PARALLEL GLASS PLATE TEST WITH THE USE OF THE PHASE SHIFT TECHNIQUE IN THE OPTICAL VORTEX INTERFEROMETER

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Abstract – The Optical Vortex Interferometer (OVI) is described in earlier paper as usefully tool to generate regular lattice of optical vortices (OVs) by interference of three plane wave. To practically use OVI in metrology method to localization of vortex points (VPs) is necessary. In this paper method with phase shift technique to VPs localisation is presented. Phase shift technique in this method is applied to additional fourth wave. Phase shift of the fourth wave doesn't change intensity in vortex points and increases intensity gradient in their vicinity. Result of localisation can be useful to analyse wave-front deformation caused by the object inserted in OVI setup.

Keywords: optical vortex, interferometry, wave-front

1. INTRODUCTION

Optical Vortices (OV's) are isolated singularities in the phase distribution of the optical wave-field [1]. They have some special properties, which have been studied in the last decade [2]. The OV's are subject of many scientific tasks. One of them is an Optical Vortex Interferometer (OVI) [3]. The OVI is a kind of a multiple beam interferometer, which uses interference of three beams (in other applications which are not described in this paper more beams may be used). In the OVI a regular lattice of OV's is generated by the interference of the three plane beams. Fig. 1 presents a part of interference pattern with the VPs marked. The hexagonal lattice is one of possibilities and another lattice of vortices can be generated and also these can be useful. [4]. Applied of the OVI in metrology is depend on precisely of the VPs localization. In last years same method to VPs localisation where created but this topic is still open [5]. In this paper is mentioned phase shift technique to localization VPs. Each optical vortex (OV) contain Vortex Point (VP) where phase is indefinite [1, 6]. By VP we mean the central point of the optical vortex where the phase is indeterminate and intensity is zero. In cross-section to wave propagation, direction around the VP phase is changing from 0 to 2π.

Direction of this changing is depend on sign of topological charge of the OV [7]. Rotation of phase around the VP is connected with helical wave-front of wave which contains the OV.
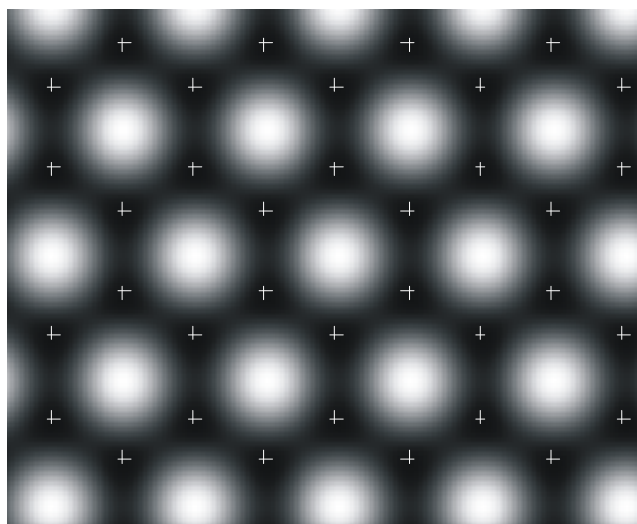

Fig. 1. Hexagonal lattice of the OVs generated in the OVI. VPs are marked by '+'.

When the plane wave interferes with the OV fork like fringe is generated. Example of interference plane wave with lattice of the OVs is presented in the Fig. 2. The direction of the fork the fringe is connected with sign of topological charge of OV. In some measurements sign of topological determination is required and to determine it the direction of forks can be used [8]. A proposition how take advantages of phase shift technique and fork-like fringes to localization of VPs is described in section 2. Next step is using result of localization to measure chosen physics quantity. For example wave-front distortion caused by parallel glass plate. The principle of the measurement using OVI is following. If one of the interfering beams is somehow distorted, then the geometry of the vortex lattice is changed. We can calculate

the change of the lattice geometry and used it to calculate the parameters of the quantity causing the wave distortion.
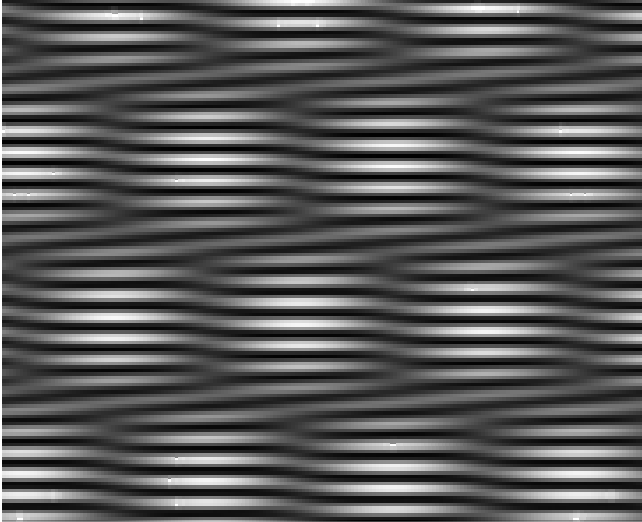

Fig. 2. The example of fork like fringe pattern created by interference of four waves in the OVI.

Usually the lattice with hundreds of optical vortices is used. How lattice geometry can be use to calculate relative phase between two waves is shown in section 3. Differences between two results of phase reconstruction may be interpreted as wave-front distortion caused by tested object.

2. VORTEX POINTS LOCALISATION

Fig. 1. presents an exemplary interferogram in the case where three plane waves (A, B, C) interfere. In the dark areas of the interferogram there are the OVs. If we add an additional reference wave (D), at the vortex points the characteristic fork like fringes appear - (Fig. 2.). To obtain interference pattern like in fig. 2. the amplitude of wave D should be equal to the sum of the amplitudes the three another beams (A, B, C). Waves A, B and C should have the same value of amplitudes. As was mentioned earlier, the direction of the fork is connected with phase rotation in vortex point vicinity.

