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PRIMARY ACCELEROMETER CALIBRATION IN UME BY SINE APPROXIMATION METHOD

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Abstract − The traceability of the vibration measurements to SI units is maintained through the calibration of the reference transducers. Primary calibration of the reference transducers in the field of mechanical vibration and shock is performed in accordance with the international standard ISO 16063-11 [1]. In some applications beside magnitude of the transducer's sensitivity, the knowledge of phase shift is also required. Magnitude and phase shift of the complex sensitivity of the reference transducer could be obtained by applying the sine approximation method, described in ISO 16063-11 standard. In general, this method is applied in many leading National Metrology Institutes in the world. However, experimental implementation of the method varies from one institute to another. The experimental setup constructed in Turkish National Metrology Institute for the realization of sine approximation technique and calibration results for reference standard accelerometers are presented in this paper.

Keywords: calibration, laser, accelerometer, sine approximation

1. INTRODUCTION

Accelerometers are mainly used in general-purpose vibration measurements. Performance check of measurement chain including accelerometer is carried out by means of hand-held calibration exciter. However, calibration of vibration transducer performed by comparing to the reference standard accelerometer calibrated by primary method.

The primary calibration of the reference standard accelerometer is performed in accordance with the international standard ISO 16063-11. Three different methods are described in the standard. One of them is fringe counting method that is applicable up to frequency 800 Hz, the second one is minimum point method, which is used in the frequency range from 800 Hz to 10 kHz and the last is sine approximation method applicable from 1 Hz to 10 kHz. The fringe counting and minimum point methods provide just information about magnitude of the complex sensitivity of the accelerometer. These two methods are applied in TÜBİTAK UME as a routine service for many years. The results of developed systems and quality of the primary accelerometer calibration performed in TÜBİTAK UME are reported in the literature [2-6].

Some of the applications require also phase information of complex sensitivity. Currently set-ups implementing sine approximation method are constructed in national metrology institutes [7-11]. TÜBİTAK UME started to work on this subject recently, and nowadays the system for primary calibration of accelerometers by sine approximation method is operational.

2. THEORY AND EXPERIMENTAL SETUP

Calibration setup used for sine approximation method is shown in Fig. 1. [1]. Interferometer used in the setup is modified Michelson interferometer - quadrature homodyne type.

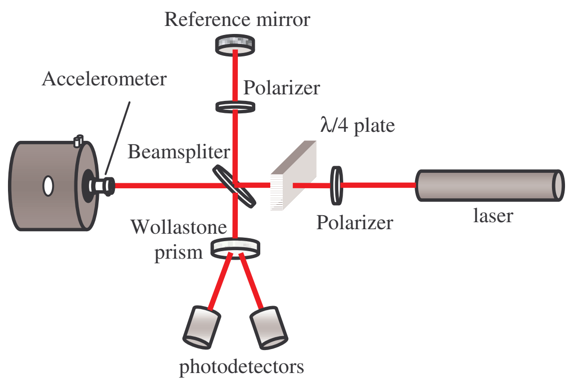

Fig. 1. Homodyne laser interferometer with quadrature output.

The interferometer setup consists of stabilized He-Ne laser, polarizer, quarter wavelength retarder, beam splitter and Wollaston prism. Wollaston prism separates the incoming beam into 90° phase shifted two beams. The quadrature outputs of photo-detectors corresponding to sinusoidal excitation of accelerometer, constituting moving arm of the interferometer, are given below:

$$u_{1f}(t) = u_1 \cos(\varphi_{Mod}) \quad (1)$$

$$u_{2f}(t) = u_2 \sin(\varphi_{Mod}) \quad (2)$$

The experimental setup for sine approximation method is given in Fig. 2. The sinusoidal signal from a signal generator is applied to the electrodynamics exciter by means of power amplifier. This leads to the linear vibration of the exciter's moving head at the frequency (f) of the electrical signal applied from the signal generator. As a result the accelerometer mounted on the top of the moving head of the exciter also vibrates at the same frequency being exposed to the acceleration with the amplitude â. Acceleration amplitude could be adjusted to any required level by varying the amplitude of electrical signal applied through signal generator and power amplifier.

