XIX IMEKO World Congress
Fundamental and Applied Metrology
September 6–11, 2009, Lisbon, Portugal

EVALUATING UNCERTAINTIES OF LASERSCANNER MEASUREMENTS BY USING A JOINT MONTE CARLO AND FUZZY APPROACH

Hamza Alkhatib [1], *Ingo Neumann* [2], *Hansjörg Kutterer* [1]

[1] Geodetic Institute, Leibniz University of Hannover, Germany, {alkhatib, kutterer}@gih.uni-hannover.de
[2] Institute of Geodesy -Geodetic Laboratory-,University FAF Munich, Germany, ingo.neumann@unibw.de

Abstract – The evaluation of uncertainties according to the ‘‘Guide to the Expression of Uncertainty in Measurement’’ is presented in this study based on a novel Fuzzy-random-variables approach. Whereas the classically proposed methods like Monte Carlo techniques treat all uncertainties as having a random nature, the fuzzy technique distinguishes between *aleatoric* and *epistemic* uncertainties. The *aleatoric* components are modeled in a Monte Carlo framework, and the *epistemic* uncertainties were treated with fuzzy techniques. The applied procedure is outlined showing both the theory and a numerical example for the evaluation of uncertainties in an application for terrestrial laserscanning.

Keywords: GUM, Monte Carlo, Fuzzy technique

1. INTRODUCTION

A central issue within computational engineering disciplines is the modeling of uncertainty. The "Guide to the Expression of Uncertainty in Measurement" (GUM) is the standard reference in uncertainty modeling in engineering and mathematical science, cf. [1]. GUM groups the occurring uncertain quantities into "Type A" and "Type B". Uncertainties of "Type A" are determined with the classical statistical methods, while "Type B" is subject to other uncertainties like experience with and knowledge about an instrument and procedure, respectively. Whereas the uncertainties of the quantities of "Type A" can be estimated based on the measurement itself, the uncertainties of the quantities of "Type B" are based on expert knowledge, e.g., the technical knowledge about an instrumental error source. GUM defines a scalar output quantity y as a function of a vector $\boldsymbol{z}$ of input quantities

$$y = f(z_1, z_2, ..., z_n) = f(\boldsymbol{z}), \tag{1}$$

with n the number of input quantities. Each component z_i can be a quantity ([1], chapter 4.1.3):

- "…, whose values and uncertainties are determined in the current measurement (original measurement)."
- "…, whose values and uncertainties are brought into the measurement from external sources, like the values from a calibration for an instrument."

According to [2], the uncertainty classification of the input quantities with respect to its sources can be grouped into *aleatoric and epistemic* uncertainty. The *aleatoric* uncertainty arises because of unpredictable variation in the performance of the modeled system or the environment. This uncertainty is referred as irreducible uncertainty. *Aleatoric* uncertainty is caused mostly because of not clearly objective description (measurement value cannot be precisely evaluated). An example is a distance measurement under non noticeably changing atmospheric conditions.

The *epistemic* uncertainty describes the reducible component of the uncertainty, which is due the lack of knowledge about the behavior of the system/object. The *epistemic* uncertainty can, in principle, be eliminated with sufficient study and, therefore, expert judgments may be useful in its reduction. A good example is a reflector-less distance measurement which has an *epistemic* uncertainty due to the surface properties. If an expert can provide the surface properties, the *epistemic* uncertainty can be significantly reduced by a calibration process.

The common procedure for the representation of *aleatoric* and *epistemic* uncertainty is the traditional probability theory. In case of knowledge about the probability density function (PDF) of the input quantities, [3] suggested to use Monte Carlo (MC) simulations instead of the classical treatment of the combined uncertainties in the classical GUM. [4] recommended the determination of the uncertainty according to GUM by a Bayesian confidence interval using MC simulation. This approach has been explained in detail and applied to the results of terrestrial laserscanning (TLS).

