XIX IMEKO World Congress
Fundamental and Applied Metrology
September 6–11, 2009, Lisbon, Portugal

ADVANCED CALIBRATION METHOD FOR PITCH ARTIFACT

Yohan Kondo [1], Kazuyuki Sasajima [1], Sonko Osawa [2], Osamu Sato [2] and Tsukasa Watanabe [2]

[1] Department of Mechanical and Environmental Informatics, Tokyo Institute of Technology, Ookayama 2-12-1, Meguro-ku, Tokyo, Japan, kondo.y.ae@m.titech.ac.jp
[2] Dimensional Standards Section, National Metrology Institute of Japan, AIST, Umezono 1-1-1, Tsukuba, Ibaraki, Japan, sonko.osawa@aist.go.jp

Abstract – The pitch measuring accuracy for gear measuring instruments (GMIs) is evaluated by measuring a calibrated artifact. We proposed a novel artifact which called multiball artifact (MBA). The MBA is composed of the pitch balls assumed to gear teeth and the reference axis assumed to a center axis of gear. The MBA can be calibrated more accurate than a conventional pitch artifact. When the MBA is calibrated, it is important to eliminate an error of measuring instrument; therefore we calibrate the MBA adapting the multiple-orientation technique. In the multiple-orientation technique, the MBA is set up in difference orientations around the reference axis and the measurement error is eliminates by averaging the measurement value for all orientations. There are, however, Fourier components of the measurement error that can not be eliminated depending on the total number of orientations. In this paper, we propose the advanced calibration method; the error separation method for the multiple-orientation technique is improved and the total number of orientations can be reduced. The superiority of the proposed method is clarified from the calibration results using the MBA.

Keywords: pitch, multiple-orientation technique, error separation technique

1. INTRODUCTION

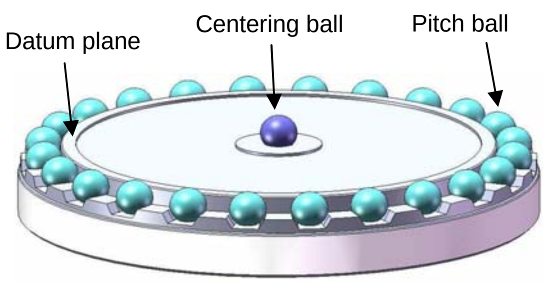
A pitch of gear teeth is measured using various instruments. The pitch-measuring accuracy of gear measuring instruments (GMIs) is evaluated by measuring a calibrated gear artifact or a gear like artifact [1-3]. Gears, however, have a form error and surface roughness [4, 5] and it is difficult to obtain a stable measurement result when the measurement position on the gear face slightly differs. In view of this situation, we proposed a novel artifact composed of equally spaced balls called a multiball artifact (MBA) as a pitch reference artifact for GMIs [6-8]. The balls can be manufactured with an accuracy of several tens of nanometer; therefore the measurement with extremely small uncertainty can be expected.

When the MBA is calibrated, it is important to eliminate an error of measuring instrument. For the elimination method, it is proposed the multiple-orientation technique [9]. These methods eliminate the systematic error while the MBA is set up in different orientations. There are, however, Fourier components of the systematic error that can not be eliminated depending on the total number of orientations. In this paper, we propose an advanced calibration method for the multiple-orientation technique. The superiority of the proposed analysis method is clarified from the calibration results using the MBA.

2. MULTIBALL ARTIFACT

Figure 1 shows a photograph and a schematic view of the manufactured MBA. The balls on the outer circumference (which are named pitch balls) are assumed to act as gear teeth. The ball at the center (which is named the centering ball) is used to set a reference axis and the reference axis is assumed to a center axis of gear. The refer-

(a) Photograph



(b) Schematic view

Fig. 1. MBA proposed as a new reference for GMIs.

-ence axis is perpendicular to the datum plane and passes through the center of the centering ball. The size of the pitch balls and the centering ball are 12.7 mm in diameter and the accuracy is grade 3 as standardized by ISO 3290. The number of pitch balls is 24. Pitch balls are arranged on a curvic coupling (type: 24180-120V, manufactured by Okubo Gear Co., Ltd.) in contacting with both tooth flanks and the pitch balls are also in contact with a cylinder manufactured to be concentric with the centering ball.

