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A NEW APPROACH TO THE DESIGN OF POST-DAC FILTERS

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Abstract –In many measurement applications, it is required to have digital-to-analog converters (DAC). All DACs use a reconstruction (anti-imaging) filter as the final step in the conversion process. Without this filter, generating a continuous-amplitude signal would not be possible. This paper presents a new theoretical concept of a post-DAC continuous-time filter. The proposed filter was designed so as to provide both a constant group delay over the desired frequency band and a maximally flat magnitude response. Moreover, the filter coefficients were varied in the time in order to accelerate the filter response. Results verifying the effectiveness of the proposed approach are presented and compared to the performance of a traditional lowpass reconstruction filter.

Keywords: analog signal processing, post-DAC filters, time-varying systems, group delay compensation.

1. INTRODUCTION

Analog circuits [1–4] are an integral part of front ends used in measurement systems as well as in other types of electronic systems such as telecommunication and signal processing systems. Although the processing of information is done efficiently using digital circuits (it suffices to increase the number of bits used to represent a single word in order to increase the signal-to-noise ratio of a given digital signal processing system, for instance), the interface with the real world has to be invariably done via analog blocks. In measurement applications, amplifiers must be used to increase the power content of a given signal of interest, analog-to-digital converters must be used to obtain a discrete representation of a signal for further processing in a digital system and digital-to-analog converters must be used to transform a carrying-information digital signal into a discrete-amplitude signal. This signal may be transformed into a continuous-amplitude one by means of a lowpass reconstruction filter.

There are many filter approximations which may be used to implement a lowpass reconstruction filter. In any case, the bandpass of the filter should match the expected bandwidth of the reconstructed signal. In this article, a new strategy for the design of reconstruction filters for digital to analog converters is presented. This strategy is based on the usage of a delay-compensated Butterworth lowpass filter with accelerated time-domain response by means of the variation of its parameters.

2. PROPOSED FILTER STRUCTURE

2.1. Maximally flat magnitude response

The main goal of the Butterworth approximation is to obtain a maximally flat magnitude response in the filter passband. The magnitude response of the low-pass Butterworth filter is written as follows:

$$|H_F(j\omega)|^2 = \frac{h_0}{1+\left(\frac{\omega}{\omega_c}\right)^{2n}} \tag{1}$$

where n is the order of the filter, h_0 is the DC gain (gain at zero frequency), ω_c is the 3dB limit frequency. For this class of filters, however, its phase response is nonlinear.

2.2. Compensation of group delay variations

The phase linearity of an arbitrary filter is specified in terms of its group delay versus frequency. A flat group delay indicates that all frequency components of a given input signal are delayed by the same amount. Therefore, the shape of the waveform in the time domain is preserved.

If the group delay of a given filter has an unacceptable variation range in its passband, its phase has to be compensated. However, this must be done in such a way that the filter's magnitude response is not altered. In order to attain group delay equalization an allpass filter with transfer function denoted as $H_A(s)$ and magnitude response equal to unity may be connected in cascade to the uncompensated filter with transfer function denoted as $H_F(s)$. The transfer function $H_C(s)$ of the compensated filter will be given by $H_C(s) = H_F(s) \cdot H_A(s)$, or alternatively,

$$\begin{aligned} H_C(j\omega) &= |H_F(j\omega)||H_A(j\omega)|e^{j\varphi_F(\omega)}e^{j\varphi_A(\omega)} \\ &= |H_F(j\omega)|e^{j[\varphi_F(\omega)+\varphi_A(\omega)]}. \end{aligned} \tag{2}$$

We see that the magnitudes multiply with no contribution from the allpass module because $|H_A(j\omega)| \equiv 1$, and that the phases add. Since the group delay is obtained from the negative derivative of the phase, the delays $D_F(\omega)$ and $D_A(\omega)$ add as well

$$\begin{aligned} D_C(\omega) &= -\frac{d[\varphi_F(\omega)+\varphi_A(\omega)]}{d\omega} \\ &= -\frac{d\varphi_F(\omega)}{d\omega} - \frac{d\varphi_A(\omega)}{d\omega} \\ &= D_F(\omega) + D_A(\omega). \end{aligned} \tag{3}$$

The total group delay $D_C(\omega)$ in this process will increase because $D_C(\omega)$ is larger than either $D_F(\omega)$ and $D_A(\omega)$.