The MC-technique assumes that *aleatoric* and *epistemic* uncertainties are appropriately handled by means of PDFs. Unfortunately, MC modeling leads to a rather optimistic evaluation of uncertainties. Optimistic evaluation means that, e.g., the confidence intervals of output quantities are too narrow in comparison to the actual uncertainty about the true value. This shall be highlighted with two references. On the one hand [5] presents in his paper a too optimistic estimation of confidence intervals for the measurement of the speed of light. On the other hand [6] illustrated a difference between confidence intervals for the measurements of the astronomical unit with the true values. Therefore in this paper the *epistemic uncertainty* will be treated in a separate form using the fuzzy set theory [7]. This representation, which will be called fuzzy approach, leads to a reluctant or even more pessimistic evaluation of the uncertainties. The fuzzy techniques have proven to be an appropriate solution for the description of uncertainties and were applied in different science and engineering applications, see, e.g., [2]. The basic idea for the fuzzy approach in this paper is related to the so-called Fuzzy-Random-Variables (FRV) which are based on a combination of probability theory and fuzzy theory, see [8]. In this paper the FRV-approach shall be discussed theoretically and using a practical example in TLS.

The paper is organized as follows: First a FRV-approach to handle measurement uncertainties is introduced. Then, in Section 3 the FRV-approach is applied to TLS and the obtained results are critically compared to the classical MC-techniques. The paper finishes with a discussion and conclusions.

2. UNCERTAINTY MODELING WITH A FRV-APPROACH

In this section, a FRV-approach to uncertainty modeling in the context of GUM is introduced. In the FRV-approach we distinguish between *aleatoric* and *epistemic* uncertainties in the propagation process of the uncertainties of the input quantities to the output quantity y. Whereas the *aleatoric* part is treated with MC techniques based on probability theory (see Section 2.1), *epistemic* uncertainty is propagated by means of a range-of-values search problem (see Sections 2.2 and 2.3). The *aleatoric* component is superposed with the *epistemic* component of uncertainty without any distribution information. Both types of uncertainty are modeled in a comprehensive way, using fuzzy intervals.

2.1. Modeling of the aleatoric uncertainty component

The *aleatoric* component of the uncertainty is treated with MC techniques. Therefore, the *aleatoric* uncertainty component is described by PDFs. The GUM suggests in some cases to select the PDF of the input quantities as rectangular, triangular, and trapezoidal [1]. In these cases, it is hard to obtain the estimate of the uncertainty for the output quantity in a closed mathematical form. With a set of generated samples the distribution function for the value of the output quantity in Eq. (1) will be numerically approximated. MC-techniques to estimate the uncertainty include the following three main steps [9]:

In the first step, a set of random samples, which have the size n, is generated from the PDF for each random input quantity $Z_1, Z_2, ..., Z_n$. The sampling procedure is repeated M times for every input quantity. In a second step, the realizations $y^{(i)}$ of the output quantity will be calculated by:

$$y^{(i)} = f\left(z_1^{(i)}, z_2^{(i)}, ..., z_n^{(i)}\right) = f\left(\boldsymbol{z}^{(i)}\right). \tag{2}$$

With the $i = 1...M$ generated samples of Y, we obtain an estimate of the PDF for Y. In the last step, particularly relevant estimates of any statistical quantities can be calculated. The most important statistical quantities are the expectation $\hat{y}$ of the output quantity:

$$\hat{y} = \frac{1}{M}\sum_{i=1}^{M} f\left(\boldsymbol{z}^{(i)}\right), \tag{3}$$

and the estimate of the variance $\hat{\sigma}_y^2$ of the output quantity as well, see [10]:

$$\hat{\sigma}_y^2 = \frac{1}{M}\sum_{i=1}^{M}\left(f\left(\boldsymbol{z}^{(i)}\right) - \hat{y}\right)\left(f\left(\boldsymbol{z}^{(i)}\right) - \hat{y}\right)^T. \tag{4}$$

Additionally, the computation of the confidence interval $y_{conf,MC} = [\underline{y}, \overline{y}]$ of the estimate of the output quantity with the significance level of γ is of major importance when evaluating of uncertainties. To compute the confidence interval by MC simulation, one has to sort the independent samples $y^{(i)}$ from the smallest to largest; an approximate $100 \cdot (1 - 2\gamma)\%$ for the output quantity y is given in [4].