For the centers of each pitch ball, the difference between the theoretical angular position and the actual angular position around the reference axis is named the angular deviation, where, the theoretical angular pitch position is the angular position for the ideal pitch ball arranged at a complete equal interval and the actual angular position is the angular position for the actual pitch ball of the manufactured MBA. GMIs can be evaluated by measuring the calibrated angular pitch deviation.

3. MULTIPLE-ORIENTATION TECHNIQUE

We calibrated the angular deviation of the MBA using a coordinate-measuring machine (CMM) to measure each pitch ball center and by adapting the multiple-orientation technique. Figure 2 shows the measurement setup. The rotary index table was placed on the CMM table and the MBA was clamped onto the rotary index table. For the multiple-orientation technique, the MBA was set up in different orientations using the rotary index table. The angular pitch deviation of the MBA was measured at each orientation. Figure 3 shows a relationship of the MBA positions at each orientation. The multiple-orientation technique eliminates the systematic error by averaging the measurement value for all orientations. The detail of the multiple-orientation technique is described at the following.

If the pitch balls are arranged at a complete equal interval, the nominal angular position of ith pitch ball is defined by

$$\theta_i = \frac{2\pi}{N}(i-1) \quad (i = 1,2,\cdots,N), \tag{1}$$

where N is the total number of pitch balls and the pitch ball number is assigned in clockwise direction. We denote the true angular position of ith pitch ball by $T(\theta_i)$. Additionally, we define the angular pitch deviation by

$$P(\theta_i) = T(\theta_i) - \theta_i , \tag{2}$$

We denote the measurement value of $P(\theta_i)$ by $M(\theta_i, \varphi_j)$, which is sum of $P(\theta_i)$, the systematic error $E(\theta_i)$ and the nonsystematic error E_{rand}, where E_{rand} is the component of the random errors of the CMM:

$$M(\theta_i, \varphi_j) = P(\theta_i) + E(\theta_i + \varphi_j) + E_{rand} , \tag{3}$$

where φ_j is the rotation angle of rotary index table at jth orientation as shown in Fig. 3. The rotary index table rotates at equal intervals as follows:

$$\varphi_j = -\frac{2\pi}{m}(j-1) \quad (j = 1,2,\cdots,m), \tag{4}$$

where m is the total number of orientations. Here, we denote the mean value of $M(\theta_i, \varphi_j)$ for all orientations by

$$\begin{aligned}\mu(\theta_i) &= \frac{1}{m}\sum_{j=1}^{m} M(\theta_i, \varphi_j) \\ &= \frac{1}{m}\sum_{j=1}^{m}\left\{P(\theta_i) + E(\theta_i + \varphi_j) + E_{rand}\right\} \\ &= P(\theta_i) + E^{(m)}(\theta_i)\end{aligned} \tag{5}$$

where the expected value of E_{rand} is zero and $E^{(m)}(\theta_i)$ is a curve composed by the sum of the multiple of mth-order Fourier component of $E(\theta_i)$. It is explained by the following law of the Fourier series:

"An arbitrary periodic curve of 2π can be expressed by the Fourier series, and when n-number of curve with a phase shift of $2\pi/n$ at a time are averaged, the averaged curve shows the sum of an integral multiple of nth-order Fourier components of the original curve."

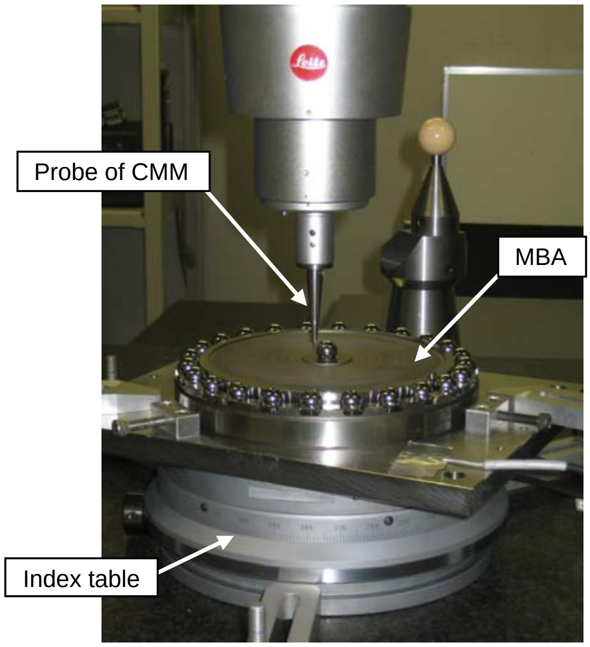


Fig. 2. Overview of MBA measurement on CMM.

