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FOUR TERMINAL-PAIR COAXIAL STANDARDS OF CAPACITANCE

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Abstract – Four terminal-pair coaxial capacitance standards with nominal values of 1 pF and 10 pF have been realized and, using a uniform transmission line model, their frequency dependences have been evaluated. 1:1 comparisons against reference capacitors with known frequency dependence (Agilent 16381A and Agilent 16382A) have shown that the values of the 1 pF and the 10 pF standard change by 0,04 % and -0,05 % respectively when frequency increases from 1 kHz to 1 MHz.

Keywords: four terminal-pair standard, coaxial capacitor

1. INTRODUCTION

Impedance standards with known frequency dependence proved to be useful tools for calibrating wideband LCR meters. Mainly the so-called calculable standards are of importance, frequency dependence of which can be calculated, with a sufficient accuracy, from the knowledge of their constructional parameters. Resistors and capacitors of coaxial design can be mentioned as examples of these standards [1].

In this paper 1 pF and 10 pF coaxial capacitors are described. Their form is shown in Fig. 1, where 1 and 2 are active electrodes, 3 and 4 being the guards. The electrodes and their supporting structures are placed in duralumin boxes which act as shields and on which coaxial connectors HC, HP, LC and LP are mounted. Connectors HC and HP are interconnected with electrode 1, connectors LC and LP with electrode 2 (all connections are situated in the same plane which is marked by a dot-and-dash line in Fig. 1). Guard electrodes are connected to the shield and, in practical use, they are at the same potential as electrode 2.

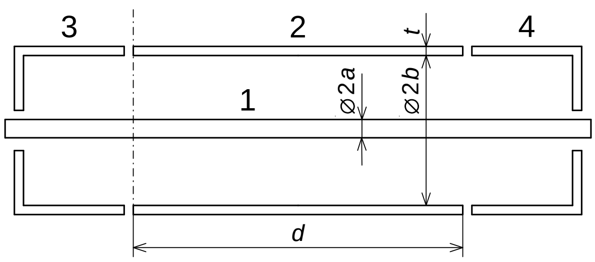

Fig. 1. Longitudinal section of a coaxial capacitor.

2. DESIGN

Capacitance C_{12} between active electrodes can be calculated from

$$C_{12} = \frac{2\pi\varepsilon\varepsilon_0 d}{\ln\frac{b}{a}} \qquad (1)$$

where ε is relative permeability of the dielectric between the electrodes (for air $\varepsilon = 1$) and $\varepsilon_0 \approx 8{,}854 \times 10^{-12}$ F/m. Using this equation, ratios b/a have been calculated for various lengths d on condition that C_{12} = 1 pF (Fig. 2) or C_{12} = 10 pF (Fig. 3). With regard to the results of these calculations, dimensions d and $2b$ have been chosen and corresponding diameters $2a$ determined (Table 1).

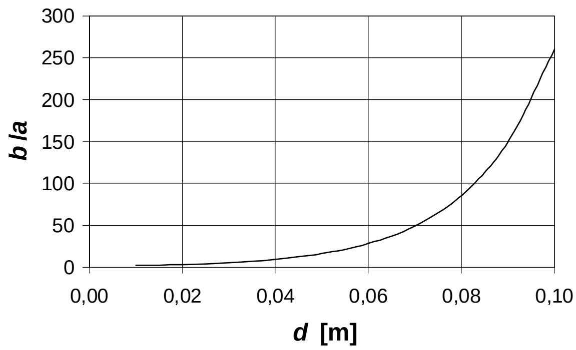

Fig. 2. Dependence of ratio b/a on the length d for C_{12} = 1 pF.

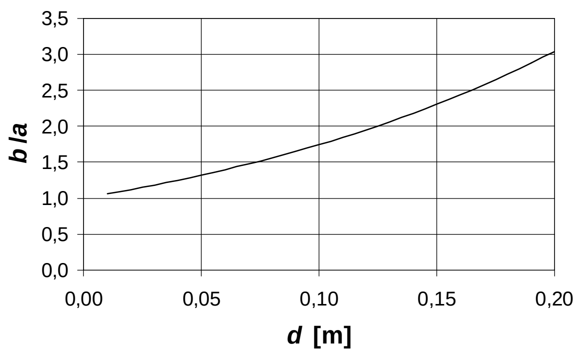

Fig. 3. Dependence of ratio b/a on the length d for C_{12} = 10 pF.