2.2. Modeling of the epistemic uncertainty component

The *epistemic* part of the uncertainty is modeled with the aid of fuzzy-theory [7]. Each uncertain quantity z_i is exclusively described in terms of fuzzy intervals. A fuzzy interval $\tilde{A}$ is uniquely defined by its membership function $m_{\tilde{A}}(x)$ over the set $\mathbb{R}$ of real numbers with a membership degree between 0 and 1:

$$\tilde{A} := \left\{(x, m_{\tilde{A}}(x)) \middle| x \in \mathbb{R}\right\} \tag{6a}$$

with $m_{\tilde{A}} : \mathbb{R} \to [0,1]$. The membership function of a fuzzy interval can be described by its left *(L)* and right (*R*) reference function (see also Fig. 1):

$$m_{\tilde{A}}(x) = \begin{cases} L\left(\dfrac{x_m - x - r}{c_l}\right), & \text{for} \quad x < x_m - r \\ 1, & \text{for} \quad x_m - r \le x \le x_m + r \\ R\left(\dfrac{x - x_m - r}{c_r}\right), & \text{for} \quad x > x_m + r \end{cases} \tag{6b}$$

with x_m denoting the midpoint, r its radius, and c_l, c_r the spread parameters of the monotonously decreasing reference functions (convex fuzzy intervals). The $\alpha-$cut with $\alpha \in [0,1]$ of a fuzzy interval $\tilde{A}$ is defined by:

$$\tilde{A}_\alpha := \left\{x \in X \middle| m_{\tilde{A}}(x) \ge \alpha\right\}. \tag{7}$$

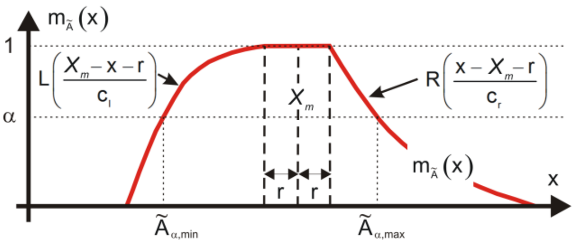

Fig. 1. Fuzzy interval and its $\alpha-$cut.

The lower $\tilde{A}_{\alpha,min}$, and the upper bound $\tilde{A}_{\alpha,max}$ of an $\alpha-$cut and its radius $\tilde{A}_{\alpha,r}$ are:

$$\tilde{A}_{\alpha,\min} = \min\left(\tilde{A}_{\alpha}\right) \text{ and } \tilde{A}_{\alpha,\max} = \max\left(\tilde{A}_{\alpha}\right)$$
$$\tilde{A}_{\alpha,r} = \left(\tilde{A}_{\alpha,\max} - \tilde{A}_{\alpha,\min}\right)/2. \tag{8}$$

The integral over all $\alpha-$cut equals $m_{\tilde{A}}(x)$:

$$m_{\tilde{A}}(x) = \int_0^1 m_{\tilde{A}_\alpha}(x)d\alpha. \tag{9}$$

2.3. The FRV-approach to handle both uncertainties

In order to combine the uncertainties with the methods described in the Sections 2.1 and 2.2, the *aleatoric* and *epistemic* components of the uncertainties are characterized as a special case of fuzzy theory, so called *Fuzzy Randomness* [2].

In this concept fuzzy intervals serve as basic quantities; their midpoints x_m are considered in the following as random variables and their spreads c_l, c_r describe the range of the uncertainty for the *epistemic* uncertainties. If one component has random uncertainty only, then this input quantity only consists of a single midpoint with radius $r = 0$ and without a left and right reference function. In contrast to the MC-techniques, the membership function of a fuzzy interval cannot be interpreted in a probabilistic meaning. Therefore the propagation of the *epistemic* uncertainties has to be modified accordingly. In the fuzzy case, we model the influence of an *epistemic* component of the uncertainty on the output quantity y. Figure 2 shows the interpretation of a fuzzy interval in the here presented approach.

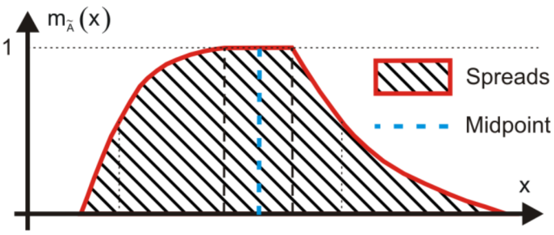

Fig. 2. Interpretation of a fuzzy interval.

