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PERIOD ESTIMATION OF THE MODULATED SIGNAL

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Abstract − In the paper, the two non-parametric algorithms for the period estimation are compared: the autocorrelation approach and the interpolated discrete Fourier transform (IDFT) approach both added with algorithm for searching of the lowest common frequency component or the largest period of the modulated signal. The direct approach by IDFT shows better results at two and more cycles of the investigated period in the measurement interval. Simulation results also show the robustness of the searching algorithm of the lowest common frequency component.

Keywords: period estimation, non-parametric approach, autocorrelation, IDFT

1. INTRODUCTION

Estimations of the periodic signal parameters, where period/frequency of the investigated component is the key parameter, are very important in many measurement applications where we are looking for frequency response functions. Many approaches are reported in literature for the period measurement of digitised signals [1]-[6]. In most of them, a first analysis gives a rough estimation of the period, while a further algorithm improves the measurement accuracy [3]-[6]. The first step has the non-parametric estimation nature and it is important for the successive signal processing procedure.

One possibility in the first step is the level crossing approach [1],[2]. This method evaluates the period of sampled signal by means of the time distance between two consecutive crossings of a trigger level (usually zero) with the same slope and it is a method adopted by most of the scopes. The zero-crossing methods require a very low computational burden, but they are not applicable to signals with more than one zero crossing for the period, and their accuracy depend on the signal-to-noise ratio. Even though they are based on an interesting approach, we know that these algorithms all fail in the period measurement of the modulated signal. Most oscilloscopes fail completely in the period measurement of the frequency modulated (FM) and the amplitude modulated (AM) signals.

As regards the autocorrelation function, it requires more processing power than the zero-crossing one and is more robust and general. The literature [3] proposed to precede a zero-crossing interpolated method with an algorithm based on the autocorrelation; a normalisation function is also suggested to compensate the edge effect owing to the limited time window. The basic idea is to detect the distance between successive peaks. If a wide band signal with a flat amplitude spectrum is analysed, this simple method gives a good estimation. It fails in practice, however, for a number of special cases [4].

Since we are looking for the periodicity of the signal for which energy is essentially concentrated around some frequency origins, it is very suitable to use the frequency domain approach. Fourier transformation is in principle the best approximation to periodicity in the signal [7]-[11], with some restrictions. A finite time of measurement is a source of dynamic errors, which are shown as leakage parts of the measurement window spectrum convolved on the spectrum of the measured (sampled) signal. Tones of the sampled signal do not generally coincide with the basic set of the periodic components of the discrete Fourier transformation (DFT). The position, i.e. the frequency/period of the measurement component, can be estimated by means of the interpolated DFT [11]. This is a non-parametric frequency domain approach.

In this paper, the two non-parametric algorithms for the period estimation are compared: by the autocorrelation and by the IDFT, both added with algorithm for searching of the lowest common frequency component or the largest period of the modulated signals (FM, AM, etc. [12]).

2. NON-PARAMETRIC PERIOD ESTIMATION

There are generally three steps associated with the digital processing of the periodical signal with period T_x. First, the signal $g(t/T_x)$ is uniformly sampled ($f_s = 1/\Delta t$ - sampling frequency) and quantised into a discrete sequence $g(k\Delta t/T_x)$. Then, a block of data ($k = 0,1,..., N-1$) with suitable weights $w(k)$ is constructed by looking at the sequence for a period of time, which should be as close as possible to the integer values of the investigated periodicity. This period of time T_M is referred to as the data window $w(k)$ or observation interval and it is suitable for normalising the time-dependent parameters with the measurement time $T_M = N\Delta t$ or in the frequency domain with the frequency resolution $\Delta f = 1/T_M$:

$$\begin{aligned} w(k)\cdot g\left(k\frac{\Delta t}{T_x}\right) &= g\left(k\frac{\Delta t}{T_x}\right)_N \\ &= g\left(\frac{T_M}{T_x}\frac{k}{N}\right) = g\left(\frac{f_x}{\Delta f}\frac{k}{N}\right) = g\left(\theta_x\frac{k}{N}\right) \end{aligned} \quad (1)$$

There are some requirements associated with the first two steps. First of all, the sampling frequency must be at least twice as high as the highest frequency of interest (practically 2,5 times higher). In the same vein, the measurement time should be long enough to resolve two components. Using a basic rectangular window shape and the non-parametric resolution approach, the measurement time should be at least around one period.

