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PHASE MEASUREMENT OF OPTICAL WAVEFRONT BY AN SLM DIFFERENTIATION FILTER

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Abstract − A system is proposed that employs a spatial light modulator as a differentiation filter to measure the phase of an optical wavefront. The pattern, position and orientation of the filter can be controlled electrically. This system measures the phase derivative of a wavefront that passes through a transparent phase object. The 3D shape of the object can be calculated by integrating the phase derivative.

Keywords: wavefront, SLM, differentiation filter

1. INTRODUCTION

Phase measurements of optical wavefronts are of considerable interest for many applications in optical testing, material diagnostics and microscopic analysis of living cells [1-11]. Many techniques are based on phase contrast [6-11]. One such technique is the Schlieren method, which employs a knife-edge [8-11] and enables qualitative visualization of the phase derivative. In order to obtain quantitative measurements over a wide range of the phase derivative, it is necessary to scan the knife-edge filter. Some studies have reported mechanical scanning of the knife-edge [1].

There are various methods that measure phase by performing optical differentiation using a differentiation filter based on Fourier optics [12-17]. Such a filter does not require scanning and it is possible to measure the phase from a single image. This method has a simple construction and is suitable for wavefronts having a wide range of phase variations. It can be used to do things such as obtaining the 3D shape of a transparent phase object and measuring optical components.

The transmission coefficient of a differentiation filter changes linearly with position along the differentiation direction. The range of the phase derivative measurement is limited by the width of the filter. Consequently, a wide filter can measure a wide range of phases; however, wide filters also have a low resolution. Therefore, an appropriate filter should be selected based on the object to be measured. Furthermore, differentiation filters can visualize the phase derivative only in a single direction. A differentiation filter thus has to be rotated in order to obtain the phase derivative in other directions.

In this paper, a spatial light modulator (SLM) is used as a differentiation filter that can be varied electrically allowing quantitative measurement of wavefronts. Using an SLM enables the filter width and rotation angle of the filter to be varied electrically. This method is used to measure the phase derivative of a wavefront passing through a transparent phase object. The 3D shape of the object can then be calculated by integrating the phase derivative.

2. EXPERIMENTAL

Figure 1 shows the experimental setup of the proposed system. The wavefront of a collimated beam from a 633-nm-wavelength He-Ne laser is measured after it passes through a transparent phase object (sample S). Sample S is located in the object plane of a Fourier-transform lens L2, which has a focal length of 300 mm and a diameter of 45 mm. The SLM is located in the focal point of lens L2 and it functions as an optical differentiation filter with polarizer P. The SLM is a reflective liquid-crystal SLM (Holoeye, LC-R 2500), which has 256 gray levels, an image array with dimensions of 19.5 × 14.6 mm^2, a nominal resolution of 1024 × 769 pixels and a pixel pitch of 19 μm. Light reflected from the SLM and a half mirror is captured using a charge-coupled device (CCD) camera (resolution: 648×494 pixels; cell pitch: 7.4 μm). The image intensity in the image plane is proportional to the square of the derivative of the phase.

The filter pattern on the SLM is controlled using a personal computer. A horizontal differentiation filter is used for measuring the phase derivative in the x-direction (Fig. 2(a)), while a vertical filter is used for measurements in the y-direction (Fig. 2(b)). The gray level (amplitude transmittance coefficient) of the SLM is proportional to the position in each direction.

The phase derivative is calculated using the following equations for the *x* and *y* directions.

$$x\text{-direction: } \frac{d\phi}{dx} = \frac{2\pi i_x}{\alpha_x A \lambda f} - \phi'_{x0} \quad (1).$$