The construction of the membership function can be based on expert knowledge. Each expert provides a range of values (an interval) for the *epistemic* uncertainty which he considers as realistic. The $\alpha-$level of one describes the range of values where all experts agree that these values are possible (most optimistic outcome). The $\alpha-$level of zero represents the most pessimistic expert opinion for the range of values for the *epistemic* uncertainty. The above described procedure to construct fuzzy intervals is based on the theory of nested sets; see [11]. For a detailed description in the context of uncertainty propagation in parameter estimation and hypothesis testing the reader is referred to [12] and [16].

2.4. Evaluating the uncertainties with an optimization

The extension principle, introduced by [7], serves as basic role to propagate both types of uncertainties in the FRV-approach. In case of the introduced fuzzy intervals the computation of the membership function $m_{\tilde{y}}(y)$ for the output quantity is based on the $\alpha-$cuts of the input quantities $\tilde{\boldsymbol{z}}_\alpha$ within an optimization problem of the following target function, see, e.g., [13]:

$$\begin{aligned}\tilde{y}_{\alpha,\min} &= \min_{z_i \in [\tilde{z}_{i_{\alpha,\min}}, \tilde{z}_{i_{\alpha,\max}}]} f(\boldsymbol{z}) \\ \tilde{y}_{\alpha,\max} &= \max_{z_i \in [\tilde{z}_{i_{\alpha,\min}}, \tilde{z}_{i_{\alpha,\max}}]} f(\boldsymbol{z})\end{aligned} \tag{10}$$

with $m_{\tilde{y}}(y) = \int_0^1 m_{\tilde{y}_\alpha}(x)d\alpha$ and $m_{\tilde{y}_\alpha} = \left[\tilde{y}_{\alpha,\min}, \tilde{y}_{\alpha,\max}\right]$. The approximate midpoint of the fuzzy interval for the output quantity y_m is:

$$y_m = f(z_{1_m}, z_{2_m}, ..., z_{n_m}) = f(\boldsymbol{z}_m). \tag{11}$$

Finally, the confidence interval $y_{\text{conf,Fuzzy}}$ in the FRV-approach (at the α-level) is obtained by the combination of both uncertainty components:

$$y_{\text{conf,Fuzzy}} = [\underline{y} - \tilde{y}_{\alpha,r}\ ;\ \overline{y} + \tilde{y}_{\alpha,r}] \tag{12}$$

Whereas the α-level of zero represents the pessimistic outcome, the optimistic outcome is obtained for $\alpha = 1$. Only the random uncertainty components from the input quantities $\boldsymbol{z}$ contribute to the lower and upper bound of the MC confidence interval $y_{\text{conf,MC}} = [\underline{y}, \overline{y}]$. Figure 3 shows a diagram with the main steps of uncertainty modeling with a different treatment of the *aleatoric* and *epistemic* uncertainties.

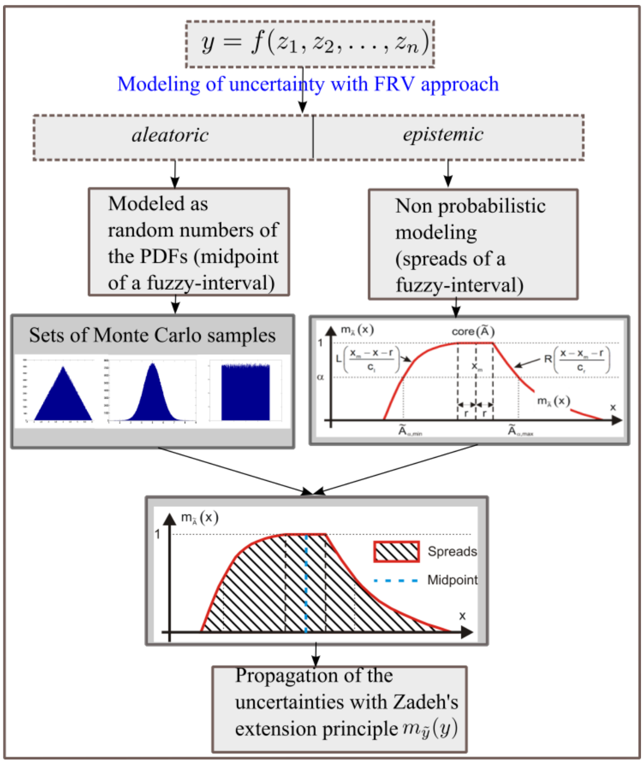

Fig. 3. Treatment of uncertainty components in FRV-approach.