The output of the accelerometer is measured by the conditioning amplifier in mV. In order to obtain the sensitivity of the accelerometer in pC/ms^{-2}, the value of a conversion factor of conditioning amplifier is required. This value for charge amplifier is determined in mV/pC unit. Therefore, it is advised to calibrate charge amplifier before accelerometer calibration. Distortion meter is used for measuring of a total harmonic distortion.

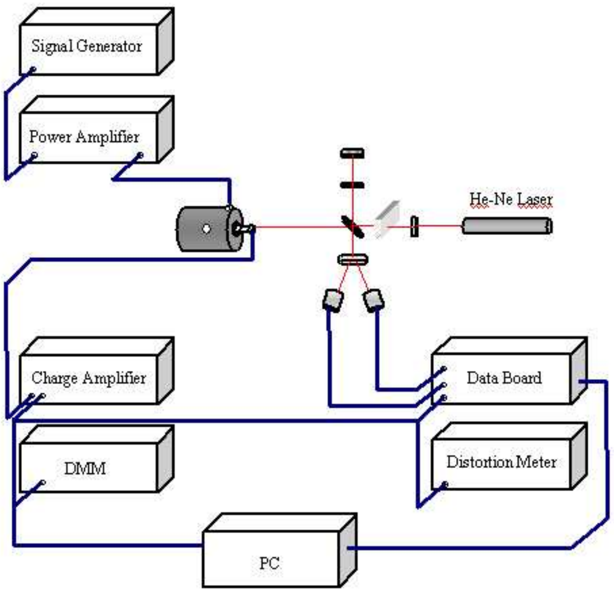

Fig. 2. Experimental setup for sine approximation method

One of the critical parts of the calibration set-up is signal-processing chain. The amplitude resolution and sampling rate shall be sufficient for calibration in the intended amplitude range with the uncertainty specified in ISO 16063-11 standard. Typically, an amplitude resolution higher than 10 bits is used for the accelerometer output and for the interferometer quadrature output signals; a resolution of more than 8 bits is sufficient [1].

In addition, synchronous sampling of the quadrature outputs of photo-detectors and accelerometer is required. The sampled series of outputs of photo-detectors and accelerometer are $\{u_{1f}(t_i)\}$, $\{u_{2f}(t_i)\}$ and $\{u_a(t_i)\}$ respectively. The sampling process shall start and end at the same points of the time. Sampling takes place during measurement period $t_0 < t < t_0+T_{meas}$ and sampling interval $\Delta t = t_{i+1} - t_i$ is constant.

Magnitude and phase shift of the accelerometer sensitivity and modulation are obtained from modulation phase values by the following steps. Modulation phase values, $\varphi_{Mod}(t_i)$ are obtained from the sampled interferometer signal $\{u_{1f}(t_i)\}$ and $\{u_{2f}(t_i)\}$ using the following relationship.

$$\varphi_{Mod}(t_i) = \arctan\left(\frac{u_{1f}(t_i)}{u_{2f}(t_i)}\right) + n\pi \qquad (3)$$

Here n is an integer number and it is chosen as n = 0, 1, 2, ... in order to avoid discontinues of modulation phase values $\{\varphi_{Mod}(t_i)\}$ for the values $n\pi$. This is called as phase unwrapping process and some of the algorithms related for that are reported in literature [11, 12].

The modulation phase values obtained experimentally describe the signal form which is used for calculation of acceleration that accelerometer exposed. However this information is not sufficient for direct calculation of acceleration. Therefore, obtained series of modulation phase values is approximated by solving N+1 equations for the three unknown parameters, A, B and C using the least-squares sum method.

$$\varphi_{Mod}(t_i) = A\cos(\omega t_i) - B\sin(\omega t_i) + C \qquad (4)$$

where

- i = 0,1, ... , N
- A = $\varphi_M \cos\varphi_s$,
- B = $\varphi_M \sin\varphi_s$,
- C : a constant,
- f : the vibration frequency,
- ω : angular frequency, $\omega=2\pi f$, in radians,
- φ_s : initial phase angle of the displacement,
- φ_M : modulation phase amplitude,
- N+1 : number of samples synchronously taken over the measurement period, T_{meas}

