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SYSTEM IDENTIFICATION OF FORCE TRANSDUCERS FOR DYNAMIC MEASUREMENTS

Alfred Link, *Bernd Glöckner*, *Christian Schlegel*, *Rolf Kumme*, *Clemens Elster*

Physikalisch-Technische Bundesanstalt, Germany, Alfred.Link@ptb.de

Abstract – Knowledge of the transfer function of a force transducer is required in order to determine a transient force from the transducer's output signal. We describe a linear least-squares fit method for system identification to estimate the transfer function from sinusoidal force calibration measurements, and we consider the evaluation of uncertainty associated with the obtained estimate. In applying this method to different calibration measurements it is demonstrated that consistent results are obtained for the transfer function.

Keywords: force transducer, transfer function, identification.

1. INTRODUCTION

The requirements for measuring forces in industrial and research applications become increasingly challenging. Currently, static methods in which transducers are calibrated by static weighting are widely accepted. However, in order to retrieve a transient force signal, the dynamic input-output behaviour of an employed force transducer has to be accounted for. Since the dynamic input-output behaviour cannot be determined from a static calibration, dynamic calibrations of force transducer are required. Here, we use the method proposed by Kumme [1] which applies the inertial force of a mass to the transducer. Since the mass is attached to the transducer, the resulting frequency response of the transducer also depends on the attached mass.

Fig. 1 shows an experimental setup for dynamic calibration of a force transducer. The setup allows acceleration measurements of the transducer base and the loading mass using high-precision accelerometers. Typically, these signals, together with the simultaneously recorded electrical transducer output, are used to characterize the dynamic input-output behaviour of a transducer.

In contrast to static measurements, the analysis of dynamic measurements requires the application of advanced tools from Digital Signal Processing (DSP) [2]. Likewise, the calculation of uncertainties [3] is much more involved as compared to the analysis of static calibration measurements. We propose methods for both tasks, and we demonstrate the benefit of the proposed procedures by their application to different dynamic calibration measurements.

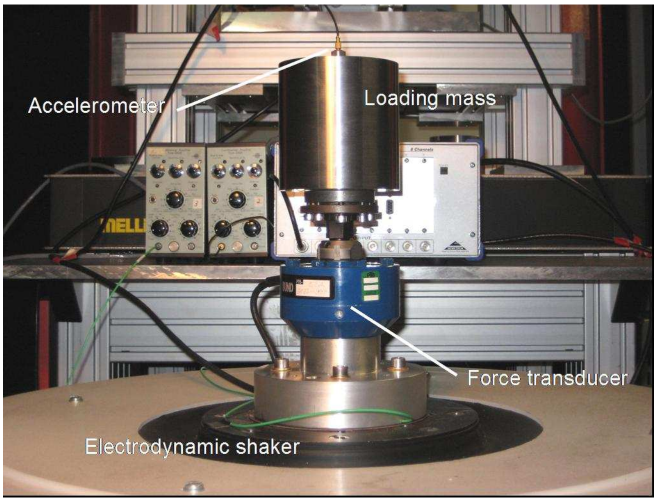

Fig. 1 Experimental setup for dynamic calibration of a force transducer.

2. IDENTIFICATION OF TRANSFER FUNCTION

Fig. 2 shows a force transducer schematically. When the transducer is accelerated at the transducer basis an initial force $F(t)$ is acting on the transducer spring element which is given by the acting mass at the head side of the transducer and by its acceleration.

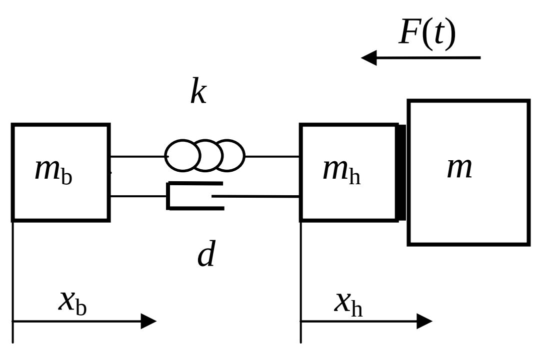

Fig. 2. Schematic diagram of a force transducer, modelled by the base and head masses m_b and m_h, and the spring elements d and k representing damping and stiffness.

