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ELEMENTS OF STATISTICAL DECISION MAKING

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Abstract – Measurements are the key to rational decision making. Measurement information generates value, when it is applied in the decision making. Normative decision making considers all decision tasks as optimization problems, typically with multiple objectives and uncertainties. Therefore in normative decision making the decision task must first be formulated mathematically and then the resulting optimization problem is solved. This paper considers the elements of decision making under uncertainty based on statistical decision theory.

Keywords decision making, statistical decision theory, uncertainty

1. INTRODUCTION

We all make hundreds of decisions in every day, some are more important than others but the structure is always similar. The daily decision making about process and product quality by operators and engineers should be supported with information systems so that the best practice of operation can be achieved continuously. Measurements, soft sensors and process simulators form the basis for such decision support by reducing the uncertainty about the present state of the process and about its future evolution.

This paper introduces a concept for structuring the decision making task. We are presenting different elements of decision making based on the statistical decision theory (SDT). When making decisions or when combining information from various sources, the uncertainty of information is decisive and must be known. This is interesting thing in the decision making – what is attitude towards risk and how the decision maker handles the uncertainty. In this paper we assume that decision making is a single-objective problem, although in practice operational decision tasks may often be of multi-objective nature.

As SDT itself structures decision making task it is logical to structure decision same systematic way so them can be analysed also a more philosophical way and collect different decision making situations to database. Advantage of such kind of knowledge database is the collection of silent information and process intelligence.

This paper is divided as follows. Chapter 2 describes a link between decision making and measurement information. Chapter 3 discusses statistical decision theory and decision making. Chapter 4 then elaborates the structure of general decision making task. Chapter 5 presents an example about broke management and control and finally chapter 6 presents conclusions.

2. MEASUREMENT INFORMATION AND DECISION MAKING WITH UNCERTAINTY

This chapter emphasizes a link between measurement information and decision making with uncertainty. Information from measurements, soft sensors and simulators generates value through improved decisions [13], because the uncertainty about the state of the process has been reduced. The amount of value generated depends on the goal set by the decision maker, including the decision maker's attitude towards risk.

The optimal measurement system is such that it maximizes the value of information generated, under a given set of scenarios on external effects to the process. A feedback control is a measurement - decision system, but usually when decision gets more expensive there is human involved in the decision making.

Commonly measurements are used to detect the need for decision making and earlier studies show statistical decision theory in optimization of measurement strategy [9] and choosing and optimization of measurements (policy) [10, 11]. Other applications could be evaluating the limits for measurement uncertainty based on information needed in the decision making.

Fig. 1 shows the chain from the process via data and information to decision and vice versa. This other direction is often overlooked and forgotten but it should be taken into consideration. Use of information, control and other decisions, defines processing of data and measurements.