To analyse the signal in the frequency domain, the DFT can be applied to the samples within the data window. The DFT at the spectral line i of the multi-component m signal

$$g(k\Delta t)_N = \sum_m A_m \sin(2\pi\theta_m k/N + \varphi_m) \tag{2}$$

where A_m, $\theta_m = f_m/\Delta f = i_m + \delta_m$, and φ_m are the amplitude, the normalised frequency and the phase of the particular harmonic or non-harmonic component in the signal, respectively, is given by

$$G(i) = -\frac{j}{2}\sum_m A_m\left[W(i-\theta_m)e^{j\varphi_m} - W(i+\theta_m)e^{-j\varphi_m}\right] \tag{3}$$

$W(*)$ is a spectrum of the window function $w(k)$. The displacement term δ_m is owed to the non-coherent sampling around the integer values i_m ($-0.5 < \delta_m \le 0.5$) and can be estimated by means of the interpolated DFT [11].

One of the most generally used non-parametric approaches to estimate the periodicity in the time domain is the autocorrelation function. The autocorrelation $R(\tau)$ of a waveform gives an indication of similarity of the waveform with its time-shifted τ version

$$R(\tau) = \frac{1}{T_M}\int_0^{T_M} g(t)g(t+\tau)\,dt \tag{4}$$

$R(\tau)$ is periodic with the same period as $g(t)$. The largest value is at $\tau = 0$ $R(0) \ge |R(\tau)|$ [13], that is, $R(\tau)$ has its largest magnitude at $\tau = 0, \pm T_x, \pm 2T_x$ etc. (Fig. 1), at which points it is equal to the average power in $g(t)$. The Fourier coefficients of $R(\tau)$ are equal $|G(i)|^2$ (Fig. 2).

In the discrete version on the N samples, the autocorrelation function is calculated as

$$R(n) = \frac{1}{N}\sum_{m=0}^{N-1} g(m)g(m+n), \quad -(N-1) < n < N-1 \tag{5}$$

The autocorrelation function duration is almost twice as wide as the duration of the input signal ($2N-1$) and the point $\tau = 0$ corresponds to the middle of the span (Fig. 1).

3. PERIOD ESTIMATION

A. Frequency modulation

To find the period of the modulated signal, the FM signal was used with frequency sweep from 1 to 10 cycles in the common period and 2,2 cycles of this period in the measurement interval (Fig. 1: $\theta_x = 2{,}2$).

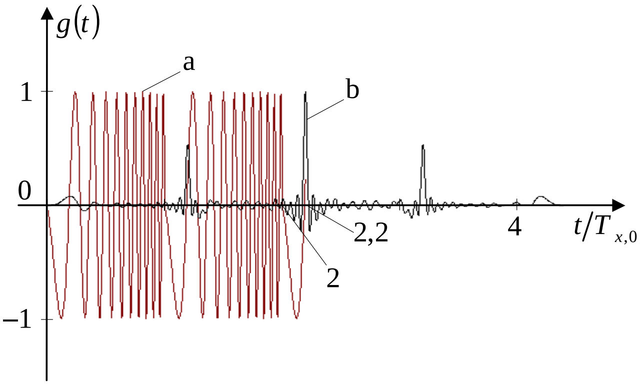

Fig. 1. Frequency modulated signal $g(t)$ (a) and its normalised autocorrelation function $R(\tau)_{norm} = R(\tau)/R(\tau)_{max}$ (b); $\theta_x = 2{,}2$

To find the periodicity of the investigated signal and its autocorrelation function the frequency domain approach has been adopted. Since we are looking for the period of the significant component, the largest amplitude DFT coefficients have to be searched (Fig. 2: at the dotted lines).