The parameter values of A and B are obtained from the sine approximation process. The modulation phase amplitude, φ_M and the initial phase angle of the displacement, φ_s is calculated from the values of A and B using the formulae given below:

$$\varphi_M = \sqrt{A^2 + B^2} \qquad (5)$$

$$\varphi_s = \arctan\frac{B}{A} \qquad (6)$$

The amplitude of the acceleration, â that accelerometer exposed and the initial phase angle of the acceleration, φ_a are calculated from the modulation phase amplitude, φ_M and the initial phase angle of the displacement, φ_s using the formulae given below:

$$\hat{a} = \pi\lambda f^2 \varphi_M \tag{7}$$

$$\varphi_a = \varphi_s + \pi \tag{8}$$

The series of the sampled accelerometer output values, $\{u_a(ti)\}$ are also approximated by the sine approximation method used for series of modulation phase values. The accelerometer output, $u(t_i)$ is then rewritten as the following:

$$u(t_i) = A_u \cos(\omega t_i) - B_u \sin(\omega t_i) + C_u , \tag{9}$$

where

A_u $= \hat{u}\cos\varphi_u$,
B_u $= \hat{u}\sin\varphi_u$,
C_u : a constant,
$\hat{u}$: amplitude of accelerometer output,
φ_u : output initial phase angle,

The amplitude of the accelerometer output, $\hat{u}$ and output initial phase angle, φ_u are calculated from the values of A_u and B_u obtained by means of sine approximation method, using the following formulae:

$$\hat{u} = \sqrt{A_u^2 + B_u^2} \tag{10}$$

$$\varphi_u = \arctan\frac{B_u}{A_u} \tag{11}$$

The magnitude of complex sensitivity S_a and phase shift $\Delta\varphi$ of the accelerometer are calculated from magnitude of acceleration and initial phase angles obtained by using sine approximation method, using the formulae given below:

$$S_a = \frac{\hat{u}}{\hat{a}} \tag{12}$$

$$\Delta\varphi = (\varphi_u - \varphi_a) = (\varphi_u - \varphi_s - \pi) \tag{13}$$

3. MEASUREMENT RESULTS AND ANALYSIS

The schematic of measurement setup is shown in Fig. 2, while the view of the realized experimental set-up is presented in Fig. 3. Brüel & Kjaer type 8305 back-to-back reference standard accelerometer was used during the measurements. The applicable frequency range of the ISO 16063-11 is stated as 1 Hz to 10 kHz. Two different vibration exciters are used for the calibrations in full frequency range. The experimental results presented in this paper are obtained in mid and high frequency range where Brüel & Kjaer type 4809 exciter was used for generation of mechanical vibrations. Commercially available 8 channels data acquisition card with sampling rate of 10 MS/s was used for data collection and sampling.

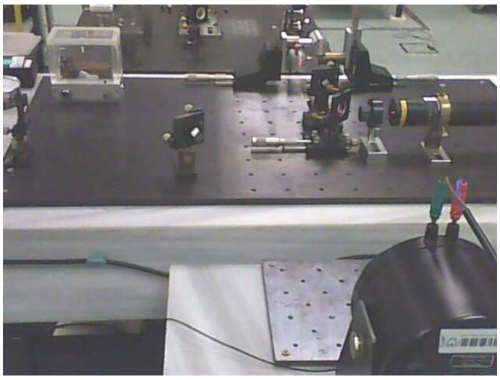

Fig. 3. View of experimental setup for sine approximation method

DC component of the signal from photo detectors were removed by the amplifier connected to them. Quadrature signals from photo detectors and accelerometer output signal over charge amplifier were sampled synchronously and equidistantly recorded into a PC. A typical waveform of the quadrature signal is shown in Fig. 4.