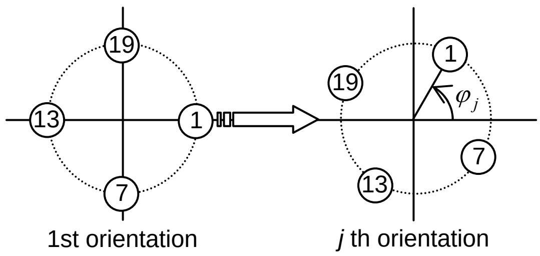


Fig. 3. Relationship of MBA positions at each orientation.

Figure 4 shows the measurement result of m = 3. The dots are the measurement value $M(\theta_i, \varphi_j)$ for each orientation and the line is the mean value $\mu(\theta_i)$. In the exiting multiple-orientation technique, the calibration value of $P(\theta_i)$ is estimated by $\mu(\theta_i)$; however, $\mu(\theta_i)$ has $E^{(m)}(\theta_i)$. We propose the advanced analysis method at the following section.

4. ADVANCED ANALYSIS METHOD

4.1. Improvement of error separation method

The origin of $E(\theta_i)$ contained in $M(\theta_i, \varphi_j)$ is different at each orientation. We shift $M(\theta_i, \varphi_j)$ so that the origin of $E(\theta_i)$ at each orientation becomes uniform as follows:

$$M(\theta_i - \varphi_j, \varphi_j) = P(\theta_i - \varphi_j) + E(\theta_i) + E_{rand} . \qquad (6)$$

We denote the mean value of $M(\theta_i - \varphi_j, \varphi_j)$ by $\mu'(\theta_i)$ as follows:

$$\begin{aligned} \mu'(\theta_i) &= \frac{1}{m}\sum_{j=1}^{m} M(\theta_i - \varphi_j, \varphi_j) \\ &= \frac{1}{m}\sum_{j=1}^{m} \{P(\theta_i - \varphi_j) + E(\theta_i) + E_{rand}\} \qquad (7) \\ &= P^{(m)}(\theta_i) + E(\theta_i), \end{aligned}$$

where $P^{(m)}(\theta_i)$ is a curve composed by the sum of the multiple of mth-order Fourier components of $P(\theta_i)$. Here, let subtract $\mu'(\theta_i)$ from $M(\theta_i - \varphi_j, \varphi_j)$ and we obtain the following equation.

$$M(\theta_i - \varphi_j, \varphi_j) - \mu'(\theta_i) = P(\theta_i - \varphi_j) + E_{rand} - P^{(m)}(\theta_i) . \ (8)$$

Figure 5 shows the calculated result of Eq. (8) for the measurement value. We can obtain $P(\theta_i)$ using the data for j = 1 ($\varphi_1 = 0$). The statistics accuracy is not enough because it is handling only the data for j = 1 despite the measurement of m-orientations. And so, we analyze to obtain $P(\theta_i)$ using the measurement value for all orientations. We make the m-numbers of $\mu'(\theta_i+\varphi_j)$ which is $\mu'(\theta_i)$ with the phase shift of φ_j. We subtract $\mu'(\theta_i+\varphi_j)$ from $M(\theta_i, \varphi_j)$ to obtain

$$\begin{aligned} & M(\theta_i, \varphi_j) - \mu'(\theta_i + \varphi_j) \\ &= \{P(\theta_i) + E(\theta_i + \varphi_j) + E_{rand}\} \\ &\quad - \{P^{(m)}(\theta_i + \varphi_j) + E(\theta_i + \varphi_j)\} \qquad (9) \\ &= P(\theta_i) + E_{rand} - P^{(m)}(\theta_i + \varphi_j). \end{aligned}$$

Figure 6 shows the calculated result of Eq. (9) for the measurement value. We denote the mean value of Eq. (9) for all orientations by

$$\begin{aligned} p_m(\theta_i) &= \frac{1}{m}\sum_{j=1}^{m} \{M(\theta_i, \varphi_j) - \mu'(\theta_i + \varphi_j)\} \\ &= \frac{1}{m}\sum_{j=1}^{m} \{P(\theta_i) + E_{rand} - P^{(m)}(\theta_i + \varphi_j)\} \qquad (10) \\ &= P(\theta_i) - P^{(m)}(\theta_i). \end{aligned}$$

