XIX IMEKO World Congress
Fundamental and Applied Metrology
September 6–11, 2009, Lisbon, Portugal

CHOICE OF THE MEAUREMENT POINTS FOR A CALIBRATION IN A RANGE

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Abstract – Calibration of instruments like modern programmable instruments is generally made in specific points within each range, even if the desired result is the general calibration of the instruments. This paper is considering the problem of how many points are needed for a correct calibration of a range, and, by means of a statistical approach, the method propose evaluates a confidence interval where the difference between the measured and the calibrated can be confined with an assigned probability.

Keywords Uncertainty, Range characterisation, Sampling.

1. INTRODUCTION

Systems for calibration and verification of instruments have been built. Generally, even in case of measurements by means of automatic system [1] – [4] the number of measurements performed is limited and, in each range, only a few points are taken. In fact, the use of an automatic systems is an advantage for the calibration and the verification of instruments, but, even if in this case, time is necessary for both the calibration of the instruments under test and the instruments of the calibration system. So, each measurement point introduced in the procedure increases the measurement time and consequently the total cost of the calibration. For this reason it is important to reach a compromise between the reliability of the measurement procedure and its cost.

For the characterisation of an instrument it is useful to think, for every quantity and range of the instrument under investigation a function that characterise the output value. In a voltmeter, for example, this function can be given as a table or a graphics where, for each calibrated voltage at the input of the instrument, the value of the voltage read by the instrument is supplied.

An alternative representation is a function *d(x)* of the difference between the value measured or generated by the instruments and the calibrated one. This difference can be a function of the value of the quantity and of other quantities of influence, like the frequency.

Using a function like this, even if in an implicit way, manufacturers generally assign the specifications of an instrument by attributing an interval around the expected value. The specification can be given, for example, as a fix term added to another proportional to the absolute value of the quantity under test.

The expression of the uncertainties in specific points of the range does not imply particular conceptual difficulties, because the theoretical framework derives mainly from the method for the expression of the uncertainty for single measurement [5]. Instead, the characterisation of the whole selected ranges requires a different approach.

A disadvantage of specifications given as a interval of values, in the traditional sense, is due to the fact that, for each measurement, the result can be only within or outside this interval. So, the hypothesis of compliance to a given specification can be rejected by a single measurement when it is out of the interval, but instead, when more results are within the interval, one cannot assume that the instrument complies the same specification.

The probabilistic approach appears to be the only acceptable compromise that also reduces the number of measurement points and then the cost of the process.

A possible trade-off between the number of the measurements, and consequently the cost of the measurements, and the compliance to the specifications is then given with a probabilistic model, where the probability of being within a specified limit is given as a result of a set of measurements.

2. THEORY OF THE PROPOSED METHOD

2.1. Relation between the samples

In the method considered here the evaluation is made in a given range of values, (for example in the range –10 V +10 V dc of a voltmeter) In this range the possible difference function *d(x)* is assumed to exist but not to be known. The problem will be to identify an interval where this function is confined, when only a finite number of samples d_i of this function can be measured each as a function to the quantity x and of other quantities (for example the frequency in an ac voltmeter).

So, for assumption, the function of one or more variables.

$$d_i = d(x_i) \tag{1}$$

$$d_{i.k} = d(x_i, y_k) \tag{2}$$

In the simpler case given by expression (1), (the extension to the multiple independent variables can be made accordingly) the additional hypothesis presupposes that the x_i are chosen as a possible output of random variables with a probability density function respectively of *W(x)* within the range. With this assumption, the results obtained by applying the functions *d(x)* can also be seen as a random variable δ. The probability density function *W(δ)* could be derived for *W(x)* if both this probability density function and the function *d(x)* were known. If we have not other information a good choice for *W(*x) seems to be the uniform distribution within the independent variable *x*.

In order to give a first solution to the problem we can assume that the measurement of each single sample of the function *d(x)* is only affected by a negligible uncertainty.

2.2. Identification of a tendency of the difference function

A first step to gain same understanding on the function difference from the data obtained by the measurements it is the identification of a possible tendency of the difference function *d(x*) in the range. For this purpose, the manufacturer can already have supplied possible models.

For example is some case of electrical instrument the offset-gain model is employed to give the specification and that means that the implicit model is a linear one and it can be represented by a function of the independent variable of the range by function like:

$$\begin{aligned} \tilde{d}(x) &= a + b \cdot x \\ \tilde{d}(x) &= a + b \cdot |x| \end{aligned} \tag{3}$$

If the manufacturer has not supplied a model for the correction function, it can be identified by using the common functions. In a group of possible functions, the best one can be chosen considering the value of the correlation factor between the samples and the functions of the set. A statistical test can reject the hypothesis of not correlation to a degree of probability (1-*P*), with the probability *P* appropriately selected (for example *P*=90% or *P*=95%)

When the structure of the function for correcting the characteristic has been selected, the parameters are computed by means of the best fit by means of the samples taken in the measurement process. The correction lead that to a new difference function:

$$d^*(x) = d(x) - c(x) \tag{4}$$

where c(x) is the function selected by the method considered before.

