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DYNAMIC MEASUREMENT UNCERTAINTY OF HV VOLTAGE DIVIDERS

Jan Peter Hessling, Anders Mannikoff

SP Technical Research Institute of Sweden, Measurement Technology,
Box 857, SE-50115 Borås, Sweden
peter.hessling@sp.se, anders.mannikoff@sp.se

Abstract − Recently we proposed an approach based on digital filtering for evaluating dynamic non-stationary contributions to the measurement uncertainty. A dynamic simulator instead of a digital filter bank is here utilized for deriving the dynamic measurement uncertainty of mixed capacitive voltage dividers. These are used for lightning impulse measurements, during a calibration measurement of a standard lightning impulse.

Keywords: Dynamic, measurement uncertainty, voltage divider

1. INTRODUCTION

The demand for dynamic analysis of non-stationary measurements [1-5] is steadily increasing. The dynamic measurement uncertainty may be strongly time-dependent, just as the measured quantity itself. The systematic contributions to the uncertainty are preferably reduced by using dynamic correction/estimators. At present, this kind of dynamic analysis is not offered in any field as a general calibration service. In particular, this applies to electric measuring system components such as the widely used high voltage dividers [6]. A specific example of a measurement of general interest is the calibration test with a standard lightning impulse.

In this study we address contributions to the time-dependent dynamic measurement uncertainty of the voltage divider during a transient lightning impulse measurement, due to the uncertainty of the dynamic model (here assumed known) and measurement noise. The contribution to uncertainty due to the dynamic estimator is described by the model uncertainty, while the noise contributes also for static estimators. The 'cost' for using a dynamic rather than a static estimator is the propagated model uncertainty. If this cost is less than the associated reduction of systematic error, the combined measurement uncertainty is reduced, as desired.

Systematic errors will be excluded from the discussion. Consequently, the unavoidable loading of the probed circuit, the impedance mismatch, stray capacitances, etc. will not be modelled [6]. Accurate modelling of a short lightning impulse requires some parameters to be distributed. Nevertheless, a lumped model will be used as an approximation. These simplifications are made to stress the approach rather than focusing on details. It should be emphasized that there is no conceptual limitation how the presented model may be extended, as long as it is linear and time-invariant.

The approach to dynamic uncertainty evaluation is here made more accessible by using a standard dynamic simulator instead of the previously proposed [4] digital filter bank. This method only requires the widely taught, practiced and known concepts of transfer functions and measurement uncertainty as described in the GUM [7], and the use of dynamic software simulators.

2. DEFINITION OF PROBLEM

In electric measuring systems, a high voltage (HV) is often estimated by using low voltage (LV) equipment connected to a voltage divider with a large voltage reduction ratio (Fig. 1).

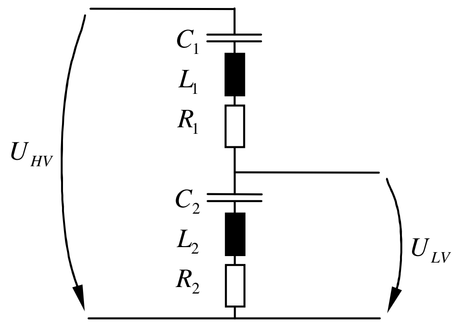

Fig. 1. Equivalent simplified circuit of a voltage divider.

The values of the components are calculated from the parameters listed in Table 1. The LC and RC time constants are given by $\sqrt{LC} = 1/2\pi f_C$ and $RC = 2\zeta\sqrt{LC}$.

Table 1. Parameters of voltage divider measurement system.

Parameter	Value	Description
K	$1/1000$	Nominal ratio of voltage division
f_C	$[2.3 \quad 0.8]$	Resonance freq. (MHz) HV, LV circuits
ζ	$[1.2 \quad 0.4]$	Relative damping HV, LV circuits
f_S	10	Sampling rate (MHz) of dyn. estimator
u_K	2	Relative unc. (%) of static gain K
u_{LV}	60	Signal to noise ratio (dB) LV
u_{P}^2	$10^{-4}\begin{pmatrix} 2^2 & 1^2 & 0 & 0 & 0 & 0 \\ 1^2 & 10^2 & 2^2 & 0 & 0 & 0 \\ 0 & 2^2 & 5^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1^2 & 0.5^2 & 0 \\ 0 & 0 & 0 & 0.5^2 & 8^2 & 1^2 \\ 0 & 0 & 0 & 0 & 1^2 & 6^2 \end{pmatrix}$	Covariance matrix