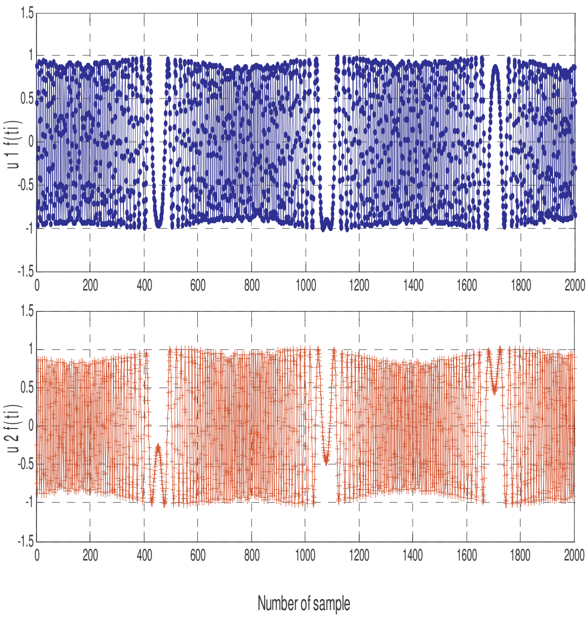

Fig. 4. Some part of quadrature signal from photo detectors for 160 Hz vibration frequency, upper trace from photodetector 1, $u_{1f}(t_i)$ lower trace from photodetector 2, $u_{2f}(t_i)$

Due to the misalignment of interferometer and difference between gains of photo-detector channels and deviation from 90° between two beams, the quadrature output deviates from exact circle. The correction algorithm described in reference [13] and used during signal processing can be explained simply as the following. The distorted coordinates of the circle can be written as:

$$u_1^d = u_1 + p , \tag{14}$$

$$u_2^d = \frac{1}{r}(u_2 \cos\alpha - u_1 \sin\alpha) + q , \tag{15}$$

where

u_{1d}, u_{2d} : sampled data from photo detector, (distorted x and y coordinates of the circle respectively)

p : offset from axis x ,

q : offset from axis y,

r : gain ratio of photo detector channels,

α : quadrature error for reference signal, i.e. deviation from 90°

Equation for a circle with the radius of R can be described by distorted x and y coordinates as the following:

$$(u_1^d - p)^2 + \left[\frac{(u_2^d - q)r + (u_1^d - p)\sin\alpha}{\cos\alpha}\right]^2 = R^2 \qquad (16)$$

The equation (16) can be rewritten as below with the new parameters in terms A, B, C, D and E.

$$(Au_1^{d^2} + Bu_2^{d^2} + Cu_1^d u_2^d + Du_1^d + Eu_2^d) = 1, \qquad (17)$$

where

$$A = (R^2\cos^2\alpha - p^2 - r^2q^2 - 2rpq\sin\alpha)^{-1}$$

$$B = Ar^2$$

$$C = 2Ar\sin\alpha$$

$$D = -2A(p + rq\sin\alpha)$$

$$E = -2Ar(rq + p\sin\alpha)$$

Those signals were corrected applying series of equations from (14) to (17) by using least-square method. In order to reproduce photo detector output signal with the radius R, algorithm presented in reference [14] is used. The signals obtained from correction process are presented in Fig. 5.

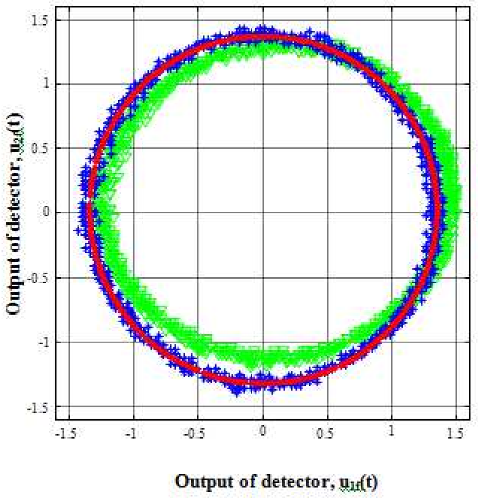

Fig. 5. Quadrature output of interferometer

▽▽▽ : Raw data obtained from measurements

*** : Corrected data after Heydemann algorithm

●●● : Normalized data to the radius R after application of Kasa algorithm.

The modulation phase value is calculated based on the equation (3). The corrected modulation phase value obtained from corrected photo detector output signals are shown in Fig. 6. It is clear from Fig. 6 that modulation phase value varies within the range from $\pi/2$ to $-\pi/2$.

The amplitude of the displacement and therefore the amplitude of acceleration is calculated form the modulation phase values applied phase unwrapping routine. Obtained displacement signal is presented in Fig. 7. The sine approximation method is also applied to the displacement signal calculated in accordance to the equation (3). Calculated displacement is straight line in blue and sine approximated signal is straight line in red color. Blue straight line presenting the displacement is not seen clearly because of the good agreement between calculated and sine approximated displacement graph. The modulation phase amplitude, φ_M and the initial phase angle of the displacement, φ_s are calculated from the values of A and B using equations (7) and (8).