2.3. Variance estimation

By the sample acquired, once the correction has been performed it is possible to estimate the variance. An estimator sufficiently robust of the variance of the distribution of *d** for a small number of samples can be obtained by computing the value that with a probability *P* is greater than the variance. This is obtained by dividing the experimental value of χ^2 by its value computed for the given probability *P* and degrees of freedom ν.

$$\sigma_{d^*}{}^2 = \frac{1}{\chi^2(\frac{1-P}{2},\nu)} \sum_{i=1}^{N} \left[d_i^* - \frac{1}{N} \sum_{i=1}^{N} d_i^* \right]^2 \tag{5}$$

where the degree of freedom ν is linked to the number of samples in the range (*N*) by:

$$\nu = N - 1 \tag{6}$$

The evaluated variance given in (5) is formally valid only for a Gaussian probability density function, The extension for other distributions can be assumed with some limitations if it can be identified as not greatly different. As a support for this assumption the probability density function of the random variable *d** can be tested to evaluate if its identification can be achieved. A possible analysis can be performed by means of a histogram method with the application of the χ^2 test. However, to be consistent with the probability assigned, this requires generally a large number of samples, which can be inconsistent with the measurement requirements. For few samples some evidence, especially to reject the hypothesis of a normal or rectangular probability density function, can be obtained for example by tests on the skewness and kurtosis of the distribution.

2.4. Estimation of the confidence interval

The confidence interval where the values of the difference function in the range where they have not been measured can be evaluated by the previous consideration.
There are two possible ways:

- If the probability density function is identified for example as the normal distribution) the proper coverage factor can be used.
- If instead, there is not sufficient information about the probability density function, the Chebyshev inequality, which is more conservative, but it is valid for every probability density function, can be used to evaluate the limit [6].

Chebyshev inequality states the probability of finding the value outside the limit $t\sigma$ as:

$$P\left\{ \left|X - E(X)\right| \geq t \cdot \sigma \right\} \leq \frac{1}{t^2} \tag{7}$$

3. APPLICATION OF THE METHOD

3.1. Operations for evaluating the limit.

Based on the previous theory the evaluation of the interval limits from a given set of measurements and with a given probability can be programmed in an automatic procedure. In fact, by assuming that the input value have been randomly selected, the operations performed to find the interval limits are:

- Evaluation of the differences between the measured and calibrated values.
- Detection of a possible correction function *c(x)* between the pools of functions considered (at the moment only the linear functions) and evaluation of its parameters by means of least square adjustment.
- Computing the values of d^*_i.
- Estimation of the maximum standard deviation for a given probability by means of relation (5).
- Estimation of the limits by relation (7).

3.2. Model for the simulation of the application of the automatic procedure

A program for the evaluation of the confidence interval where the values are contained with a given probability has been built for simulation purpose.

This program goes through the following operations:

- Requires the definition of the characteristic function *d(x)*. The characteristic functions in the ranges can be given both by means of functions of the input value or by means of tables. In the latter case the analytical function is evaluated by means of an interpolation between the points given in the table.
- Randomly selects a number *N* of samples of *x*.
- Computes the corresponding values of d^*_i.
- Compute the estimated variance and the limits for predefined values of the probability *P*.
- Verify the rate of *x* axis where the difference is outside the evaluated limits.

In this way it is possible to explore different type of functions and also simulate to the effect of the introduction of the uncertainty of the measurements.

4. PRELIMINARY RESULTS

The analysis has been performed with different characteristic functions given by means of analytical expressions. The intervals have been evaluated for different probability (the values investigated were mostly 90% and 95%).

As an example, the determination of the error and of the uncertainty in a voltmeter of range 1 V is given. The unknown function is supposed to be described by a linear function superimposed to a sinusoidal one, as in the equation:

$$d(x) = a \cdot x + b + c \cdot sin(2\pi \cdot x) \quad (8)$$

with a=0.01 %, b= 0.02 % and c=0.002 %.

The uncertainty in each measurement performed by the calibrator is assumed to be at the level of 0.005% (k=2).

The manufacturer of the instruments suggests a linear model by specifying a gain error and an offset error.

Fig. 1 and Fig. 2 gives the results using the theory reported respectively for 5 samples and for 15 samples in the range.