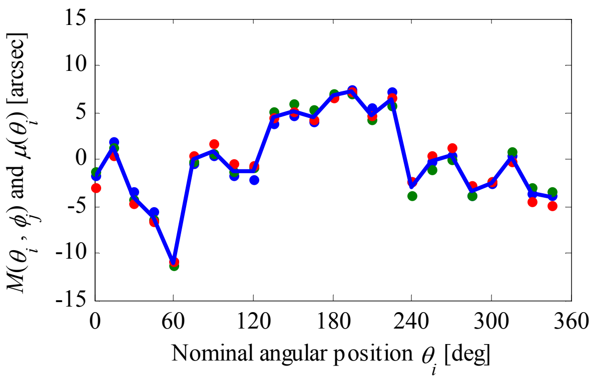


Fig. 4. Measurement results of $M(\theta_i, \varphi_j)$ and $\mu(\theta_i)$.

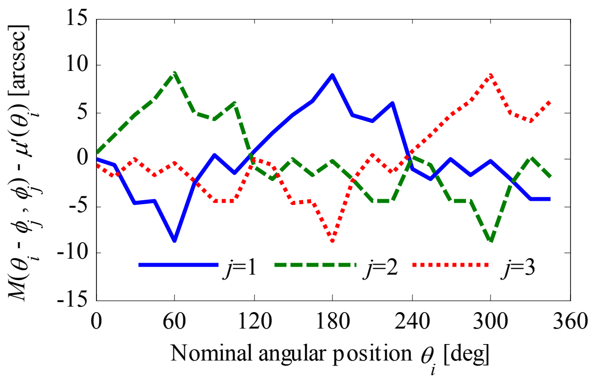


Fig. 5. Calculated result of $M(\theta_i - \varphi_j, \varphi_j) - \mu'(\theta_i)$.

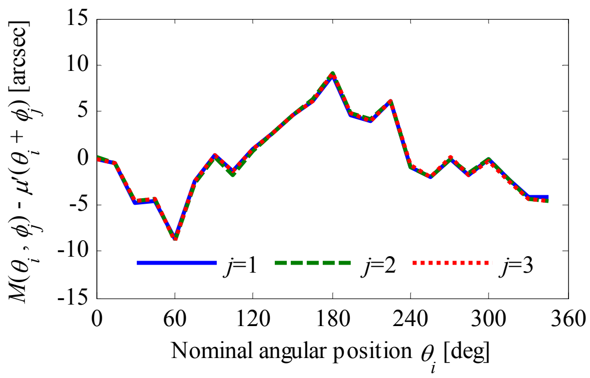


Fig. 6. Calculated result of $M(\theta_i, \varphi_j) - \mu'(\theta_i + \varphi_j)$.

$p_m(\theta_i)$ is expressed the form without $E(\theta_i)$; however, it does not contain $P^{(m)}(\theta_i)$.

4.2. Improvement of orientation number

GMIs are not evaluated about Fourier components larger than the total number of pitch balls according to the sampling theorem. When we perform the calibration at the total number of orientations as the number of pitch balls, $p_m(\theta_i)$ contain the required Fourier components for the evaluation of GMIs. It, however, takes very long time and it is difficult to keep a stable environment during the measurement. In addition to the *section 4.1*, we propose the improved method so that the total number of orientations can be reduced.

$p_m(\theta_i)$ can not be obtained when the total number of orientations is m. We compensate the deficient Fourier component from the measurement of another total number of orientations. When the total number of orientations is n ($\neq m$), we denote the calculated result of Eq. (10) by $p_n(\theta_i)$.

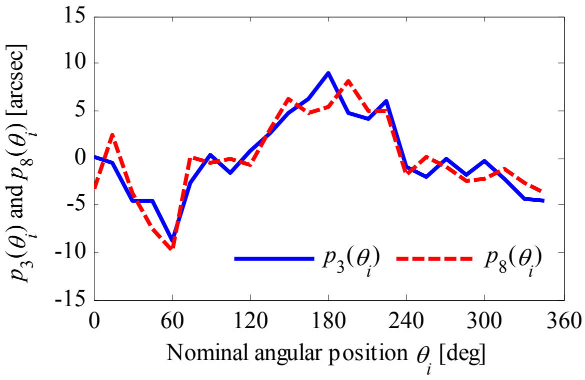


Fig. 7. Analyzed results of $p_3(\theta_i)$ and $p_8(\theta_i)$.