A block diagram of the compensation process is presented in Fig. 1.

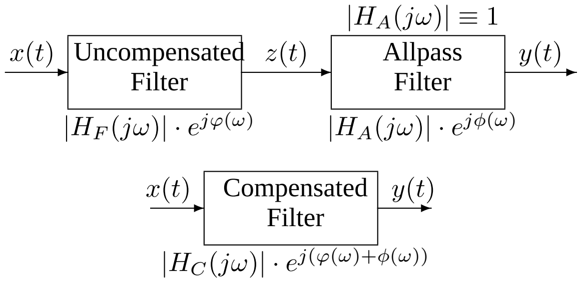

Fig. 1. General scheme of the group delay compensation process.

The aforementioned compensation process [2, 3] is always carried out at the cost of increasing the duration of the transient behavior of the filter. This problem will be solved with the aid of the parameter-varying approach.

2.3. The parameter-varying approach

As it was demonstrated in [5–9], it is possible to obtain significant changes of the duration of the transient behavior of a lowpass filter by the variation of the filter passband. In the most general case, an even-order lowpass filter may be described by means of the following transfer function

$$H_{F_{even}}(s) = \frac{1}{\prod_i^{n/2}\left(\omega_{0i}^{-2}s^2 + 2\beta_i\omega_{0i}^{-1}s + 1\right)} \qquad (4)$$

whereas the transfer function of an odd-order lowpass filter may be expressed as

$$H_{F_{odd}}(s) = \frac{1}{(Ts+1)\prod_i^{n/2}\left(\omega_{0i}^{-2}s^2 + 2\beta_i\omega_{0i}^{-1}s + 1\right)}. \qquad (5)$$

The dynamic properties of the filters described by the transfer functions given in expressions (4) and (5) are defined in terms of the damping factors β_i, $i = 1, 2, \ldots, n/2$, the characteristic frequencies ω_{0i}, and the time constant T (for odd filter orders). In a similar way, the dynamic behavior of a second-order allpass filter having the following transfer function

$$H_{A_2}(s) = \frac{\omega_{0p}^{-2}s^2 - 2\beta_p\omega_{0p}^{-1}s + 1}{\omega_{0p}^{-2}s^2 + 2\beta_p\omega_{0p}^{-1}s + 1}. \qquad (6)$$

is described in terms of its characteristic frequency ω_{0p} and the damping factor β_p

A parameter-varying filter is the result of modeling by means of a system of ordinary differential equations with varying coefficients the systems whose transfer functions in the frequency domain are described by expressions (4) or (5) and (6). In order to speed up the filter response, the function responsible of the change of each of the filter parameters $F = \left\{\omega_{0i}, \beta_i, \frac{1}{T}, \omega_{0p}, \beta_p\right\}$ has been assumed to adopt the following form:

$$F(t) = F(0) \cdot \left[1 - \frac{d-1}{d} \cdot h(t)\right], \quad d = \frac{F(0)}{\overline{F}} \qquad (7)$$

where $\overline{F}$ is the value of the previously mentioned filter parameters which has been derived from from the Butterworth approximation and the coefficient estimation of the required allpass filter transfer function, and d is the variation range of the function $F(t)$. The function $h(t)$ in (7) describes the step response of the second-order support system $H_S(s)$

$$\begin{aligned} h(t) &= \mathcal{L}^{-1}\left[s^{-1} \cdot H_S(s)\right] \\ &= \mathcal{L}^{-1}\left[s^{-1} \cdot \frac{1}{\omega_{0f}^{-2}s^2 + 2\beta_f\omega_{0f}^{-1}s + 1}\right] \end{aligned} \qquad (8)$$

where $\mathcal{L}^{-1}$ is the inverse Laplace transform, and β_f and ω_{0f} determine the oscillations and the variation rate of the function $F(t)$, respectively. A block diagram of the proposed filter structure is presented in Fig. 2. It is assumed that function

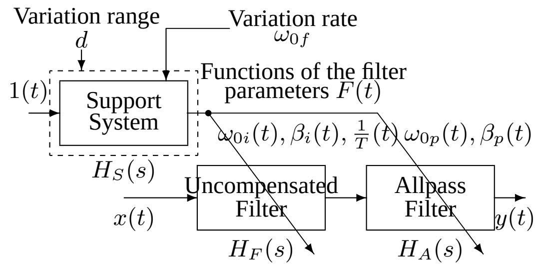

Fig. 2. Block diagram of the proposed parameter-varying filter.