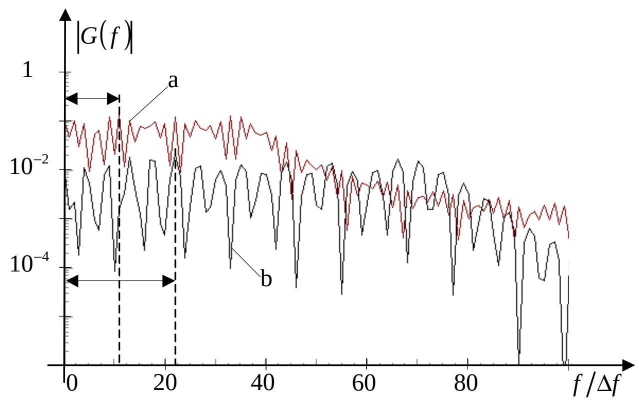

Fig. 2. Spectra of the frequency modulated signal $g(t)$ (a) and its normalised autocorrelation function $R(\tau)_{norm}$ (b)

The effect of the non-coherency for this local component can be reduced by the interpolation of the DFT coefficients. It has been shown [11] that the best estimation results in reducing long leakage effects gives the three-point estimation using the Hann window. In the estimation of the particular component *m*, the three largest local DFT coefficients $|G(i_m-1)|$, $|G(i_m)|$, and $|G(i_m+1)|$ are used for the frequency estimation.

$$\delta_m \cong 2\frac{|G(i_m+1)| - |G(i_m-1)|}{|G(i_m-1)| + 2|G(i_m)| + |G(i_m+1)|} \tag{6}$$

Since we are looking for the lowest common frequency, the interval from the zero to the estimated frequency of the largest component has to be investigated. There could be lower periodic peaks at positions with integer divisors of the position of the largest one. Again, the DFT of the waveform of the amplitude spectrum from zero to the maximal peak can be used to estimate this period (Fig. 3).

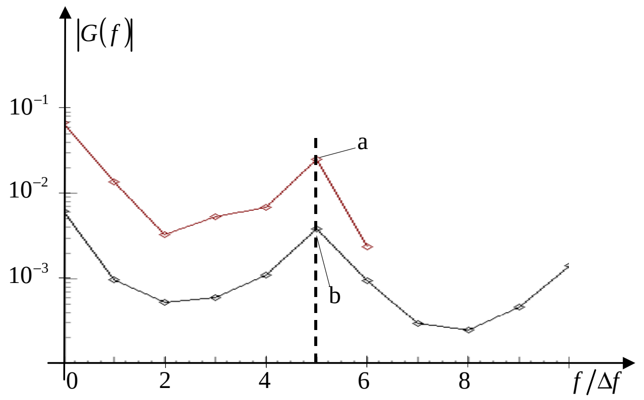

Fig. 3. Spectra of the amplitude spectra waveforms in the intervals from zero to the maximum peak at dotted lines in Fig. 2: a - signal $g(t)$; b - autocorrelation function $R(\tau)_{\mathrm{norm}}$

If there is any lower significant frequency component, the amplitude DFT spectrum shows a peak different from the zero DC component and with the value of the frequency of this peak (Fig. 3: in our case $j(|G|_{\max}) = 5$, $j = f/\Delta f$) and the estimated frequency of the previous step should be divided:

$$f_{\mathrm{x}} = \frac{i_m + \delta_m}{j}\Delta f \;\rightarrow\; T_{\mathrm{x}} = \frac{1}{f_{\mathrm{x}}} \qquad (7)$$

To show the effectiveness of the proposed algorithm with advanced searching of the lowest common frequency component for both non-parametric approaches of the period estimation (a - by the IDFT of the signal and b - by the IDFT of the signal autocorrelation function), the maximal values of errors were searched with double scan (Fig. 4: $N = 1024$; $1 \le \theta_{\mathrm{sweep}} \le 10$, $1 \le \theta_{\mathrm{x}} \le 6$ and at each frequency the phase angle has been changed $-\pi/2 \le \varphi \le \pi/2$, $\Delta\varphi = \pi/18$). The systematic errors of the frequency estimations $E = \theta_{\mathrm{est}} - \theta_{\mathrm{true}}$ (θ_{true} is the true value of the frequency) are phase-dependent. φ is the phase of the first cycle in the sweep.