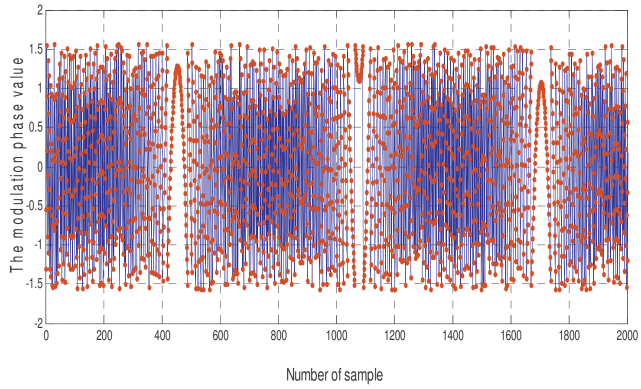

Fig. 6. Corrected modulation phase values obtained from corrected photo detector output signals

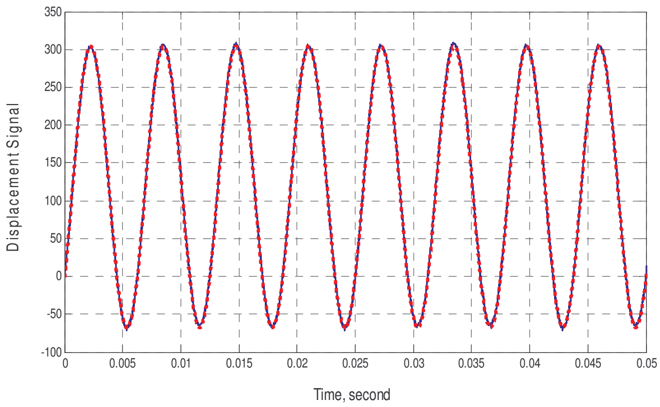

Fig. 7. Displacement signal vs. time obtained by sine approximation method. Calculated displacement is straight line in blue and sine approximated signal is dashed line in red

The sampled accelerometer output signal, $\{u_a(ti)\}$ and its sine approximated signal is shown in Fig 8.

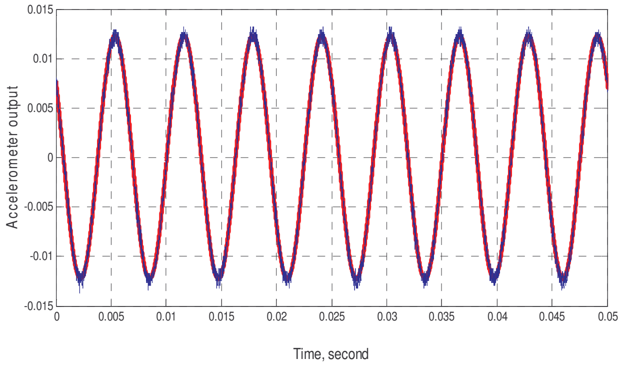

Fig. 8. Accelerometer output signal. Sampled value is straight-line in blue and sine approximated signal is straight-line in red.

The accelerometer output amplitude, û and the initial phase angle of the acceleration, φ_u are calculated from the values of A_u and B_u using equations (10) and (11). Finally, all the parameters required for the determination of the magnitude and phase shift of the complex sensitivity of the accelerometer are obtained using the equations (12) and (13). The obtained magnitude of the sensitivity is given in Fig. 9.

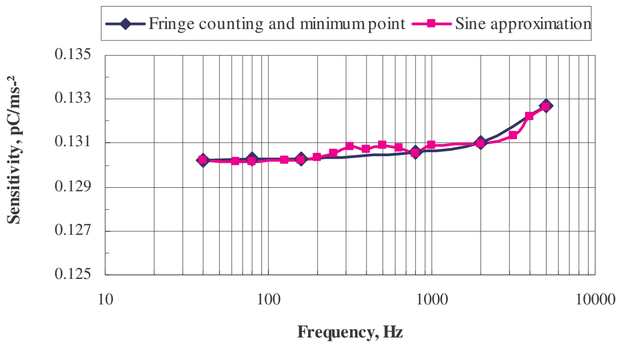

Fig. 9. Magnitude of complex sensitivity vs. frequency

4. DISCUSSION OF MEASUREMENT RESULTS

Capabilities of TÜBİTAK UME for primary calibrations of accelerometers cover all three methods described in ISO 16063-11 standard. The results of calibrations performed by sine approximation method were compared with those obtained by fringe counting and minimum point method in the frequency range from 40 Hz to 5 kHz. As one can see from graph on Fig. 9 all values are in reasonable agreement.