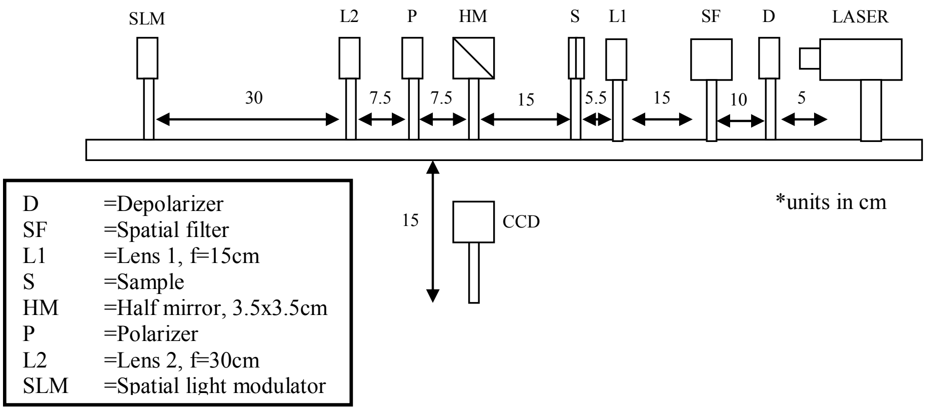
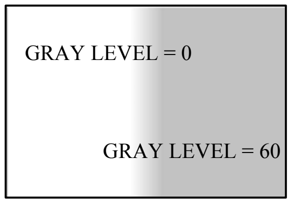
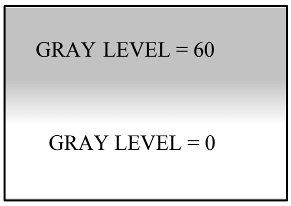


Fig. 1 Experimental setup of the optical phase measurement system using an SLM as a differentiation Fourier filter.



(a) *x*-direction



(b) *y*-direction

Fig. 2 Linear filter (SLM differentiation Fourier filter) for the (a) *x* direction and (b) *y* direction.

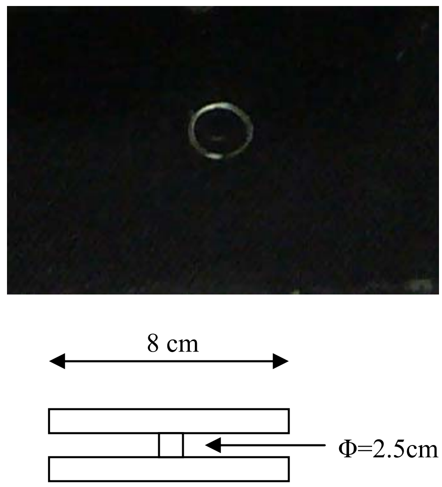


Fig. 3. Object used in the experiment.

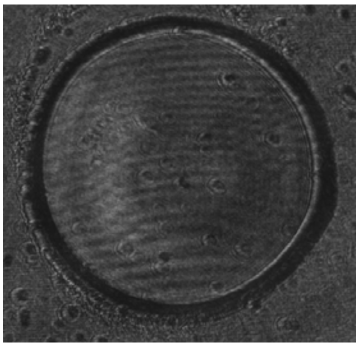
y-direction: $$\frac{d\phi}{dy} = \frac{2\pi i_y}{\alpha_y A\lambda f} - \phi'_{y0} \qquad (2)$$

where A is the amplitude of the laser beam incident on the object, λ is the wavelength of the beam, f is the focal length of the Fourier transform lens, and α_x and α_y are the proportional coefficients of the amplitude transmittance of the spatial differentiation filters for the x and y directions, respectively. ϕ'_{x0} and ϕ'_{y0} are the bias values of the phase derivative corresponding to the focal point of the plane beam on the filters. i_x and i_y are the amplitudes of the image on the image plane for the differentiation filters in the *x* and y directions, respectively.

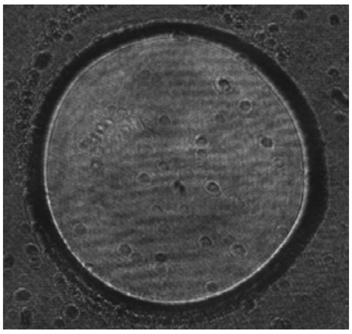
The wavefront profile was measured after the beam had passed thorough the phase object (sample S). The phase object used was a thin glass cylinder (diameter: 2.5 mm) wedged between two plates having refractive indices of 1.5. Figure 3 shows the object used in this experiment. Since lens L2 has a focal length of 30 cm and a diameter of 45 mm, the diameter of the Airy disc on the filter is approximately 10 μm.