Table 1. Uncertainties for the input quantities.

Input quantity z_i	Uncertainty component	pdf / membership function	Uncertainty	Type
z_1	*aleatoric*	normal	$\sigma = 3$ mm	A
z_2	*epistemic*	triangular	$a_+ - a = 3$ mm $\tilde{z}_{\alpha=0,r} = 3$ mm	B
z_3	*aleatoric*	normal	$\sigma = 0.9$ mm (8987)	B
z_4	*aleatoric*	normal	$\sigma = 7.2$ mm (8987)	B
z_5	*aleatoric*	normal	$\sigma = 20$ mgon	A
z_6	*epistemic*	triangular	$a_+ - a = 20$ mgon $\tilde{z}_{\alpha=0,r} = 20$ mgon	B
z_7	*epistemic*	rectangular	$a_+ - a = 10$ mgon $\tilde{z}_{\alpha=0,r} = 10$ mgon	B

3. APPLICATION OF THE FRV-APPROACH TO TLS

In this section a short numerical example for the approach, presented in Section 2, is shown. The aim of the application is to detect the vertical displacements of a bridge under load, e.g., due to car traffic or train crossings. For this reason, a laserscanner of type *Leica* HDS 4500 was placed beneath the bridge; the measurements in the "Profiler Mode" span the green plane in Fig. 4.

The laserscanner carries out very fast distance and angle measurements and the measurements are influenced by vibrations due to traffic load of the bridge. The time series of the vertical height h_t of the bridge at the points 1831 and 8987 can be expressed in the local coordinate system of the laserscanner by the following equation:

$$h_t = s_t \cdot \cos(\zeta_t) \tag{13}$$

with the slope distance s_t and the zenith angle ζ_t measured by the laserscanner. The output quantity:

$$y = w_t = h_t - \frac{1}{q}\sum_{t=0}^{q} h_t \tag{14}$$

depends on several input quantities z_i. The number of measured epochs in Eq. (14) is q. The input quantities can be divided into two main groups: input quantities depending on the distance or on the zenith angle, respectively.

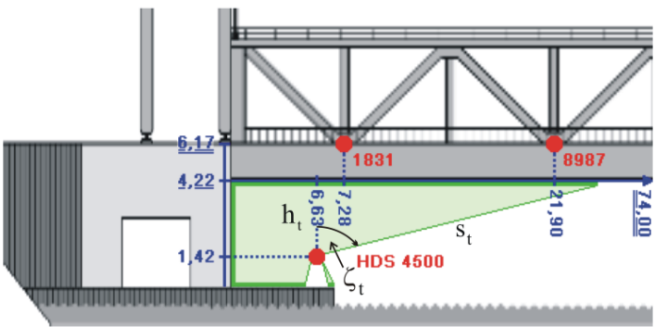

Fig. 4. Position of the laserscanner beneath the bridge.

In detail, the output quantity $y \triangleq w_{scan}(x,t)$ depends on the following input quantities z_i:

- Uncertainty of the distance (z_1, Type A), and their additional constant (z_2, Type B)
- Distance depending term for the uncertainty of the distance measurement (z_3, Type B)
- Incidence angle of the measured distance under the bridge (z_4, Type B)
- Uncertainty of the zenith angle (z_5, Type A) and the vertical index error (z_6, Type B)
- Vertical resolution for the zenith angle (the step width of the motor) (z_7, Type B)

The uncertainties and the pdf / membership function for the input quantities z_i are given in Table 1. The assumptions for the uncertainties of z_1, z_5 and z_6 are based on the technical data from the manufacturer and for the uncertainties of z_2, z_3 and z_4 on [14] and for z_7 on [15]. The input quantities z_3 and z_4 have a correlation of 0.5, according to [4]. The numbers 8987 and 1831 in the brackets represent the point number (see Fig. 2).