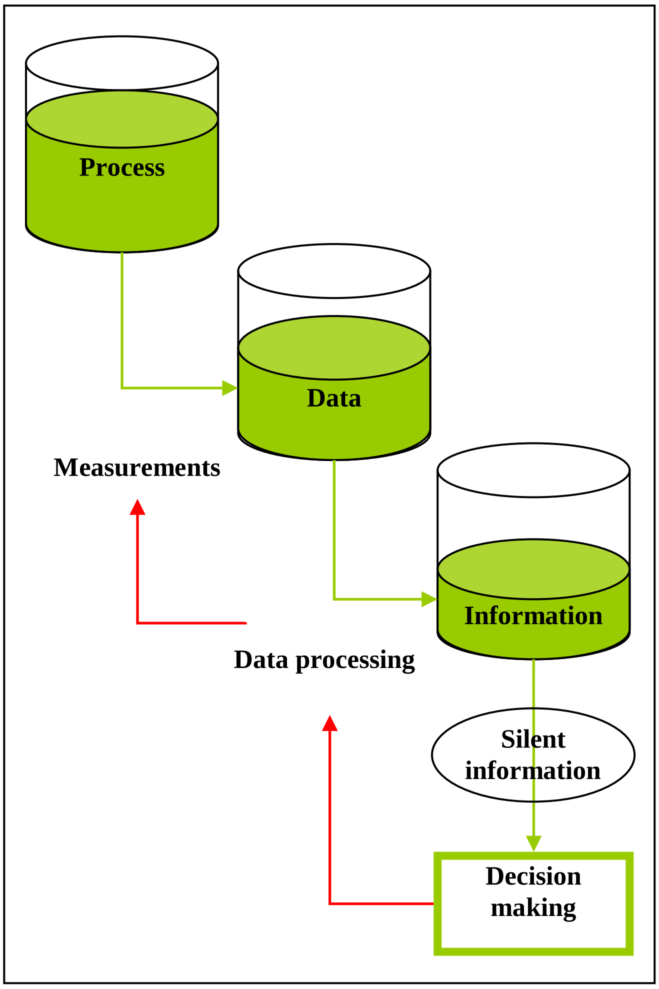

Fig. 1. From process to decisions and back.

Measuring the silent information, that is, in broad sense process knowledge of the operator, is important application of this statistical decision making tool. With this kind library of silent information it is possible to formulate decision support system for operators in wide area of industry.

3. STATISTICAL DECISION THEORY

Statistical Decision Theory (SDT) is a mathematical theory on how to make rational decisions when there is uncertainty in consequences of potential actions and such uncertainties may vary greatly from action to action. Fully structured SDT is a deterministic optimization problem with the objective, constraints, and system and observation models. Decisions are based on available information about the target system – current measurements, a priori information in form of models and tacit knowledge.

The formal statistical decision making problem consists of the following elements: a priori information about the state of the system, models of measurements, model for predicting the consequences of decision alternatives, and the expectation value of utility of the consequences.

To define these elements, the system state (x), the set of consequences (c) and the set of allowable decisions (actions, a) must be described. Note that x, c and a, are multidimensional and that they may be past time series (x) or future time series (c,a). Fig. 2 presents the decision making task: given the measurement value $x^{(obs)}$, and the probabilistic models what is the action that yields maximal expected utility for decision maker (DM) [1,2].

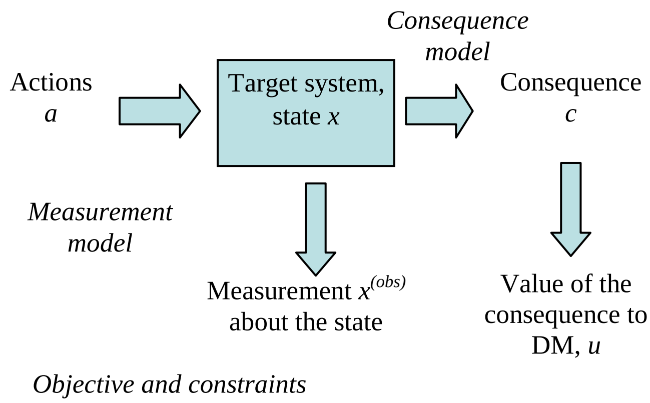

Fig. 2. The key descriptors of the decision making problem and their relationship.

3.1. Making the decision

Decision maker knows the state of the system, x, only probabilistically through uncertain measurements and possibly through a priori information. The consequence c of the action a, given that system state is x, is known probabilistically as a priori information. DM evaluates the system performance in terms of consequences. The utilities of consequences c, if the consequence were certain, are given as $u(c)$ [3]. Then the best action a^* is the one with highest expected utility. The utility is a description of both DM's preference order and attitude towards risk. If utility exists, DM is guaranteed rational in the sense that he does not have circular preferences in pair wise comparisons of decision alternatives.

Formally, the elements of a priori information, measurement models and prediction models are then, respectively, the probability density functions:

$$f_X^{(ap)}(x) \qquad (1a)$$

$$f_{X^{(obs)}|X}^{(meas)}(x^{(obs)} \mid x) \qquad (1b)$$

$$f_{C|a,x}^{(pred)}(c \mid a, x) \qquad (1c).$$

Here $x^{(obs)}$ refers to the measured value of x. The probability density function of consequence c, given that $x^{(obs)}$ has been measured and DM would decide a is then according to Bayes formula [4,5]

$$f^{(pred)}_{C|a,x^{(obs)}}(c\,|\,a,x^{(obs)}) = N * \int_{domain(X)} f^{(pred)}_{C|a,x}(c\,|\,a,x)\, f^{(meas)}_{X^{(obs)}|x}(x^{(obs)}\,|\,x)\, f^{(ap)}_{x}(x)\, d^n x \qquad (2)$$

where N is a normalization factor and n is the dimensionality of system state space description.