The measurand is the high voltage input U_{HV}, while the measured signal is the low voltage output U_{LV}. By the standard rule of voltage division, the transfer function is readily found,

$$\begin{aligned} H(s) &\equiv \frac{U_{LV}}{U_{HV}} = K \cdot H_{DYN}(s) \\ K &\equiv \frac{C_{HV}}{C_{LV}}, \quad H_{DYN}(s) \equiv \frac{1+(RC)_{LV}s+(LC)_{LV}s^2}{1+(RC)_{HV}s+(LC)H_{HV}s^2} \end{aligned} . \quad (1)$$

Usually, a *static* estimator $H_S^{-1} = K^{-1}$ is used. In a more general setting, a *dynamic* estimator $H_D^{-1}(s)$ should also be applied. It is derived from a prototype given by the inverse transfer function, $H_{DYN}^{-1}(s)$. For any physical system the application of this prototype is ill-conditioned due to the bandwidth limitation of the measurement system and results in a noise level of the estimated measurand without any definite bound. The prototype can also be unstable. In general it has to be regularized by means of filtering, stabilized as well as mapped to discrete time for sampled data [3]. Here however, the simple model does not have any bandwidth limitation as stray capacitances etc. have been excluded. The model (1) is therefore not strictly proper and only correct up to the bandwidth. As an unusual exception, the estimator will be chosen equal to its prototype $H_{DYN}^{-1}(s)$. To this estimator there should also be an associated measurement uncertainty.

3. MEASUREMENT UNCERTAINTY

Just as the estimator is split into a static and a dynamic part, so will also the contributions to measurement uncertainty be,

$$u_C^2 = u_G^2 + u_M^2 + u_N^2 . \quad (2)$$

Linear static (u_G), dynamic model (u_M), and measurement noise (u_N) contributions are here included. This study will address the practical evaluation of u_G^2, u_M^2 and u_N^2, but will exclude all systematic errors as well as non-linear static uncertainty contributions. The general form of these contributions was previously derived [4]:

$$\begin{aligned} u_G^2(t) &= c_K^2 u_K^2(t) = u_K^2(t) \\ u_M^2(t) &= \mathrm{Tr}\left(c_{\mathrm{P}}^2(t) \cdot u_{\mathrm{P}}^2\right) \\ u_N^2(t) &= c_N^2 u_{LV}^2(t), \quad c_N = \left\| h_D^{-1} \right\| \end{aligned} . \quad (3)$$

All uncertainties and co-variances will here be given relative to nominal values for the corresponding parameters. The static sensitivity c_K is then equal to unity. The adiabatic [4] measurement noise is $u_{LV}(t)$ with related constant sensitivity given by $\left\| h_D^{-1} \right\|$, the quadratic norm of the impulse response of the dynamic estimator. The relative covariance matrix of the dynamic model parameters $\mathrm{P} = \{R_1, L_1, C_1, R_2, L_2, C_2\}$ is denoted u_{P}^2 and is given in Table 1. The diagonal elements are thus the squared relative uncertainty for all parameters $\{R_1, L_1, C_1, R_2, L_2, C_2\}$. The associated sensitivities are generally strongly time-dependent ***signals*** $\xi(t)$. For every instant of time, $\xi(t)$ is a column vector which is expanded into an outer product matrix $c_{\mathrm{P}}^2(t) = \xi(t)\xi^T(t)$.

As in the GUM, the sensitivity signals will be found by a linearization of the model equation of the measurement. For a dynamic measurement, the model equation is a differential equation [1,4]. Applying the Laplace transform, an algebraic equation results which can be linearized,

$$\frac{\Delta H_D^{-1}}{H_D^{-1}} = \sum_{\mathrm{P}} \frac{\Delta \mathrm{P}}{\mathrm{P}} \frac{d \ln H_D^{-1}}{d \ln \mathrm{P}} \equiv \sum_{\mathrm{P}} \frac{\Delta \mathrm{P}}{\mathrm{P}} E_{\mathrm{P}}(s) \quad (4)$$

The sensitivity signals $\xi(t)$ which propagate the dynamic model uncertainties are found by convolving the corresponding impulse responses $e_{\mathrm{P}}(t)$ of the error systems $E_{\mathrm{P}}(s)$ with the estimated measurand $\hat{u}_{HV}(t)$,

$$\xi(t) = (e_{\mathrm{P}} * \hat{u}_{HV})(t), \quad e_{\mathrm{P}}(t) = [e_{R1}(t), e_{L1}(t), \ldots e_{C2}(t)]^T . \quad (5)$$

To evaluate these convolutions, digital filtering may be used as in [4]. Another alternative which will be pursued here is to use a dynamic simulator (section 3.1).