3.1. Uncertainties obtained by the FRV-approach

In the FRV-approach the treatment of the *aleatoric* and the *epistemic* component in the propagation process of the uncertainties is different, see Section 2. The uncertainties were treated with the techniques presented in the Sections 2.3 and 2.4.

According to Section 2.4 we obtain the epistemic component of the uncertainty of the output quantity w_t for $\alpha = 0$ and $\alpha = 1$ with Eq. (8), (10) and (11). The results are given in Table 2.

Within the propagation process of the *epistemic* component, the radius $\tilde{z}_{\alpha,r}$ of all *aleatoric* components z_i from Table 1 is zero. In the presented propagation process an *epistemic* uncertainty component cannot be reduced by repeated measurements due to the mathematical rules of fuzzy-theory [12].

Table 2. *Epistemic* uncertainty obtained by the FRV-approach.

Fuzzy result (*epistemic* component)	Point 8987
$\tilde{y}_{\alpha=1,r} = (\tilde{y}_{\alpha=1,max} - \tilde{y}_{\alpha=1,min}) / 2$	4.8 mm
$\tilde{y}_{\alpha=0,r} = (\tilde{y}_{\alpha=0,max} - \tilde{y}_{\alpha=0,min}) / 2$	16.1 mm

For the propagation process of the aleatoric components with the methods described in section 2.1, the uncertainty of the input quantities with an *epistemic* uncertainty component is set to zero, and we obtain the *aleatoric* uncertainty of the output quantity for 100000 MC-runs. The results are shown in Table 3.

Table 3. *Aleatoric* uncertainty obtained by the fuzzy technique.

Fuzzy result (*aleatoric* component)	Point 8987
$\hat{\sigma}_y$	5.4 mm
$y_{conf,MC} = [\underline{y}, \overline{y}]$	[-10.6,10.7] mm

Finally, we obtain the confidence interval for the fuzzy approach with Eq. (11) for $\alpha = 0$ and $\alpha = 1$. Table 4 represents the computations for the confidence interval.

Table 4. Confidence interval obtained by the fuzzy technique.

Fuzzy result (confidence interval)	Point 8987
$y_{conf,Fuzzy} = [\underline{y} - \tilde{y}_{\alpha=1,r} \ ; \overline{y} + \tilde{y}_{\alpha=1,r}]$	[-15.4,15.5] mm
$y_{conf,Fuzzy} = [\underline{y} - \tilde{y}_{\alpha=0,r} \ ; \overline{y} + \tilde{y}_{\alpha=0,r}]$	[-26.7,26.8] mm

3.2. A best and worst case scenario

The last example deals with a case study for different magnitudes for the occurring uncertainties. In order to evaluate the consequences of changing magnitudes for the uncertainties, two scenarios are realized: A best case scenario with a small uncertainty of the input quantity of $z_7 = (a_+ - a_-) / 2 = 20$ mgon and a worst case scenario with high uncertainties of $z_7 = 50$ mgon. Please note, that the value of the assumed uncertainty is still realistic in case of rapidly registered measurements of a laserscanner.

A geometrical interpretation of the *epistemic* uncertainty of the height difference w_t (output quantity) in the FRV-approach is given in Fig. 5. The range of values for the *epistemic* uncertainty can be seen as a shift in the distribution. For a clear representation the distribution of the random errors of the output quantity (obtained by the MC technique) is shown at the lower and upper bound of the *epistemic* uncertainties. The left part of the Figure shows the best case scenario with small uncertainties and the right represents the worst case scenario with high uncertainties for input quantity z_7. Additionally, in the Fig. 5 we see the comparison between the confidence intervals of the FRV-approach and the GUM for the quantity $y = w_t$. The results for the $\alpha -$levels 0.1 (pessimistic outcome, lower part of Fig. 5) and 1 (optimistic outcome, upper part of Fig. 5) are given.

When comparing the results of the fuzzy confidence interval with the results of the classical GUM one can clearly see that the fuzzy confidence intervals are significantly larger. This is due to the reason that the *epistemic* component dominates the uncertainty budget in the example presented here.