3. RESULTS

Figures 4(a) and (b) show the images obtained with the *x*-direction and *y*-direction filters, respectively. The filter for differentiation in the *x*-direction has a width of 90 pixels (1.71 mm) and the center of the filter is 30 pixels (0.57 mm) from the edge of the filter pattern. The filter for differentiation in the *y*-direction has a width of 130 pixels (2.47 mm) and the center of the filter is 120 pixels (2.28 mm) from the edge. The gray level of the filters is 0-60 and 60 gradations. The measurement range of the phase derivative is calculated using

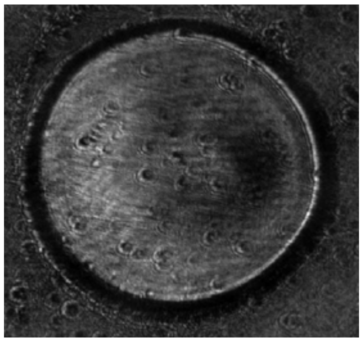


(a)

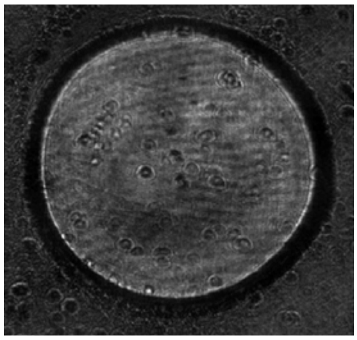


(b)

Fig. 4 Images obtained using the (a) *x*-direction filter (width 90 pixels) and (b) *y*-direction filter (width 130 pixels). Gray level: 0-60.



(a)



(b)

Fig. 5 Images obtained using the (a) *x*-direction filter (width 90 pixels) and (b) *y*-direction filter (width 130 pixels). Gray level: 0-130.

$$\Delta\gamma \approx \frac{2\pi}{\lambda f} W \ . \qquad (3)$$

Here, W is the filter width. The ranges of the phase derivative measurement are 60 rad/mm and 80 rad/mm in the x and y directions, respectively. These correspond to measurement ranges of the wavefront inclination of $\Delta\gamma \frac{\lambda}{2\pi}$ = 6 × 10^{-3} rad and 8 × 10^{-3} rad, respectively. The biases of the filter for the phase derivative are ϕ'_{x0} = 18.8 rad/mm (wavefront inclination of 1.9 × 10^{-3} rad) and ϕ'_{y0} = 75.4 rad/mm (wavefront inclination of 7.6 × 10^{-3} rad), respectively. The resolution of the phase derivative is calculated using

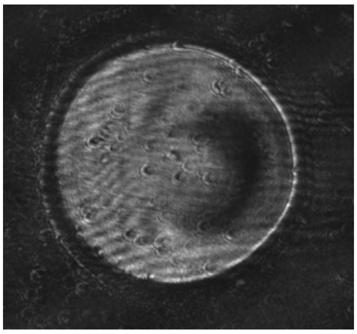
$$\frac{2\pi}{\lambda f} \times \frac{P_{pixel} N_{pixel}}{N_{GrayLevel}} \ . \qquad (4)$$

Here P_{pixel} is the pixel pitch (19 μm), N_{pixel} is the filter width in pixels, and $N_{GrayLevel}$ is the number of gray levels in the filters (60). The resolutions in *x*-direction and *y*-direction are calculated to be 1 rad/mm and 1.4 rad/mm, respectively.

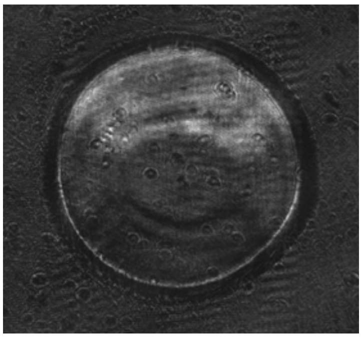
Figures 5(a) and (b) show the images obtained with the *x*-direction and *y*-direction filters with the gray levels of 0-130, respectively. The width and center of the filters are same with those of the filters used in the figure 4. The pictures in figure 5 show the same darkness direction with those in figure 4. However, they have higher contrast than pictures in figure 4.

Figures 6(a) and (b) show the images obtained with the *x*-direction and *y*-direction filters with the filter width of 0mm (knife-edge filter), respectively. The gray levels are 0 and 130. The pictures in figure 6 show the same darkness direction with those of figure 4 and 5. The contrast is very high. However, saturation of the intensity is observed.