Defining the objective of decision making, and in particular the attitude towards risk, is quite often the main challenge when applying the formal decision theory to operational decision making about production, for example in papermaking and in other industrial processes. Although the utility function exists for a rational decision maker, its most general identification method through finding certainty equivalents of “gambling cases” [3] is tedious and often not intuitive for the decision maker. We shall employ utility function as a normative decision model and assume that DM is able to express it in spite of it has been criticized for not corresponding to human decision making in all respects [2, 6].

The optimal decision is then the one that maximizes the expected utility, and the corresponding expected utility is the measure of performance [1-2, 7-8]:

$$\begin{aligned} a^*(x^{(obs)}) &= \arg\max_{a\in A} \int_{domain(C)} u(c)\, f^{(pred)}_{C|a,x^{(obs)}}(c\,|\,a,x^{(obs)})\, d^m c \\ U^*(x^{(obs)}) &= \int_{domain(C)} u(c)\, f^{(pred)}_{C|a^*(x^{(obs)}),x^{(obs)}}(c\,|\,a^*(x^{(obs)}),x^{(obs)})\, d^m c \end{aligned} \qquad (3).$$

4. ELEMENTS OF A DECISION MAKING TASK

This chapter discusses more about the structure and the elements of decision making task.

The key descriptors of SDT are state, measurement and consequence, see fig. 3. State is a unique description of the current status of the target system, as it is not directly observable but we obtain information about the state through measurement data and earlier experience. Measurement is a means of providing measurement data that is informative about the state. The available information assigns probability densities to state values according to how likely the state is to be at that value. If Gaussian distribution is used (often practical) then the mean value is referred as the state estimate and the variance as the uncertainty of the estimate. Consequence is a collection of attributes of how we judge the success of action made. The set of potential actions is the descriptor setting the degrees of freedom for the decision maker.

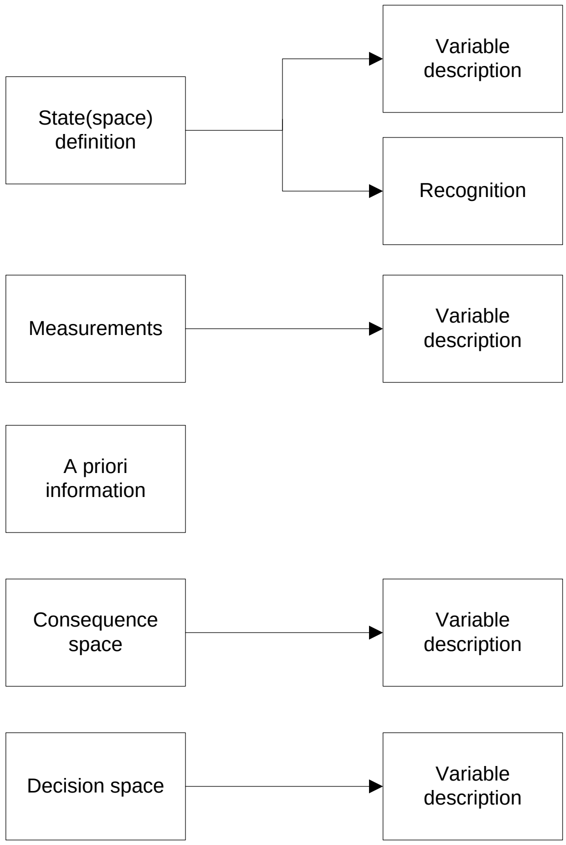

Fig. 3. Elements of decision making.

4.1. Model, objectives and constraints in decision making

SDT requires two models: a measurement model and a consequence model, see fig. 4. The measurement model assigns at each possible state of the system the probability to obtaining a given measurement result. If the model is a Gaussian distribution and the measurement is about a state component, the mean value of unbiased measurement is the actual state value and the variance is measurement uncertainty. Consequence model assigns a probability to ending up at a consequence when a given action is made at a given state. It is self-evident that the consequences of actions cannot be predetermined hence consequence models are inherently probabilistic.