ISO 16063-11 standard provides general survey on uncertainty components to be included in the uncertainty budget for magnitude and phase calibration. Careful evaluation of all uncertainty components specified in the standards resulted in the expanded uncertainty from 0,5% to 1,0% for the magnitude of the complex sensitivity in the whole applicable frequency range. Expanded uncertainty for phase calibration varies from 0,5° to 1,0° depending on a frequency of calibration.

Few systematic effects on the measurements results were evaluated by performing additional investigations. The effect of the hum and noise on the displacement for the frequencies starting from 4000 Hz was investigated and reported in [15]. Similar work was carried out at TÜBİTAK UME to evaluate an effect of the hum and noise at moderate acceleration of 50 m/s^2 for the 5000 Hz vibration frequency. The photo detector output signal for 100 ms duration is shown in Fig. 10. In order to get the frequency component of the time signal, FFT analysis was carried out. Obtained spectrum is shown in Fig. 11.

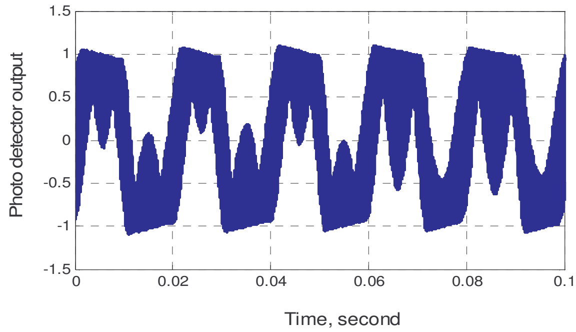

Fig. 10. Photo detector output signal obtained at about 50 m/s^2 acceleration at 5000 Hz.

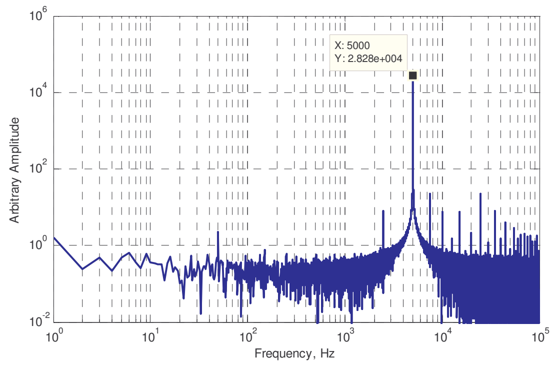

Fig. 11. FFT spectrum of the photo detector output signal obtained at about 50 m/s^2 acceleration at 5000 Hz.

The first component of the spectrum is 50 Hz and it clearly seen from Fig. 11. The main peak occurs at 5000 Hz and expanding scale of the graph enables to observe clearly the vibration signal of the 5000 Hz. It is shown in Fig. 12. It is possible to get rid of the effects of drift by low pass filtering. This investigation is not applied to the present work, but left for future work.

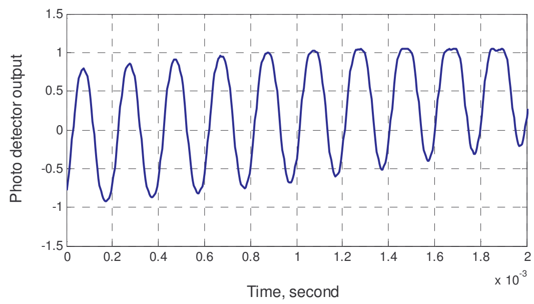

Fig. 12. 5000 Hz vibration motion affected by drift.

5. CONCLUSIONS

Experimental setup for sine approximation method was constructed and calibrations of reference standard accelerometer were carried out according to ISO 16063-11 standard in TÜBİTAK UME. All required algorithms for calculations and correction was constituted. The obtained sensitivities were compared to those obtained by fringe counting and minimum point methods. The reasonable agreement between calibration results by different methods has been achieved.

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