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Table 1. Dimensions and parasitic inductances of the standards.

	1 pF standard	10 pF standard
d [m]	0,05	0,15
$2b$ [mm]	40	40
$2a$ [mm]	2,48	17,4
t [mm]	0,05	0,05
L_1 [H]	$2{,}5 \times 10^{-9}$	$7{,}5 \times 10^{-9}$
L_d [H]	$2{,}78 \times 10^{-8}$	$2{,}50 \times 10^{-8}$
L_2 [H]	$8{,}3 \times 10^{-12}$	$2{,}5 \times 10^{-11}$
L [H]	$3{,}03 \times 10^{-8}$	$3{,}25 \times 10^{-8}$

In addition to the dimensions, calculated values of parasitic inductances of the standards are given in Table 1.

Inductance L uniformly distributed along the active electrodes can be expressed as the sum

$$L = L_1 + L_d + L_2 \tag{2}$$

where the components L_1, L_d and L_2 correspond to magnetic fields penetrating electrode 1, dielectric between the active electrodes and electrode 2, respectively. Table 1 shows DC values of these components, which have been calculated from

$$L_1 = \frac{\mu_1 \mu_0}{8\pi} d \tag{3}$$

$$L_d = \frac{\mu_d \mu_0}{2\pi} d \ln\frac{b}{a} \tag{4}$$

$$L_2 = \frac{\mu_2 \mu_0}{6\pi} d \frac{t}{b} \tag{5}$$

where $\mu_0 = 4\pi \times 10^{-7}$ H/m, μ_d is relative permeability of the dielectric and μ_i, $i = 1, 2$ are relative permeabilities of active electrodes ($\mu_d = \mu_1 = \mu_2 = 1$). While L_d is frequency independent, the values of L_1 and L_2 decrease with frequency. Consequently, the value of L asymptotically decreases to L_d when frequency increases but, as can be seen from Table 1, this decrease does not exceed 8 % for the 1 pF standard and 23 % for the 10 pF one.

Each of the standards can be regarded as a low-loss transmission line of the length d and with uniformly distributed capacitance C_{12} and inductance L [2]. Let $\boldsymbol{U}_1$ and $\boldsymbol{I}_1$ be input voltage and current of the line, than

$$\begin{bmatrix} \boldsymbol{U}_1 \\ \boldsymbol{I}_1 \end{bmatrix} = \begin{bmatrix} \cosh(\gamma d) & \boldsymbol{Z}_0 \sinh(\gamma d) \\ \frac{1}{\boldsymbol{Z}_0}\sinh(\gamma d) & \cosh(\gamma d) \end{bmatrix} \begin{bmatrix} \boldsymbol{U}_2 \\ \boldsymbol{I}_2 \end{bmatrix} \tag{6}$$

where $\boldsymbol{U}_2$ and $\boldsymbol{I}_2$ are output voltage and current respectively,

$$\boldsymbol{Z}_0 = \sqrt{L/C_{12}} \tag{7}$$

$$\gamma d = \mathrm{j}\omega\sqrt{L C_{12}} \tag{8}$$

and ω is angular frequency.

In case that such a line is unloaded ($\boldsymbol{I}_2 = 0$), its input impedance is

$$\boldsymbol{Z}_{in} = \frac{\boldsymbol{U}_1}{\boldsymbol{I}_1} = \boldsymbol{Z}_0 \coth(\gamma d) = -\mathrm{j}\,\boldsymbol{Z}_0 \cot\left(\omega\sqrt{L C_{12}}\right) \tag{9}$$

By expanding $\cot\left(\omega\sqrt{LC_{12}}\right)$ into a Taylor series we obtain

$$\boldsymbol{Z}_{in} = -\mathrm{j}\sqrt{\frac{L}{C_{12}}}\left[\frac{1}{\omega\sqrt{LC_{12}}} - \frac{\omega\sqrt{LC_{12}}}{3} - \frac{\left(\omega\sqrt{LC_{12}}\right)^3}{45}\cdots\right] =$$
$$= \frac{1}{\mathrm{j}\omega C_{12}}\left(1 - \frac{\omega^2 L C_{12}}{3} - \frac{\omega^4 L^2 C_{12}^2}{45} - \cdots\right) \tag{10}$$

and the corresponding apparent capacitance of the standard is

$$C_{app} \approx \frac{C_{12}}{1 - \frac{\omega^2 L C_{12}}{3}} \approx C_{12}\left(1 + \frac{\omega^2 L C_{12}}{3}\right) \tag{11}$$