The combination of both uncertainty components on the level of $\alpha -$cuts can be interpreted in such a way that the *aleatoric* uncertainty can obtain a translation. The range of this translation is described through the *epistemic* uncertainty (refer to Fig. 5). The FRV-approach combines both types of uncertainty in one single quantity. It is always interpretable whether the *epistemic* or the *aleatoric* components account for the main part of the total uncertainty.

4. CONCLUSIONS

In this paper a measurement equation was analyzed with multidimensional input quantities and a one dimensional output quantity. A FRV-approach was used to handle and to propagate *aleatoric* and *epistemic* uncertainties. Two important outcomes can be stated from the results obtained in the paper: First, the FRV-approach allows dealing with a pessimistic and optimistic outcome for the uncertainty of the output quantity. Second, it turns out that the difference between classical uncertainty modeling with MC-techniques and the FRV-approach increase significantly if some of the input quantities have a noticeable *epistemic* component. This is due to the fact that the *epistemic* uncertainty in the FRV-approach cannot be reduced by repeated measurements due to the mathematical rules of fuzzy-theory. Therefore the FRV-approach provides new important techniques for the modeling of the measurements uncertainties, especially in the field of laserscanning.

ACKNOWLEDGMENTS

The presented paper shows in parts results and new ideas developed during the research projects KU 1250/4-1 and 1250/4-2 "Geodätische Deformationsanalysen unter Berücksichtigung von Beobachtungsimpräzision und Objektunschärfe", which are funded by the German Research Foundation (DFG). This is gratefully acknowledged by the authors.

REFERENCES

[1] ISO, *Guide to the expression of uncertainty in measurements (GUM)*. International Organization for Standardization, Geneva, 1995.

[2] B. Möller and M. Beer, *Fuzzy Randomness -Uncertainty in Civil Engineering and Computational Mechanics-*, Springer, Berlin and New York, 2004.

[3] K. D. Sommer and B. R. L. Siebert, *Systematic approach to the modeling of measurements for uncertainty evaluation*, Metrologia, vol. 43, pp. 200-210, 2006.