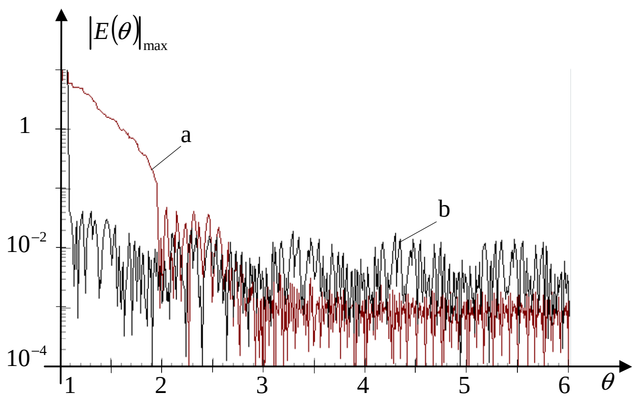

Fig. 4. Absolute maximal errors of the period estimations of the frequency modulated signal (a) and its autocorrelation function (b)

Between one and two cycles in the measurement interval ($1 \le \theta_{\mathrm{x}} \le 2$) the approach with the autocorrelation function gives better results. Above two cycles, the direct approach with IDFT only shows lower errors since the frequency resolution interval condition $2 \le \Delta\theta$ of the used Hann window in the interpolation is satisfied [11].

The same behaviour can be noticed with added noise to the signal from Fig. 1 at the level of 10 per cent (Fig. 5). The standard deviation level is lower with the autocorrelation function (Fig. 6).

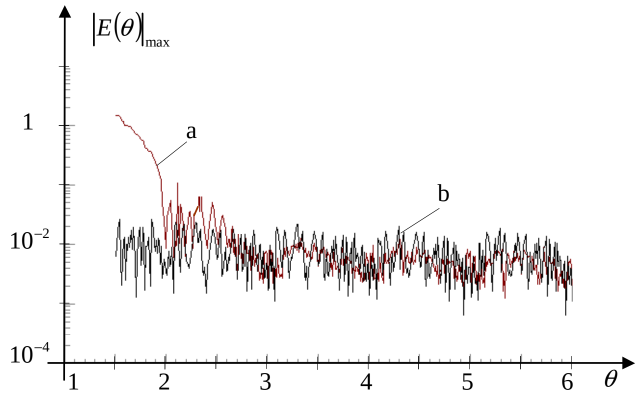

Fig. 5. Absolute maximal errors of the period estimations of the frequency modulated signal (a) and its autocorrelation function (b); $\varphi = 0$; $A_{\mathrm{noise}}/A = 0{,}1$; 100 iterations at each frequency $1 \le \theta_{\mathrm{x}} \le 6$

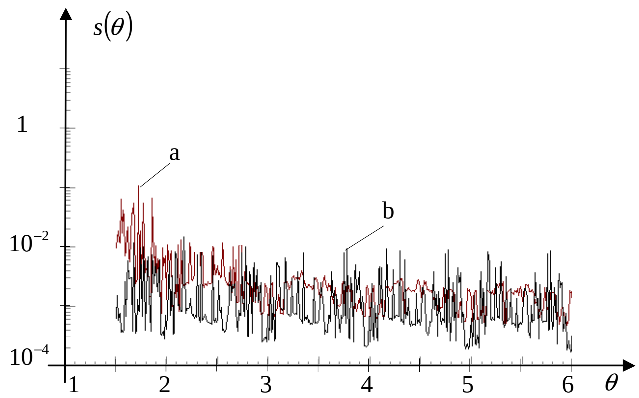

Fig. 6. Standard deviations of the period estimations of the frequency modulated signal (a) and its autocorrelation function (b); $\varphi = 0$; $A_{\mathrm{noise}}/A = 0{,}1$; 100 iterations at each frequency θ_{x}