In reality, $\boldsymbol{I}_2 \neq 0$ due tu nonzero capacitance C_{14}, which is distributed between the electrodes 1 and 4. When also this capacitance is taken into account,

$$C_{app} \approx C_{12}\left[1 + \omega^2 L\left(\frac{C_{12}}{3} + C_{14}\right)\right] \tag{12}$$

and the relative change δ_{app} in apparent capacitance of the standard due to parasitic inductance L and capacitance C_{14} (C_{14} = 4 pF for the 1 pF standard and C_{14} =15,5 pF for the 10 pF one) is

$$\delta_{app} = \frac{C_{app} - C_{12}}{C_{12}} \approx \omega^2 L\left(\frac{C_{12}}{3} + C_{14}\right) \tag{13}$$

In Figs. 4 and 5 changes δ_{app} calculated from (13) for both the 1pF and the 10 pF standard are plotted as function of frequency.

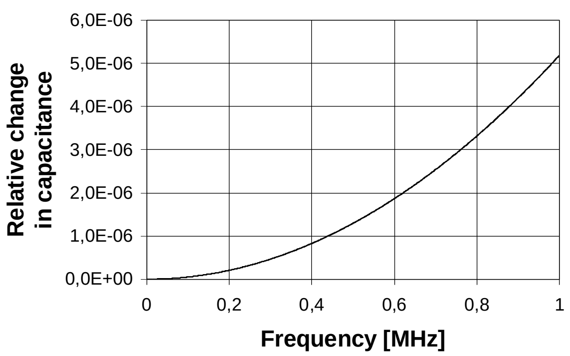

Fig. 4. Frequency dependence of δ_{app} for the 1 pF standard.

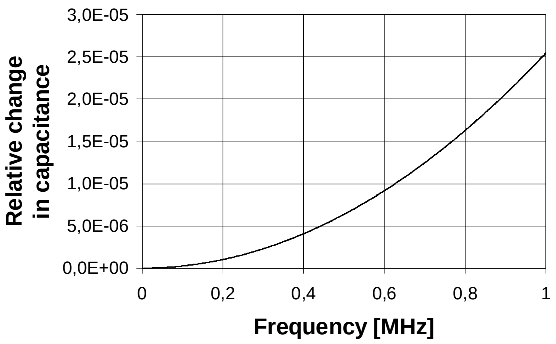

Fig. 5. Frequency dependence of δ_{app} for the 10 pF standard.

3. EXPERIMENTAL STANDARDS

Inner electrodes of these standards being formed by brass rods, parts of openable outer electrodes have been realized by gluing 0,05 mm copper foils on insulating supports of hardened fabric and hartmatte with grooves of semicircular profile (Figs. 6 and 7). By bolting the supports together, the parts of electrodes have then been assembled to form thin copper cylinders.

Frequency dependences of the completed standards have

Fig. 6. Electrodes of the 1 pF standard.

Fig. 7. Electrodes of the 10 pF standard.

been measured by 1:1 comparisons against Agilent 16381A (1 pF) and Agilent 16382A (10 pF) capacitors, frequency dependences of which have been determined by their producer using a metod described by Suzuki [3]. Comparison has been made by means of an Agilent 4285A LCR meter and its results are shown in Figs. 8 and 9.

Due to the effect of relatively high admittances between electrode 2 and the guards, and due to the effect of nonzero mutual inductances between connecting leads of electrodes,

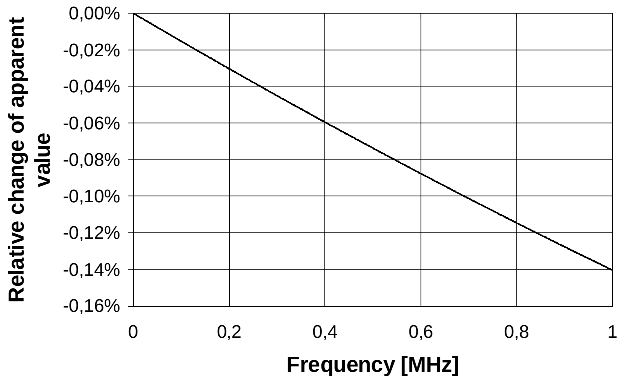

Fig. 8. Frequency dependence of the 1 pF standard.