The resulting displacement $x(t) = x_b(t) - x_h(t)$ generates the transducer's electrical output signal which is proportional to the applied force.

2.1. Model

From the model illustrated in Fig. 2 we obtain the following relation between force and displacement

$$m_h \ddot{x}_h = k(x_b - x_h) + d(\dot{x}_b - \dot{x}_h) - F(t) . \quad (1)$$

When the acting force $F(t)$ is generated as inertial force of the attached mass m, we write $F(t) = m\ddot{x}_h$ and obtain with $M = m_h + m$ the transfer function

$$G(s) = \frac{\rho X(s)}{s^2 X_b(s)} = \frac{\rho}{s^2 + s\frac{d}{M} + \frac{k}{M}} , \quad (2)$$

which describes the relationship between the acceleration $\ddot{x}_b$ and the transducer output signal $\rho x(t)$. The constant ρ realizes the transformation of the displacement $x(t)$ into a force signal. The input (i.e. $\ddot{x}_b$) and the output (i.e. $\rho x(t)$) can be measured, and hence the frequency response of the transducer determined, using sinusoidal acting inertial forces [1].

2.2. Estimation of model parameters

We refer to the transfer function (2) for which measured data provide amplitude and phase values of the corresponding frequency response $G(j\omega)$ at selected frequencies [1,4]. For this kind of frequency response the linear relation [5]

$$G^{-1}(j\omega) = \mu_1 + j\omega\mu_2 - \mu_3\omega^2 = \boldsymbol{f}^T(\omega)\boldsymbol{\mu} \quad (3)$$

is obtained for the parameter vector

$$\boldsymbol{\mu}^T = (\mu_1, \mu_2, \mu_3) = (k\rho^{-1}/M, d\rho^{-1}/M, \rho^{-1}) \quad (4)$$

and

$$\boldsymbol{f}^T = (1, j\omega, -\omega^2) . \quad (5)$$

The relation (3) is used to estimate the parameters μ_i by a linear fit and then to calculate the transfer function parameters, k, d, ρ. It is presupposed that the combined mass, $M = m + m_h$, is known from separate measurements. Further it is assumed that reliable uncertainties have been assigned for the measurements of the inputs and outputs. Let $S_i = S(\omega_i)$ and $\varphi_i = \varphi(\omega_i)$ denote the resulting amplitude and phase values of the frequency response $G(j\omega)$ with associated uncertainties $u(S_i)$ and $u(\varphi_i)$ at frequencies ω_i, $i = 1, 2, \ldots, L$. These measurements are combined by the vector

$$\begin{aligned} \boldsymbol{y}^T = (&\mathrm{Re}\, S_1^{-1} \exp(-j\varphi_1), \ldots, \mathrm{Re}\, S_L^{-1} \exp(-j\varphi_L), \\ &\mathrm{Im}\, S_1^{-1} \exp(-j\varphi_1), \ldots, \mathrm{Im}\, S_L^{-1} \exp(-j\varphi_L)) \end{aligned} \quad (6)$$

and the associated uncertainties constitute the variance-covariance matrix $\boldsymbol{V}_y$. The parameters μ_i are estimated according to

$$\begin{aligned} \hat{\boldsymbol{\mu}} &= \arg\min_{\mu} \{(\boldsymbol{y} - \boldsymbol{H\mu})^T \boldsymbol{V}_y^{-1} (\boldsymbol{y} - \boldsymbol{H\mu})\} \\ &= (\boldsymbol{H}^T \boldsymbol{V}_y^{-1} \boldsymbol{H})^{-1} \boldsymbol{H}^T \boldsymbol{V}_y^{-1} \boldsymbol{y} \end{aligned} , \quad (7)$$