The experimental validation of the non-parametric frequency estimations of the FM signals confirms the above conclusions. In the experimental set-up, a signal generator (Agilent 33220A) and digital storage oscilloscope (HP 54600) were used. The linearly changing sweep signal generated had the constant sweep time $t_{\mathrm{sweep}} = 40\,\mathrm{ms}$ with the start and the stop frequency $f_{\mathrm{start}} = 100\,\mathrm{Hz}$ and $f_{\mathrm{stop}} = 1\,\mathrm{kHz}$. On the DSO, the time base range was changed from $T_{\min} = 100\,\mathrm{ms}$ to $T_{\max} = 240\,\mathrm{ms}$ in steps of $\Delta T = 4\,\mathrm{ms}$. The relative frequency was changed from $\theta_{\min} = 100\,\mathrm{ms}/40\,\mathrm{ms} = 2{,}5$ to $\theta_{\min} = 240\,\mathrm{ms}/40\,\mathrm{ms} = 6$, respectively. The maximal errors on the 30 trials with random initial phases at each frequency were around $|E(\theta)|_{\max} \approx 0{,}1$ (Fig. 7).

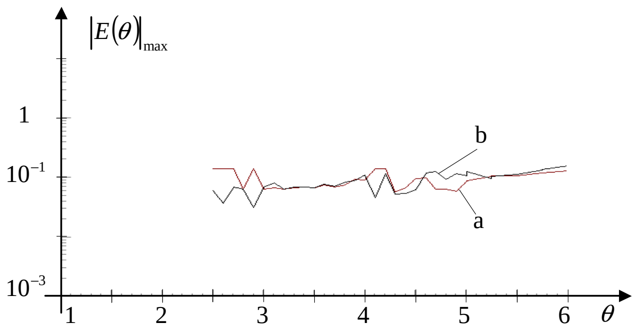

Fig. 7. Absolute maximal errors of the period estimations of the frequency modulated signal (a) and its autocorrelation function (b); $A = 1\text{V}$, φ - random; 30 iterations at each frequency $2{,}5 \le \theta_x \le 6$

B. Amplitude modulation

The amplitude spectra of the AM signals show the reduction of a number of the significant components. The carrier signal with the frequency θ_1 has the amplitude modulation with the lower frequency θ_2 (an example: eq. (8) and Fig. 8) and the spectrum shows at least two peaks at $\theta_1 - \theta_2$ and $\theta_1 + \theta_2$ but generally three peaks if there is the DC component in the AM signal (Fig. 9).

$$g(t) = (0{,}5 + 0{,}5\sin(2\pi\theta_2 t + \varphi_2)) \cdot \sin(2\pi\theta_1 t + \varphi_1) \qquad (8)$$

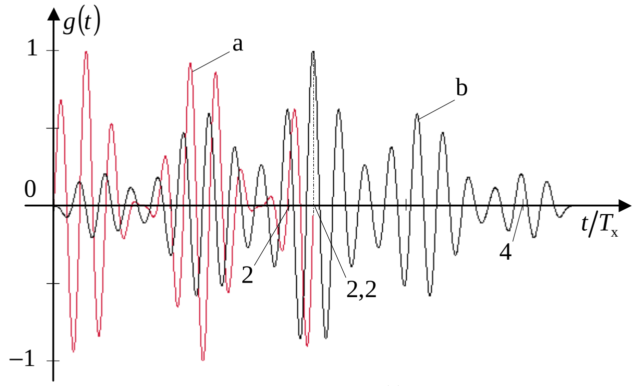

Fig. 8. Amplitude modulated signal $g(t)$ (a) and its normalised autocorrelation function $R(\tau)_{\text{norm}} = R(\tau)/R(\tau)_{\max}$ (b); $N = 1024$; $\theta_1 = 10$, $\theta_2 = 2{,}2$; $\varphi_1 = 0$, $\varphi_2 = 0$