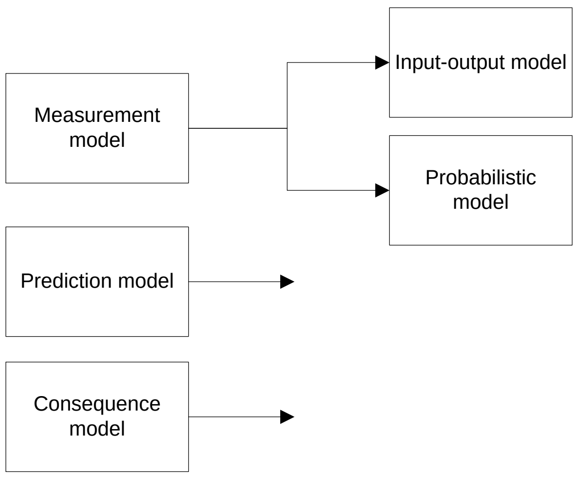

Fig. 4. Models needed at the decision making.

Assume that the decision maker seek to maximize one of the consequences (primary objective). Using measurement and consequence models and pre-existing information it is possible to calculate the probability for each value of the primary objective for any given action. In general, different actions may lead vastly differing consequence probability distributions, hence the decision maker is choosing between probability distributions, a task that has proven to be difficult in all areas of decision making.

Decision making is risk-neutral if the action with maximal expected value of primary objective is chosen and uncertainties is neglected, opportunistic if the decision making favours large uncertainties and risk-averse if it favours alternatives with small uncertainties. As the consequences of actions are known only probabilistically and the decision maker is choosing between distributions, the attitude towards uncertainty is an element in operational decision making. At present the attitude is more or less intuitive, which often leads to that two decision makers with exactly the same facts end up at different decisions. Obviously this leads to confusion within the organization.

There are several ways to derive the SDT objective given the primary objective and attitude towards uncertainty. Theoretically the soundest approach is the utility function that exists for rational and consistent decision maker. Then the optimal decision is the one maximizing the expectation value of utility function. Attitude towards risk can also be expressed as maximizing the expected primary objective under additional constraints.

One example of the heuristic ways of describing attitude towards risk is the risk premium which states that the objective is to maximize the expected value of primary objective plus/minus a term proportional to the standard deviation of primary objective. See fig. 5.

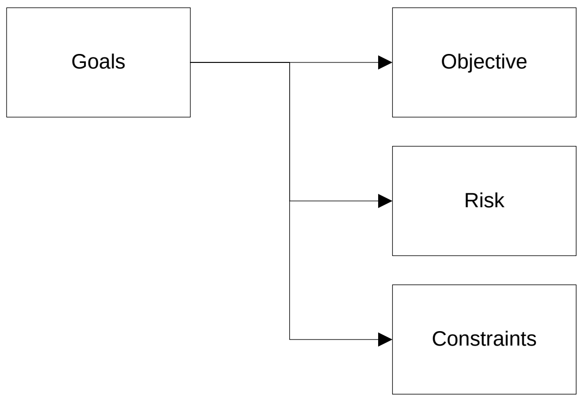

Fig. 5. Definition of goals at decision making

4.2. Need for making the decision

In operations, not making a decision is a decision itself. However, most of the decision tasks are triggered by events such as new measurement data becoming available, foreseeable internal/external events and unforeseeable internal/external events, see fig. 6.

When new measurement data becomes available, information about the system state is updated. With the new updated information the predictions about the consequences of potential actions are with less uncertainty, therefore decisions should be revised when new information becomes available.

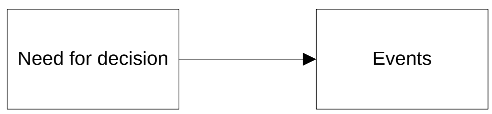

Fig. 6. Triggering the need for decision.

4.3. Decision about more information before deciding

There is a huge amount of data (and information) about the system state, and the new information is fed constantly. However, there are also information sources beyond the regular data generation and collection that may be invoked on the need basis. Hence within a decision making process about the actions on the operations, there may arise decision subtask about whether to acquire further information before making the actual decision about the action. Furthermore, there may be several alternative information sources to choose between. Prewarnings about unforeseeable internal events typically have such decision subtasks about acquiring further information.