How the covariance matrix u_{P}^2 can be determined is not addressed here as it is assumed known. Within the framework of dynamic metrology [1] it is considered as a part of system identification [8,9] of a calibration measurement. Traditional methods of identification are preferably used for this task. Only the propagation of uncertainty from input to output variables specific to metrology is considered here.

3.1. Sensitivity to dynamic model uncertainty

The simulation of the sensitivity signals for the dynamic estimator as well as the estimator itself was made with the general dynamic simulator Simulink of Mathworks inc. [10].

The models are shown in Fig. 2. The sensitivity signals depend on the measured signal and are displayed in section 4.

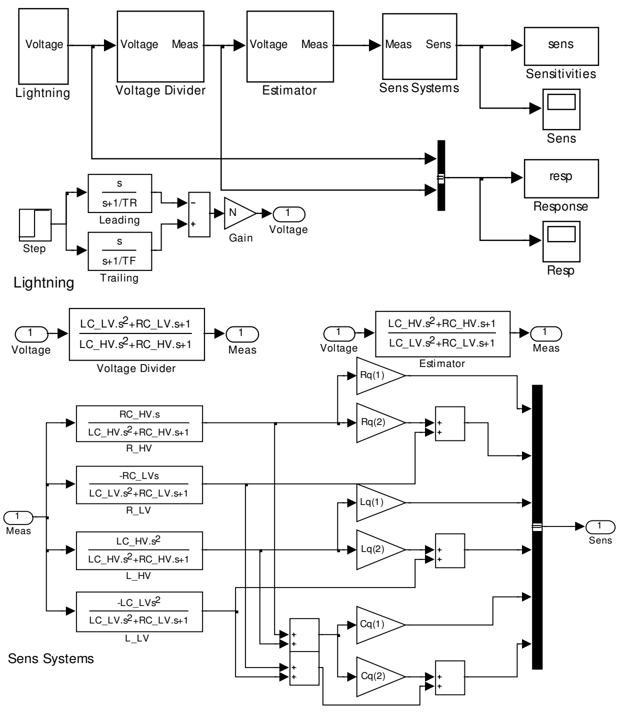

Fig. 2. Simulink model for estimation and generation of dynamic model sensitivity signals. The relative parameters are defined by $\mathrm{Xq} \equiv X_q / X_{HV}$, where $X = \{R, L, C\}$, $q = \{1,2\}$ and X_{HV} the total for the HV circuit. The RC and LC constants for the HV/LV circuits are labelled RC_HV/RC_LV and LC_HV/LC_LV, respectively.

3.2. Sensitivity to measurement noise

The measurement noise is here assumed to be un-correlated/white. As the estimator is not strictly proper, the bandwidth of estimation will be set by the Nyquist frequency rather than the conventional low-pass noise filter [3]. The sensitivity c_N will hence depend on the sampling rate and its calculation is also more complex than usual. First, extract the proper part H_P^{-1} of the dynamic estimator H_D^{-1},

$$H_P^{-1} = \frac{1+(RC)_{HV}\,s - [1+(RC)_{LV}\,s](LC)_{HV}/(LC)_{LV}}{1+(RC)_{LV}\,s + (LC)_{LV}\,s^2}. \qquad (6)$$

The impulse response (Fig. 3) is then robustly found by differentiating its simulated step response.

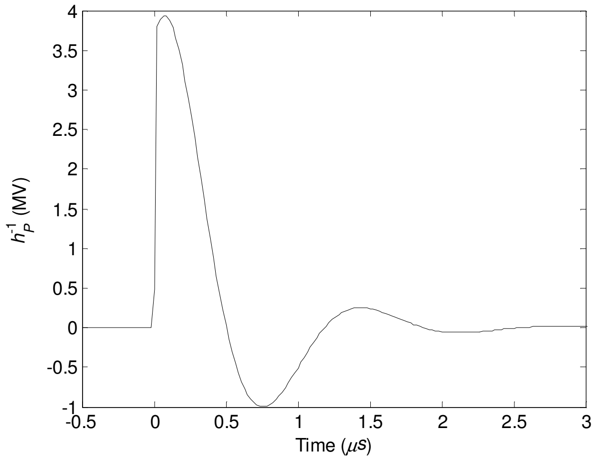

Fig. 3. Simulated impulse response of the estimator $h_P^{-1}(t)$.