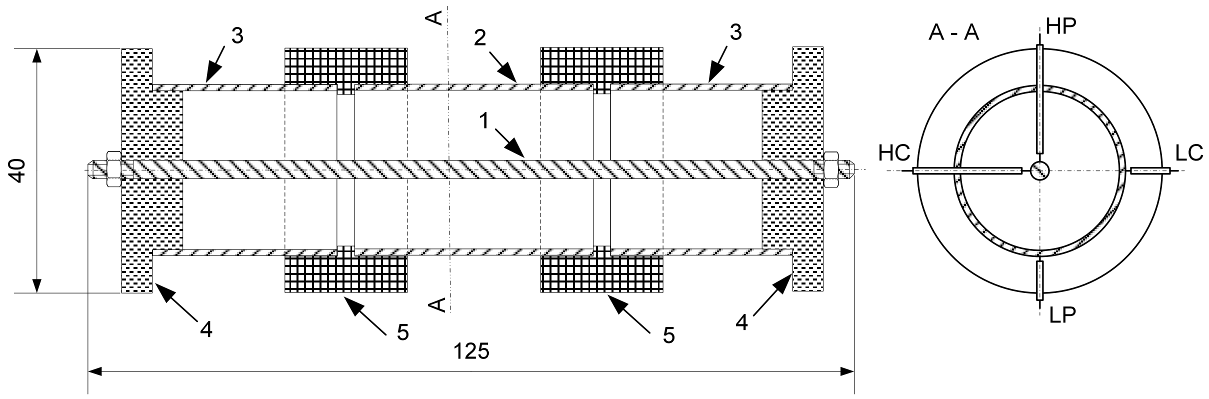

Fig. 10. Electrode system of the improved 1 pF capacitor (dimensions in milimeters).
1 – inner active electrod (brass rod), 2 - outer active electrod (copper tube), 3 – guard electrode (copper tube),
4 – supporting ring (polyamide), 5 – spacer (PTFE).

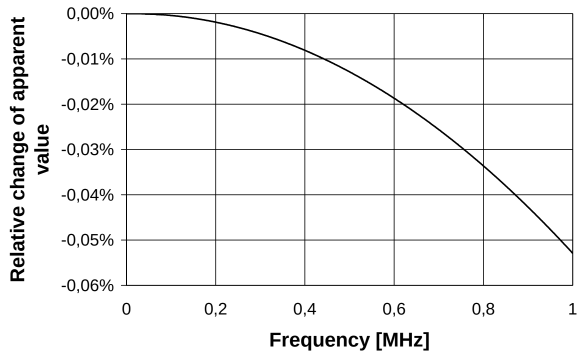

Fig. 9. Frequency dependence of the 10 pF standard.

frequency characteristics of Figs. 8 and 9 differ considerably from those shown in Figs. 4 and 5. Of course, these effects can be reduced and frequency performance of the standards can be improved by using more suitable insulating materials and by changing the design of electrode connections and supports.

4. IMPROVED 1 pF STANDARD

With an intention to reduce frequency dependence of the 1 pF capacitor, a new version of it has been realized in which two polyamide rings and two PTFE spacers serve as electrode supports (Fig. 10). In Table 2 dimensions of the electrodes are shown.

Table 2. Dimensions of the improved 1 pF standard.

d [m]	0,04
$2b$ [mm]	26
$2a$ [mm]	3,14
t [mm]	1

It has been found by a 1:1 comparison against an Agilent 16381A capacitor that the value of the realized standard changes by 0,04 % only when frequency increases from 1 kHz to 1 MHz.

5. CONCLUSIONS

Four terminal-pair capacitance standards with nominal values of 1 pF and 10 pF have been realized to be used as references in measurement of frequency dependences of low-valued capacitors in a frequency range from 1 kHz to 1 MHz.

Recalibration of these standards is normally made at 1 kHz only [4], their values for higher frequencies being calculated from their values for 1 kHz and from their known frequency dependences.

ACKNOWLEDGMENTS

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