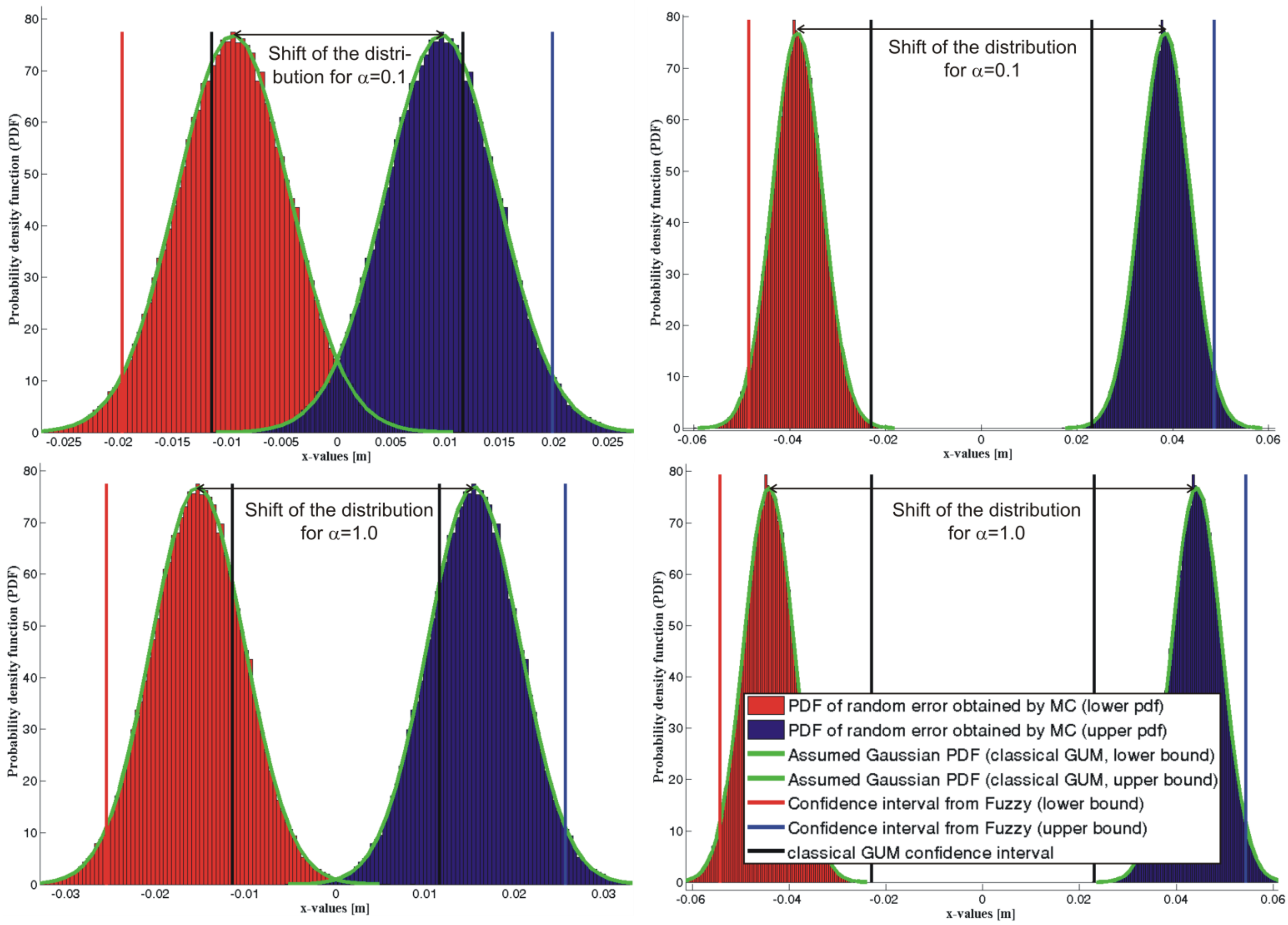

Fig. 5. Comparison between the confidence intervals of the FRV-approach and the classical GUM.

[4] K. R. Koch, *Evaluation of uncertainties in measurements by Monte Carlo simulations with an application for laserscanning*, J Applied Geodesy, vol. 2, pp. 67–77, 2008.

[5] A. G. McNish, *The speed of light*, IRE Transaction on Instrumentation 1:11 (1962), pp. 138-148, 1962.

[6] S. M. Stigler, *Statistics and the Question of Standards*, Journal of Research of the National Institute of Standards and Technology, 101:6, pp. 779–789, 1996.

[7] L. Zadeh, *Fuzzy sets*, Information Control, vol. 8, pp. 338–353, 1965.

[8] H. Kwakernaak, *Fuzzy random variables I: definitions and theorems*. Information Sciences, vol. 15, pp. 1-29, 1978.

[9] ISO, *Evaluation of measurement data - Supplement 1 to the GUM - Propagation of distributions using a Monte Carlo method*. Joint Committee for Guides in Metrology, Bureau International des Poids et Mesures, JCGM 101, 2007.

[10] H. Alkhatib, *On Monte Carlo methods with applications to the current satellite gravity missions*, PhD thesis, Institute for Geodesy and Geoinformation, Bonn University, 2007.

[11] H. T. Nguyen and V. Kreinovich, *Nested Intervals and Sets: Concepts, Relations to Fuzzy Sets, and Applications*, in: Kearfott, R. B. and V. Kreinovich (Eds.), Applications of Interval Computations, Dordrecht, pp. 245–290, Kluwer Academic Publishers, 1996.

[12] I. Neumann (2009), *Modeling of an extended uncertainty budget in parameter estimation and hypothesis testing*, PhD thesis (in German), Geodetic Institute, Leibniz University of Hannover, to appear, 2009.

[13] I. Neumann and H. Kutterer, *The probability of type I and type II errors in imprecise hypothesis testing with an application to geodetic deformation analysis*, to appear in Journal of Reliability and Safety, 2009.

[14] T. Schulz and H. Ingensand, *Influencing Variables, Precision and Accuracy of Terrestrial Laser*, in: Intergeo East, Bratislava, Slovakia, 2004.

[15] Y. Reshetyuk, *Investigation and calibration of pulsed time-of-flight terrestrial laser scanners*, Licentiate thesis in Geodesy, Sweden, 2006.

[16] H. Kutterer, *To the handling of uncertainties in Geodesy: Modules for a new error theory (in German)*, Vol. 553, Deutsche Geodätische Kommission, Series C, Munich, 2002.