To find the period of the AM signal, one has to estimate the frequency difference between the two components' peaks, as was the case with FM. The procedure requires the estimation of the frequencies of the two largest peaks by (6) in the investigated interval from the zero frequency to the maximal spectrum peak (Fig. 9.).

$$f_x = |\theta_1 - \theta_2| \cdot \Delta f = \Delta\theta \cdot \Delta f \;\; \rightarrow \;\; T_x = \frac{1}{f_x} \qquad (9)$$

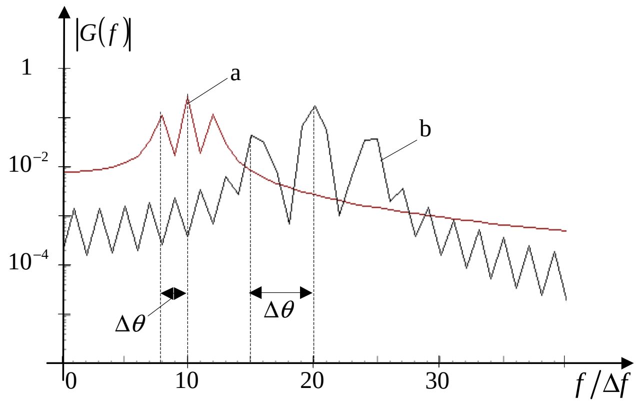

Fig. 9. Spectra of the amplitude modulated signal $g(t)$ (a) and its normalised autocorrelation function $R(\tau)_{\text{norm}}$ (b)

To analyse the effectiveness of the proposed algorithm for both non-parametric approaches of the period estimation (a - by the IDFT of the signal and b - by the IDFT of the signal autocorrelation function) the maximal values of errors were searched with double scan (Fig. 10: $N = 1024$; $\theta_1 = 10$; $1 \le \theta_2 \le 10$ and at each AM frequency the phase angle was changed $-\pi/2 \le \varphi_2 \le \pi/2$, $\Delta\varphi_2 = \pi/18$).

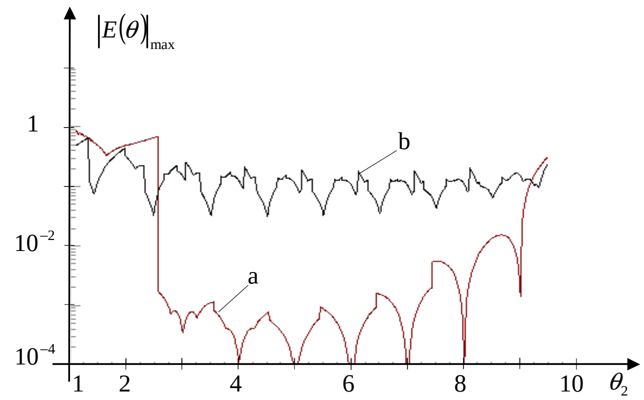

Fig. 10. Absolute maximal errors of the period estimations of the amplitude modulated signal (a) and its autocorrelation function (b) $\theta_1 = 10$, $1 \le \theta_2 \le 10$, $-\pi/2 \le \varphi_2 \le \pi/2$

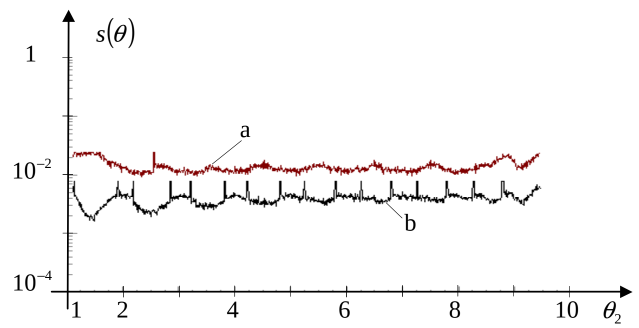

Fig. 11. Standard deviations of the period estimations of the AM signal (a) and its autocorrelation function (b); $\varphi = 0$; $A_{\text{noise}}/A = 0{,}1$; 100 iterations at each frequency θ_2

Above two and a half cycles, the direct approach with IDFT gives lower errors. This procedure shows worse results when the frequency distance drops under the two cycles $\Delta\theta < 2$ owing to the width of the Hann window spectrum main-lobe.