where $\boldsymbol{H}$ denotes the matrix

$$\boldsymbol{H} = \begin{pmatrix} \mathrm{Re}\, \boldsymbol{f}^T(\omega_1) \\ \vdots \\ \mathrm{Re}\, \boldsymbol{f}^T(\omega_L) \\ \mathrm{Im}\, \boldsymbol{f}^T(\omega_1) \\ \vdots \\ \mathrm{Im}\, \boldsymbol{f}^T(\omega_L) \end{pmatrix} . \quad (8)$$

The uncertainties of the parameter estimates are then given by the variance-covariance matrix

$$\boldsymbol{V}_{\hat{\mu}} = (\boldsymbol{H}^T \boldsymbol{V}_y^{-1} \boldsymbol{H})^{-1} . \quad (9)$$

Finally, from $\hat{\boldsymbol{\mu}}$ and $\boldsymbol{V}_{\hat{\mu}}$, estimates of the physical parameters k, d, ρ are then calculated including their associated uncertainty matrix. Note that a χ^2-test [5] should be applied in order to check that the model and the data are consistent.

3. RESULTS

The proposed identification procedure has been applied to dynamic calibration measurements.

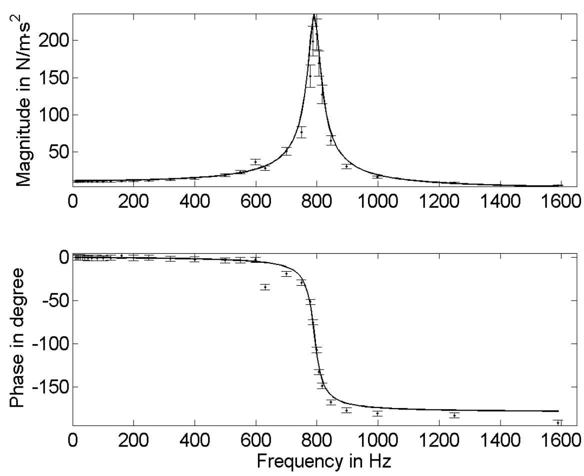

Fig. 3. Magnitude and phase of the measured (error bars) and estimated (solid line) frequency response $G(j\omega)$ from a calibration experiment ($m = 8.923\,\mathrm{kg}$, $m_h = 0.463\,\mathrm{kg}$).

Different masses were attached to the transducer to realize different acting inertial forces. Fig. 3 shows measurements of the frequency response $G(j\omega)$ at discrete frequencies ω_i for $m = 8.923$ kg within the frequency range from 10 Hz to 1600 Hz. Magnitude and phase measurements of $G(j\omega)$ are characterized by standard uncertainties of 2% and 1 degree, respectively. The measurement results are displayed together with associated standard uncertainties.

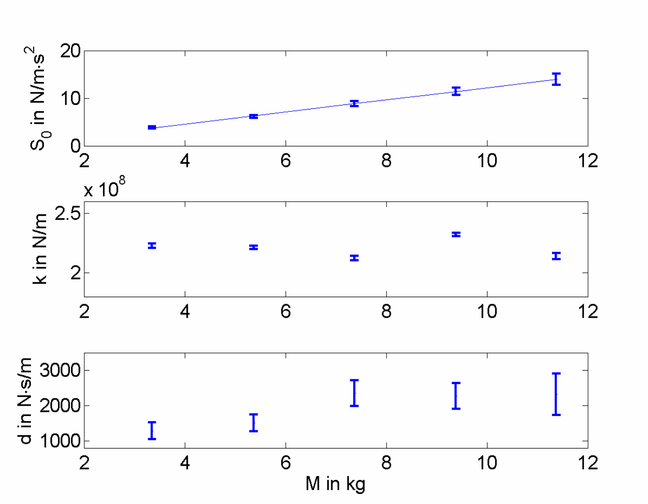

Fig. 4. Model parameter estimates including expanded uncertainties for $S_0 = \rho M / k$, k and d obtained from calibration measurements with different masses M.