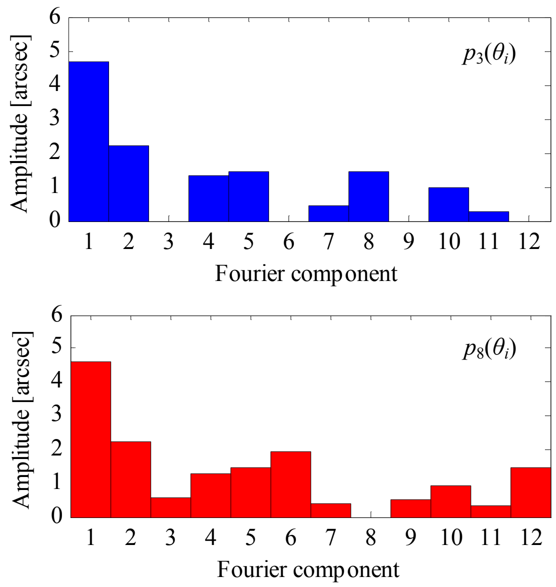


Fig. 8. Fourier components of $p_3(\theta_i)$ and $p_8(\theta_i)$.

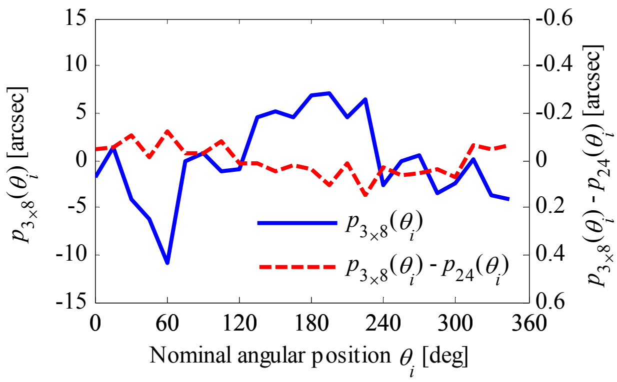


Fig. 9. Analyzed results of $p_{3\times8}(\theta_i)$ and $p_{3\times8}(\theta_i) - p_{24}(\theta_i)$.

Figure 7 shows the analyzed results of $p_m(\theta_i)$ for m = 3 and $p_n(\theta_i)$ for n = 8, respectively. Figure 8 shows the Fourier components for two curves, respectively. It can be confirmed that $P^{(3)}(\theta_i)$ for $p_3(\theta_i)$ and $P^{(8)}(\theta_i)$ for $p_8(\theta_i)$ are not contained.

Here, we make $P^{(m)}(\theta_i)$ from $p_n(\theta_i)$. We make the m-numbers of $p_n(\theta_i+\varphi_j)$ which is $p_n(\theta_i)$ with the phase shift of $\varphi_j = -(2\pi/m)\times(j-1)$. We denote the mean value of $p_n(\theta_i+\varphi_j)$ by $\mu_{(m,n)}(\theta_i)$ as follows:

$$\begin{aligned}\mu_{(m,n)}(\theta_i) &= \frac{1}{m}\sum_{j=1}^{m} p_n(\theta_i+\varphi_j) \\ &= \frac{1}{m}\sum_{j=1}^{m}\left\{P(\theta_i+\varphi_j) - P^{(n)}(\theta_i+\varphi_j)\right\} \\ &= P^{(m)}(\theta_i) - P^{(m\times n)}(\theta_i),\end{aligned} \tag{11}$$

where $P^{(m\times n)}(\theta_i)$ is a curve composed by the sum of the multiple of $m\times n$th-order Fourier components.