$F(t)$ must settle to the original value of the parameter under variation during the expected duration of the transient behavior of the original linear time-invariant filter. This condition can be written as

$$\forall_{t>t_{s\alpha}} \; |F(t) - \overline{F}| \leq \alpha \qquad (9)$$

where $t_{s\alpha}$ is the settling time (with assumed accuracy $\alpha = 5\%$) of the original filter.

The best results in the reduction of the transient behavior of compensated Butterworth filters were obtained when the parameters ω_{0i}, $\frac{1}{T}$, and ω_{0p} were varied using the same function $F(t)$ (and with the same variation range denoted by d_ω), while β and β_p were varied according to the different functions (with different variation ranges denoted d_β and d_{β_p}, respectively). $\beta(t)$ causes a stronger damping in the initial phase of the filter work as well as the suppression of undesirable overshoot in the step response, whereas $\beta_p(t)$ eliminates the undershoot from the step response.

3. RESULTS

Parameter-varying filters are very useful when the time instants in which the filter parameters should be varied are known beforehand. This situation appears during the transformation process of a zeroth-order hold signal into an analog signal. In this process, the time instants in which zeroth-order hold signal changes its level are already known, so it is possible to vary periodically the value of each of the filter coefficients. A simplified diagram presenting the aforementioned process is shown in fig. 3.

Fig. 4 presents an sample original analog signal and its zeroth–order hold equivalent, and figs. 5-7 show the functions considered for the variation of the parameters of a third-order phase–compensated Butterworth filter.

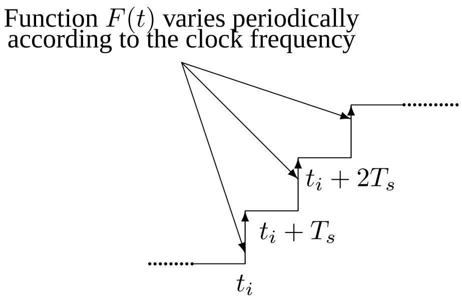

Fig. 3. Periodic generation of the function $F(t)$.

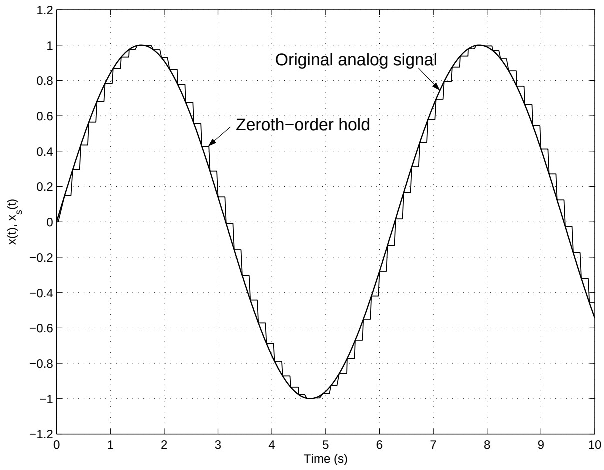

Fig. 4. Original analog signal and its zeroth-order hold equivalent.

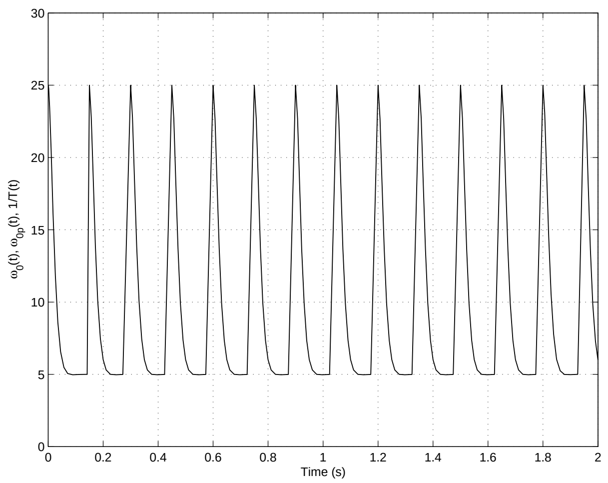

Fig. 5. Functions $\omega_0(t)$, $\omega_{0p}(t)$, and $\frac{1}{T}(t)$.