5. BROKE MANAGEMENT AND CONTROL

The broke system is a complex part of the paper production system. The function of the broke system is to handle broke from different parts of a paper machine at different moisture ratios so that broke can be reused as raw material. One of the main objectives with broke management is to prevent any broke-related runnability problems at the paper machine. Reusing broke should be as efficient as possible to minimize the costs that come from broke handling. [12]

Broke management is one of the key issues affecting quality in paper production. The reason for this is that the material components in broke differ from the corresponding fresh dry material components. Furthermore, the broke contains filler and thus high and fast variation in broke dosage leads to filler content variation in the paper produced. The filler content control at paper machine is rather slow and thus only partly compensates for the disturbance caused by the varying amount of broke.

In broke management the most demanding task occurs during a web break, when all the material is fed in to the broke tank. Even with this sudden, large and unpredictable flow the broke tank volume must be kept within its physical limit in order to avoid overflow. Thus a web break often necessitates a higher broke flow rate to the blend chest because of the volume management. Higher broke dosage increases the possibility that the web break will continue longer because of the quality disturbances caused by the broke.

In the volume management broke tank the objective is to keep the tank as empty as possible which leads to high broke dosage. The objectives in broke management are contradictory since the volume management requires occasional high broke dosage and the paper quality management requires keeping the broke dosage as constant as possible.

Let us now take an example about this volume management of broke tank. The goal is to avoid overflow and keep the tank as empty as possible. Each time step we choose new dosage for broke. We have some information and knowledge about the system and the process. But we have no explicit break probability information although we have some kind of *a priori* information about it. As in real paper machine we have volume and break on/off info all the time. Then we have information about filler when break is off and information about the filler gain with uncertainty.

Need for decision making is at every time step, objective, constraints and decision space are simple – attitude towards risk is depends on the decision maker. *A priori* information with measurements provides some understanding about the state of the process. Results are shown at the Fig. 7. At the top is volume of the broke tank with maximum of 200. Second subplot shows the web break info (1 if break is on). Third subplot shows our broke dosage and fourth subplot shows information about the filler when there is no web break and fifth subplot is the filler gain.

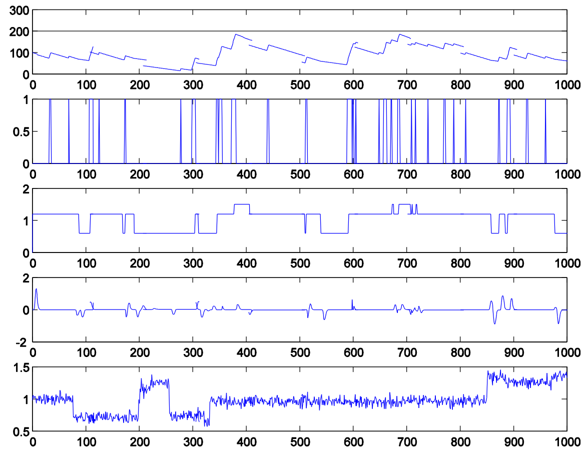

Fig. 7. Broke management and control. Measurements - at the top is volume of the broke tank with maximum of 200. Second subplot shows the web break info (1 if break is on). Third subplot shows our broke dosage and fourth subplot shows information about the filler when there is no web break and fifth subplot is the filler gain.

6. CONCLUSIONS

This paper presents the idea of structuring a decision making task, finding different elements and models which are present at every decision. Also making the decision is discussed with an example about the broke management and control at the paper mill.

We claim the strength of this approach to be - the SDT structure provides a rational and transparent basis for decision making separating measurement and response models, facts, from objectives and constraints, preferences. When the SDT structure is incomplete, the structure clarifies missing elements and guides the user to seek information form improving decision making. And because of the portfolio approach, including new decision tasks is straight forward and redistributing decision tasks during organizational changes requires only small changes within the system.

Some critical points include, such as, the SDT structure does not safeguard against thought errors, such as false measurement or response models, or incorrect interpretation of strategic objectives to optimization objectives and constraints. The SDT structure and explicit uncertainties in particular are rather abstract. Uncertainty as a piece of metadata is structurally complex and has not been made explicit in any decision support system yet.

Explicating the uncertainty in operational decision making can be considered one of the key tasks of a normative decision support system. With explicit analysis of uncertainties, a joint attitude towards uncertainty may be derived. We claim that attitude towards uncertainty in decision tasks is a strategic decision itself and should not be left for individual decision makers, if consistent and rational operation is to be achieved.

Future plans include an industrial application where a XML application is introduced for collecting (measuring) silent information about different decision making situations.

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