Finally, the sensitivity to measurement noise is found for any sampling rate $f_S = T_S^{-1}$ by sampling, scaling and concatenating with the non-proper part h_C^{-1} of h_D^{-1} which results in a Dirac-delta impulse response,

$$c_N = \left\| h^{-1} \right\| = \left\| \left[h_C^{-1} \quad T_S h_P^{-1}(kT_S) \right] \right\|, \quad h_C^{-1} = \frac{(LC)_{HV}}{(LC)_{LV}} = K. \qquad (7)$$

For the chosen sampling rate $f_S = 10\,\mathrm{MHz}$, $c_N = 0.70$.

4. LIGHTNING IMPULSE MEASUREMENT

The general results for the voltage divider will here be applied to the measurement of a normalized standard $1.2\mu s/50\mu s$ lightning pulse [11]. It is here constructed from an ideal step by means of high pass filtering. The resulting dynamic model sensitivities are shown in Fig. 4, while the dynamic measurement uncertainty components and the total are displayed in Fig. 5.

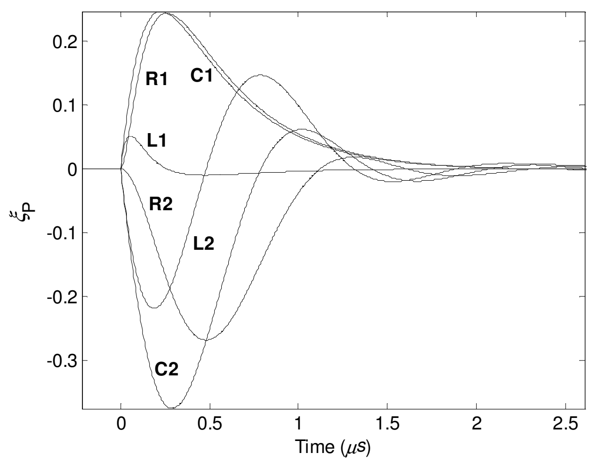

Fig. 4. Time-dependent sensitivities to dynamic model parameters.

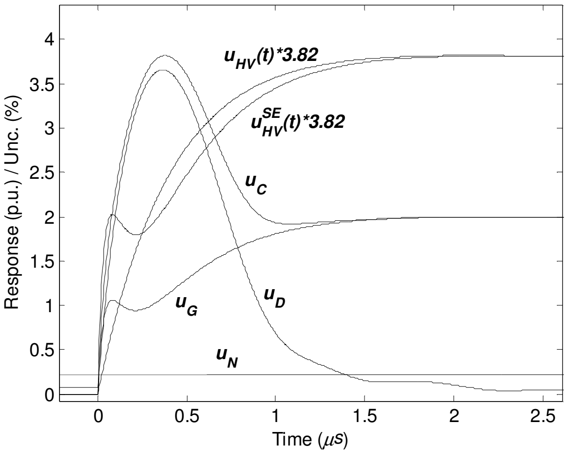

Fig. 5. The leading part of normalized lightning u_{HV} and related static estimate u_{HV}^{SE} (both rescaled), and the combined measurement uncertainty u_C built of u_G, u_M, u_N .

Additional steps contained in dynamic metrology [1] are required to verify the measurement uncertainty. The voltage divider must first be completely characterized in a calibration measurement from which its model (Eq. 1) and covariance matrix u_P^2 is identified, or determined by other means. The lightning impulse then needs to be realized and measured by the voltage divider. From several such measurements the standard uncertainty may be estimated and compared to Fig. 5. One alternative is to use the same lightning impulse set up but different repeated measurements, for characterization and verification. Finally, note that the simulation of uncertainty presented here can be repeated for *any* signal and not only the lightning impulse.

5. CONCLUSIONS

A modified version of a recently proposed method for propagation of the time-dependent measurement uncertainty has been proposed. Based on the capability of common dynamic software simulators, it effectively avoids all complications of digital filter synthesis used in the original method. The procedure was applied to a simple generic model of voltage dividers used in electric measuring systems for the standard calibration measurement of lightning impulse. The simplicity and transparency of the method supports a general acceptance of the approach to the evaluation of dynamic measurement uncertainty.

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