By applying the identification procedure to the frequency response data we obtain estimates of the transfer function parameters together with associated uncertainties. Using these estimates together with the model, the frequency response can then be evaluated at any frequency, cf. Fig. 3.

Fig. 4 shows the parameter estimates of $S_0 = \rho M / k$, k and d obtained for five different masses M within the range from 3 kg to 11 kg. Taking into account the associated uncertainties of the parameter estimates, the employed model appears to be suitable, and the model parameter estimates for damping and stiffness, d and k, are essentially consistent with rather small uncertainties. Furthermore, the parameter estimates for $S_0 = \rho M / k$ are consistent with the expected linear dependence on the mass M.

Note, that uncertainties obtained for the transfer function parameters are based on the uncertainties which have been assigned to the magnitude and phase measurement results. For each of the individual measurements related to a loading mass m, the consistency of the fit results could be approved by a χ^2-test. Additional uncertainty sources such as mounting of different loading masses have not been considered, and it has been assumed that the chosen transducer model is not affected by different attached masses and their mounting.

4. OUTLOOK

Assuming the employed transducer model to be valid, the dynamic behaviour of the transducer can be calculated for different experimental setups. The dynamic behaviour of the transducer depends on the mass acting on the transducer within a given measurement setup [1,4]. For instance when the transducer is fixed to a base and the loading mass m is mounted, we can describe the equation of motion by

$$M\ddot{x} + d\dot{x} + kx = F(t)\,, \tag{10}$$

where $F(t)$ is the external force to be measured. With respect to the acceleration $a(t) = F(t)/M$, the frequency response then reads

$$H(j\omega) = \frac{\rho X(j\omega)}{A(j\omega)} = \frac{\rho}{(j\omega)^2 + j\omega\frac{d}{M} + \frac{k}{M}}\,. \tag{11}$$

Given the estimates, $\hat{k}$, $\hat{d}$ and $\hat{\rho}$, an estimate of the transient force $F(t)$ including an associated uncertainty can then be retrieved from the output signal of the force transducer.

5. CONCLUSIONS

A method for the analysis of dynamic calibration measurements using sinusoidal forces has been proposed. The analysis is based on a second-order model to describe the input-output behaviour of the force transducer. A system identification procedure based on linear least-squares has been described including the evaluation of uncertainties associated with the estimated transfer function. Application to different measurements yielded consistent results which encourages the use of the proposed methods.

REFERENCES

[1] R. Kumme, “Untersuchungen eines direkten Verfahrens zur dynamischen Kalibrierung von Kraftmessgeräten – ein Beitrag zur Verringerung der Messunsicherheit“, Ph.D. thesis, Technical University Braunschweig (PTB-Report MA-48), ISBN 3-89429-744-1, 1996.

[2] A. V. Oppenheim, R. W. Schaffer and J. R. Buck, *Discrete-Time Signal Processing*, Prentice Hall, New Jersey, 1999.

[3] BIPM, IEC, IFCC, ISO, IUPAC, IUPAP and OIML *Guide to the Expression of Uncertainty in Measurement* Geneva, Switzerland: International Organization for Standardization ISBN 92-67-10188-9, 1995.

[4] R. Kumme, “Dynamic force measurement in practical applications”, In: Proceedings of the 16th IMEKO World Congress, Wien (Österreich), Vol. III , 25.-28. Sept., 2000.

[5] A. Link, A. Täubner, W.Wabinski , T. Bruns, C. Elster, “Modelling accelerometers for transient signals using calibration measurements upon sinusoidal excitation”, *Measurement*, **40** 928-935, 2007.