When we add the noise to the signal from Fig. 10 at the level of 10 per cent, the autocorrelation function gives lower values of the standard deviations (Fig. 11).

C. Signal with one zero crossing per period

The proposed algorithms were also tested by the triangular shape signal as a representative signal with one zero crossing per period where the largest DFT coefficient is the first in the row of the signal harmonics and there is no significant lower frequency component between zero and the largest DFT coefficient. In this case, the direct approach with IDFT only gives better results (lower systematic errors) even at lower frequencies $1{,}5 \le \theta_x$ (Fig. 12: errors were searched with double scan ($N = 1024$; $1 \le \theta_x \le 6$ and at each frequency $-\pi/2 \le \varphi \le \pi/2$, $\Delta\varphi = \pi/18$).

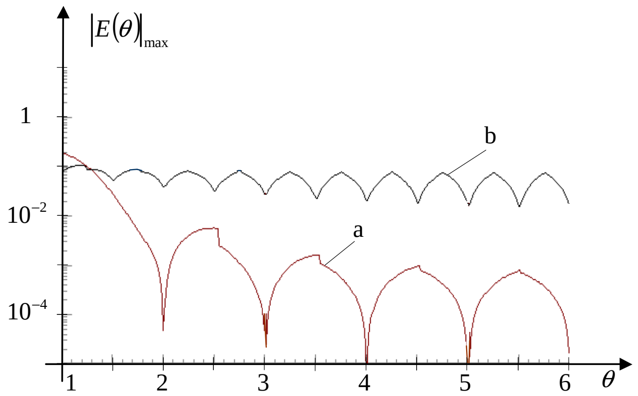

Fig. 12. Absolute maximal errors of the period estimations of the triangular signal (a) and its autocorrelation function (b) $A_{noise}/A = 0$

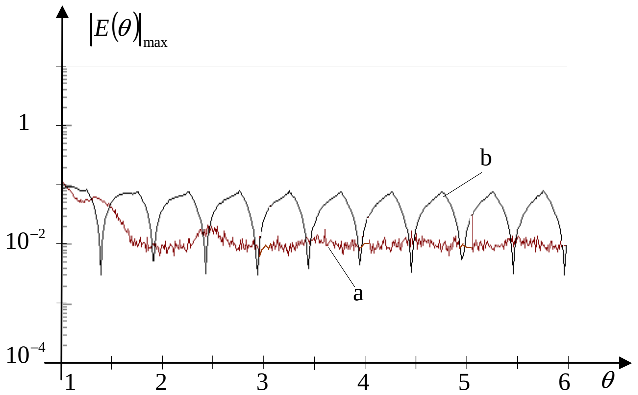

Fig. 13. Absolute maximal errors of the period estimations of the triangular signal (a) and its autocorrelation function (b); $\varphi = 0$; $A_{noise}/A = 0{,}1$; 100 iterations at each frequency θ_x

The same behaviour can be noticed with added noise to the triangular signal at the level of 10 per cent (Fig. 13 and Fig. 14). The standard deviation level of the autocorrelation approach is about four times lower than the direct approach by IDFT only.

