (where $F_c' = 3.93 < F = 5.07$ or $F = 5.86$).

6. CONCLUSIONS AND FURTHER WORK

The simple model here proposed is able to predict the measurement error of static energy meters to an accuracy of a few tenths of percent, that is well within the uncertainty limits required by the MID harmonised standards for the calibration of these devices. A reduction of calibration uncertainty is envisaged due to the good repeatability of the meters. Additional measurements will be performed in order to further test the validity of the model and the sufficiency of the parameters involved.

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