The angles (phase derivative of the wavefront) are calculated using Eqs. 1 and 2 for each direction and image position in figure 4. Figure 7 shows the *x*-*y* plane view of the wavefront angles in the *x* and *y* directions. The wavefront is inclined at angles between –3.5 × 10^{-3} rad and 4 × 10^{-3} rad in the *x* direction and between –8 × 10^{-3} rad and 2 × 10^{-3} rad in the *y* direction. Since the refractive index of the object is 1.5 and the measurement is performed in air, the inclination

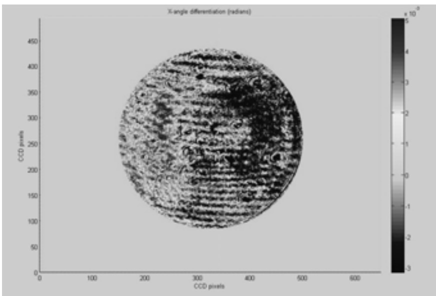


(a)

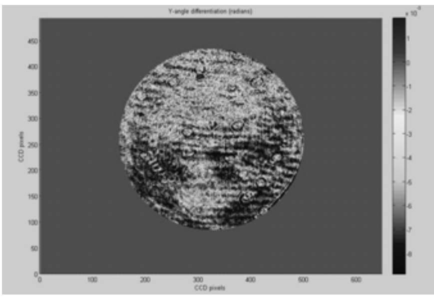


(b)

Fig. 6 Images obtained using the (a) *x*-direction filter and (b) *y*-direction filter. Gray level : 0-130. Filter width: 0 (knife-edge filter).



(a)



(b)

Fig. 7 *x*-*y* plane view of the wavefront angles in the (a) *x*-direction filter and (b) *y*-direction filter.

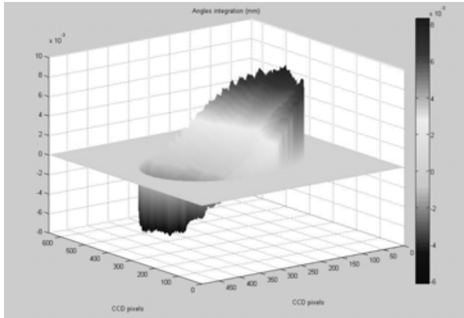


Fig. 8 Wavefront profile obtained by taking the line integral of the surface inclination.

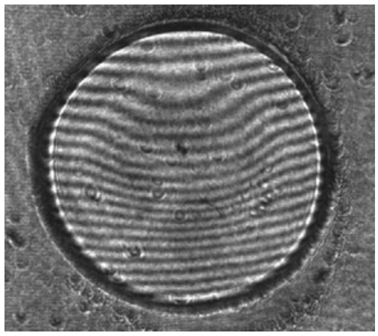


Fig. 9. Fringe pattern of the object observed using a point diffraction interferometer

of the surface of the object is between -7×10^{-3} rad and 8×10^{-3} rad in the *x* direction and between -16×10^{-3} rad and 4×10^{-3} rad in the *y* direction. The average inclination angles in the *x* and *y* directions are about 0.5×10^{-3} rad and -5×10^{-3} rad, respectively. This is equivalent to about a 5 μm change in thickness per 1 mm.

The wavefront profile is obtained by taking the line integral of the surface inclination. Figure 8 shows the profile. Since there is the phase is discontinuous at the edge of the sample, the constant of integration could not be determined. Therefore, only the difference in the optical path length between the center of the object and each point in the image was obtained. It is principally inclined in the y direction. The difference between the maximum and minimum phases in the y direction is 8 μm – (–5 μm) = 13 μm. Therefore, the inclination is 13 μm / 2.5 mm = 5.2 μm per 1 mm.

In order to verify these experimental results, the fringes in the object were observed using a point diffraction interferometer (Fig. 9). The product of the number of fringes and the wavelength of the laser should equal the difference between the maximum and minimum points in Fig. 8. The wavelength of the laser beam is 633 nm and the number of fringes is 20. Therefore, the difference is 12.7 μm and the surface angle is 5 μm/mm. These results are in good agreement with those obtained above.

4. CONCLUSIONS

In conclusion, a system was proposed for measuring the phase of an optical wavefront using an SLM differentiation filter. The pattern and orientation of the filter can be controlled by a computer. The phase derivative of a wavefront passing through a transparent phase object has been measured. The 3D shape of the object was calculated by integrating the phase derivative. The resolution of the phase derivative in the *x*-direction is 1 rad/mm with a measurement range of 60 rad/mm. The resolution of the phase derivative in *y*-direction is 1.4 rad/mm with a measurement range 80 rad/mm. The resolution of the phase derivative can be improved by employing a higher resolution (i.e., a smaller cell pitch, more gray levels) SLM and a longer focal length lens.

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