Here, let add $\mu_{(m,n)}(\theta_i)$ to $p_m(\theta_i)$ and we denote its curve by $p_{m\times n}(\theta_i)$ as follows:

$$\begin{aligned}p_{m\times n}(\theta_i) &= p_m(\theta_i) + \mu_{(m,n)}(\theta_i) \\ &= P(\theta_i) - P^{(m\times n)}(\theta_i).\end{aligned} \tag{12}$$

Figure 9 shows the result of $P_{3\times8}(\theta_i)$ analyzed from the measurement value for m = 3 and n = 8. The total number of orientations was 10 because we performed that one of orientations for m = 3 and n = 8 was a common orientation. On the other hand, we obtained $P_{24}(\theta_i)$ analyzed from the measurement value for 24 orientations which is the total number of the pitch balls. The difference between $P_{3\times8}(\theta_i)$ and $P_{24}(\theta_i)$ is shown in Fig. 9 and was less than ±0.15 arcsec. The superiority of the proposed analysis method which is calculated from the small total number of orientations is clarified. On the other hand, we obtained $P_{24}(\theta_i)$ analyzed from the measurement value for 24 orientations which is the total number of the pitch balls. The difference between $P_{3\times8}(\theta_i)$ and $P_{24}(\theta_i)$ is shown in Fig. 9 and was less than ±0.15 arcsec. The superiority of the proposed analysis method which is calculated from the small total number of orientations is clarified.

5. CONCLUSION

It is important to eliminate the error of measuring instruments when we perform the calibration for the pitch artefact. For elimination method, the multiple-orientation technique is effective. There are, however, Fourier components of the measurement error can not be eliminated depending on the total number of orientations. In this paper, we proposed the advanced analysis method. The multiple-orientation technique is improved as following two points.

- The improvement of the error separation method
- The improvement of the total number of orientations

The proposed analysis method expresses the calibration value without the measurement error. The analyzed result which was the combination of different orientations (m and n orientations) and the analyzed result which was large orientations ($m \times n$ orientations) were equated at less than ±0.15 arcsec. The proposed analysis method that can be calibrated in high accuracy and in a short time is effective.

ACKNOWLEDGMENTS

This study was supported by the New Energy and Industrial Technology Development Organization (NEDO) and the Japan Society for the Promotion of Science (JSPS).

REFERENCES

[1] G. Goch, “Gear Metrology”, *Annals of the CIRP*, vol. 52, pp. 659-695, 2003.

[2] W. Beyer, “Traceable Calibration of Master Gears at PTB”, *Technical Paper of American Gear Manufacturers Association*, 96FTM04, Alexandria, 1996.

[3] M. P. Sammartini, L. D. Chiffre “Development and validation of a new reference cylindrical gear for pitch measurement”, *Precision Eng*, Vol. 24, nº. 4, pp. 302-309, 2000.

[4] F. Takeoka, M. Komori, A. Kubo, H. Fujio, S. Taniyama, T. Ito, T. Takatsuji, S. Osawa and O. Sato, “Design of Laser Interferometric Measuring Device of Involute Profile”, *J. of Mechanical Design*, vol. 130, nº. 5, pp. 052602, 2008.

[5] K. Minoura, A. Ming, T. Kanamori, M. Kajitani, T. Sato and M. Yuzaki, “Development of Evaluation System for Gear Measuring Instruments (2nd Report) –Experimental Evaluation of Prototype Machines-”, *The Machine Design and Tribology Division meeting in JSME*, 2001(1), pp. 59-62, 2001.

[6] Y. Kondo, K. Sasajima, S. Osawa, O. Sato and M. Komori, “Traceability strategy for gear-pitch-measuring instruments: development and calibration of multiball artifact”, *Meas. Sci. and Tech.*, 20(6), pp. 065101, 2009.

[7] Y. Kondo, K. Sasajima, S. Osawa, O. Sato and M. Komori, “Pitch measurement of a new multi-ball artifact”, *Proceedings of the euspen international conference*, Vol.1, pp.348-352, Zurich, 2008.

[8] M. Komori，F. Takeoka，S. Osawa, O. Sato, T. Kiten, D. Shirasaki and Y. Kondo, “Design and Error Analysis of Multiball Artifact Composed of Simple Features to Evaluate Pitch Measurement Accuracy”, *J. of Mechanical Design*, 131(4), pp. 041006, 2008.

[9] S. Osawa, K. Busch, M. Frank and H. Schwenke, “Multiple orientation technique for the calibration of cylindrical work pieces on CMMs”, *Precision Eng*, Vo1.29, nº. 4, pp. 56-64, 2005.