It is easy to notice from fig. 8 that the introduction of time-varying coefficients to the reconstruction filter causes significant acceleration of the filter response. The cutoff frequency of the reconstruction filter was selected so as to notice significant distinction between the proposed time-varying filter and its constant parameter equivalent.

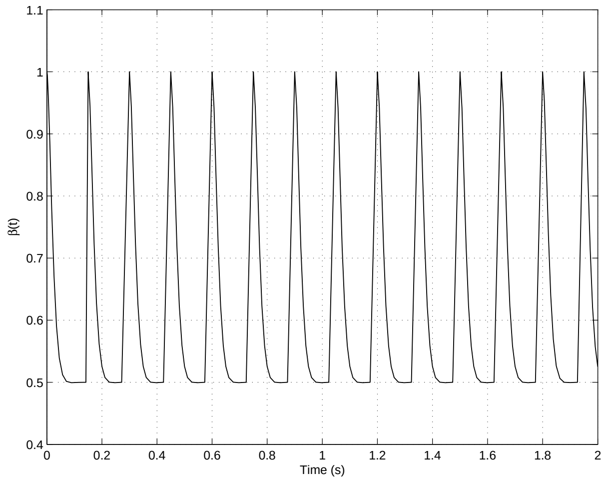

Fig. 6. Function $\beta(t)$.

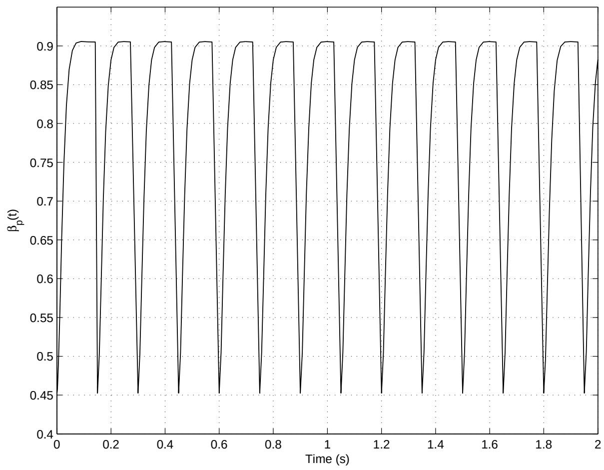

Fig. 7. Function $\beta_p(t)$.

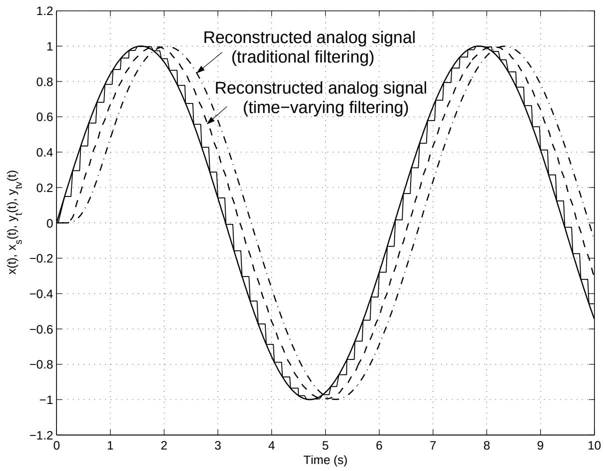

Fig. 8. Simulation results of the parameter-varying filter and the linear time-invariant filter used in the reconstruction of the signal shown in Fig. 4.

4. CONCLUSIONS

In this paper, the application of the parameter-varying approach in the design of post-DAC reconstruction filters has been presented. By using the described filtering approach, it is possible to obtain an efficient filter that ensures a maximally flat magnitude response, and at the same time provides a constant delay over the desired frequency band. Besides, the designed filter is considerably faster than the traditional time-invariant one.

ACKNOWLEDGEMENTS

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