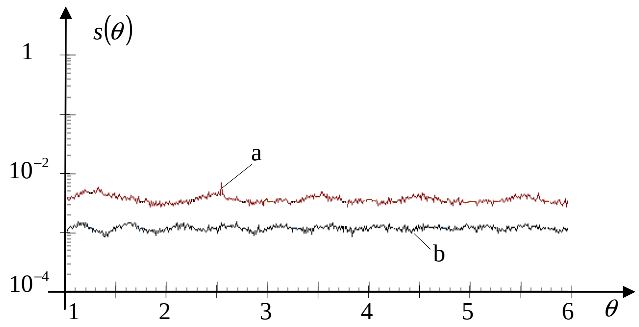

Fig. 14. Standard deviations of the period estimations of the triangular signal (a) and its autocorrelation function (b); $\varphi = 0$; $A_{noise}/A = 0{,}1$; 100 iterations at each frequency

4. CONCLUSIONS

In the paper, the two non-parametric algorithms for the period estimation are compared: by the autocorrelation and by the IDFT both added with algorithm for searching of the lowest common frequency component or the largest period of the modulated signals. The direct approach by IDFT only shows better results (lower systematic errors) at two and more cycles of the investigated period in the measurement interval. Between one and two cycles in the measurement interval the approach with the autocorrelation function gives better results.

REFERENCES

[1] N. Geckinli, D Yavuz, “Algorithm for pitch extraction using zero-crossing interval sequence”, *IEEE Trans. Acoust. Speech, Signal Process.*, vol. ASSP-25, no. 6, pp. 559–564, Dec. 1977.

[2] S. M. Mahmud, “Error analysis of digital phase measurement of distorted waves”, *IEEE Trans. Instrum. Meas.*, vol. 38, no. 1, pp. 6–9, Feb. 1989.

[3] H. W. Wong-Lam, M. Naley, “A robust and accurate algorithm for time measurements of periodic signals based on correlation techniques”, *IEEE Trans. Instrum. Meas.*, vol. 50, no. 5, pp. 1181–1189, Oct. 2001.

[4] M. G. D’Elia, C. Liguori, V. Paciello, A. Pietrosanto, “Software customization to provide digital oscilloscope with enhanced period-measurement features”, *IEEE Trans. Instrum. Meas.*, vol. 55, no. 2, pp. 493-500, April 2006.

[5] J. Schoukens, Y. Rolain, G. Simon, R. Pintelon, “Fully automated spectral analysis of periodic signals”, *IEEE Trans. Instrum. Meas.*, vol. 52, no. 4, pp.1021–1024, Aug. 2003.

[6] J. Xi, J. F. Chicaro, “A new algorithm for improving the accuracy of periodic signal analysis”, *IEEE Trans. Instr. Meas.*, vol. 45, no. 4, pp. 827–831, Aug. 1996.

[7] M.D. Sacchi, T.J. Ulrych, C.J. Walker, "Interpolation and extrapolation using a high-resolution discrete fourier transform", *IEEE Trans. Signal Process.*, vol. 46, no. 1, pp. 31-38, Jan. 1998.

[8] G. Andria, M. Savino, A. Trotta: "Windows and interpolation algorithms to improve electrical measurement accuracy", *IEEE Trans. Instrum. Meas.*, vol. 38, no. 4, pp. 856-863, Aug. 1989.

[9] R. Pintelon, J. Schoukens, "Measurement of frequency response functions using periodic excitations, corrupted by correlated input/output errors", *IEEE Trans. Instrum. Meas*., vol. 50, no. 6, pp. 1753-1760, Dec. 2001.

[10] R. M. Hidalgo, J. G. Fernandez, R. R. Rivera, H. A. Larrondo, "A simple adjustable window algorithm to improve FFT measurements", *IEEE Trans. Instrum. Meas*., vol. 51, no. 1, pp. 31-36, Feb. 2002.

[11] D. Agrež, "Dynamics of Frequency Estimation in the Frequency Domain", *IEEE Tran. Instrum. Meas.*, vol. 51, no. 2, pp. 287-292, Dec. 2007.

[12] G. Bucci, E. Fiorucci, F. Ciancetta, "The performance evaluation of IEC flicker meters in realistic conditions", *IEEE Trans. Instrum. Meas*., vol. 57, no. 11, pp. 2443 - 2449, Nov. 2008.

[13] W. McC. Siebert, *Circuits, Signals and Systems*, The MIT Press, McGraw-Hill, Cambridge, New York, ... , 1986.