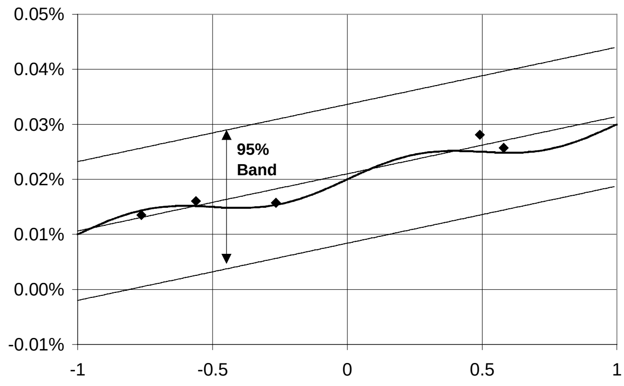

Fig.1 Determination of the 95 % error band with 5 points measured in the range. The curve is assumed to be the real characteristic of the instrument and the points the measurement performed. The identification of the error in the gain and in the offset (straight line in the centre) is respectively $\Delta a=3.9\ 10^{-6}$, and $\Delta b=9.1\ 10^{-6}$.

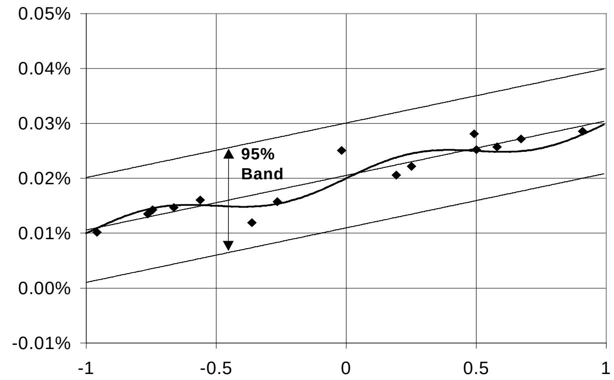

Fig.2 Determination of the 95% error band with a total of 15 point measured in the range. The identification of the error in the gain and in the offset is respectively $\Delta a=-0.7\ 10^{-6}$, and $\Delta b=4.3\ 10^{-6}$.

In this example the good result with only 5 samples is also due to the model supplied to the manufacturer that could not have been validated otherwise by the limited number of samples.

A synthesis of the results for a generalisation is not easy. The exact results, of course, depend on the specific functions selected, but, in the examples examined up to now the limit of the intervals computed always contained the assumed part of the characteristic. Due to the effect of the Chebyshev inequality the evaluation is sufficiently conservative to overcome the fact that the probability density function is not necessarily Gaussian.

An evaluation of the trade-off for the number of samples can be obtained in the same way by examining experimentally different distributions as in the example given for a different number of samples. However, in order to have a hint of the effect of the number of samples on the evaluation one can assume, for example, that the standard deviation computed by the samples is always the same. Then, the result normalised to this standard deviation is given by combining relations (5) and (7). So, the amplitude of the confidence interval containing with probability *P* is given as a function of the number of samples *N* as:

$$CI(P,N)=\sqrt{\frac{1}{\chi^2(\frac{1-P}{2},N-1)}N}\cdot\sqrt{\frac{1}{(1-P)}} \quad (9)$$

The function *CI* is given in Fig. 3 as a function of the number of sample from *N*=4 to *N*=50.

This function show that the reduction of the limits is very effective for a low number of samples, but the improvement is lower when the number of samples is higher than 10.

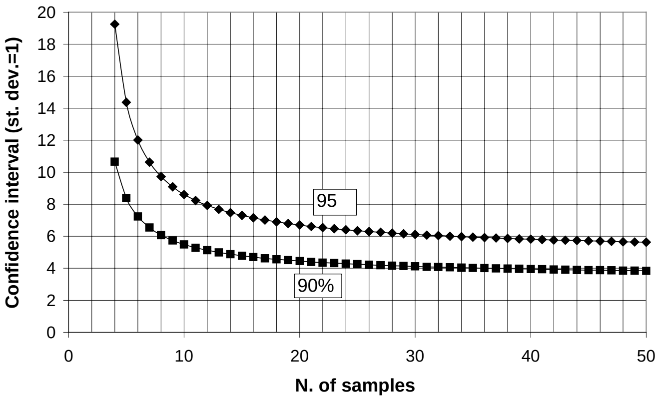

Fig.3 Relative evaluation of the limits for 95% (X) and 90% (+) probability as function of the number of samples.

5. CONCLUSIONS

A method based on statistical approach for the verification in the whole range has been developed.

From a set of measurements in a given range, this method evaluates the confidence interval for the function of the difference between the measured and the calibrated values. The method operates on the basis of systematic tests on the samples, their probability density function and the correlation with a limited number of functions.

The preliminary results obtained by a program operating on many selected analytical functions show that the method estimates the confidence limit of the difference function. The results are generally quite conservative if the probability density function of the results cannot be identified.

A compromise between the accuracy and the number of samples to be employed can be evaluated by the method and is shown in the preliminary results.

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