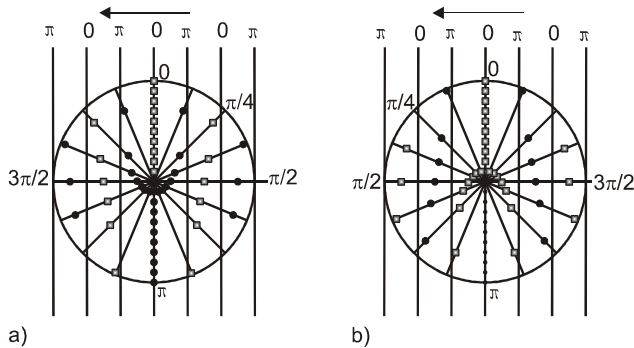a)   b)

Fig. 3. The phase distribution around optical vortex in the interference field with the additional reference wave. Two cases depend on the sign of topological charge of vortex point: a) – positive, phase circulates clockwise, b) – negative, phases circulates counter-clockwise.

Fig. 3. presents the phase distribution in two optical vortices which differs in topological sign. The projections of wave-vector of fourth wave (UD) on the screen is marked by black arrows on the top of figure. If the sign of topological charge of optical vortex is positive then fork is downward and for the negative sign is the opposite. In Figure 3 dots indicates the maximum intensity points, and squares indicates the point of intensity minimum. Upward and downward direction in figure 3. correspond to left and right direction of the forks in Figure 2. Was mentioned earlier than at the vortex point the phase is undetermined and the amplitude is equal to zero (UA+UB+UC=0). When we add the fourth wave then at the vortex point the intensity of the A+B+C+D interferogram is equal to the intensity of the wave D. In the all other points of A+B+C+D interferogram the intensity depends on the phase of the wave D. Thus, changing the phase of the wave D causes the changes in the intensity in all regions of the interferogram except vortex points. We used this fact for vortex points localization. The procedure is as following. We detect a series of interferogram with different phase shift of the wave D. Then we proceed this interferogram according the formula:

$$\Delta I = |I_2 - I_1| + |I_3 - I_2| + \ldots + |I_n - I_{n-1}|, \qquad (1)$$

where, ΔI – change of the intensity, I_k – intensity in the k-th interference pattern (from 1 to n), n – number of interferograms. Phase of wave UD in the k-th step of acquisition is calculated from equation:

$$\varphi_k = \phi_0 + (k-1) \cdot \varphi_s, \qquad (2)$$

where φ_s – step of the phase change. The result of applying such a procedure to the numerically generated interferograms is presented in Fig. 4. This intensity distribution is similar to the case of three beam interference (Fig. 1), but the dark areas containing vortices are "more visible". Around the vortex point the gradient of the intensity changes is bigger, what allows for better localization of the vortex point.
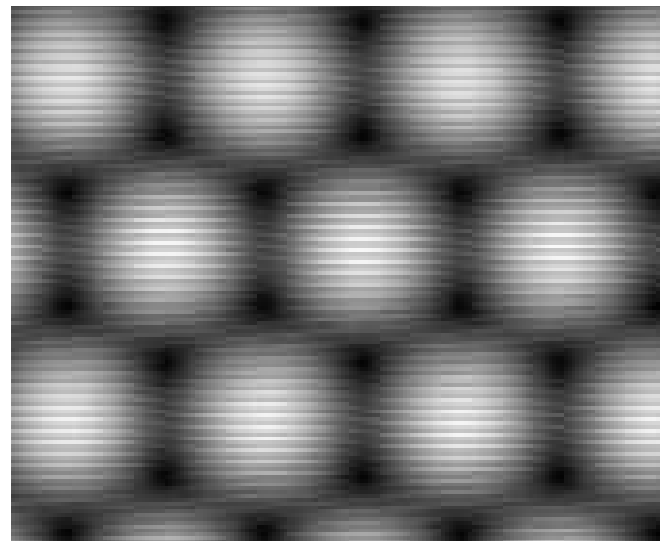

Fig. 4. Intensity distribution after applying the Equation (1).

Fig. 5 presents the gradient of the intensity changes in two cases: A+B+C interferogram (Fig. 5a) and after applying the Equation (1) (Fig. 5b).
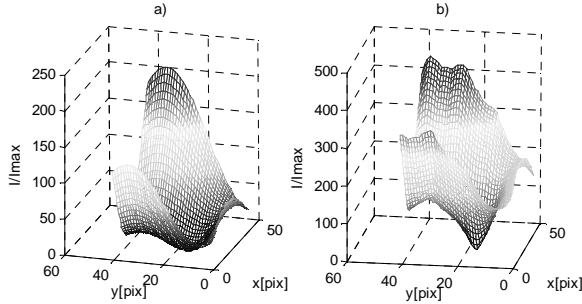

Fig. 5. Intensity in neighborhood of the vortex point. a) - three wave interference, b) - calculated from formula 1 (phase step - π/9 and no. of patterns 18).

As was shown phase shifting of additional plane wave doesn't change intensity in the VPs thus is possible to obtain intensity pattern with bigger gradient of the value in neighbourhood of the VP them gradient in intensity pattern from three waves interference. The characteristic feature of pattern calculated from more many patterns is helpful to localization the VPs more precisely. Presented method was tested in simulation for several value of OV lattice constant. By lattice constant is mean distance between the nearest VPs with the same sign. The best precision of localisation is obtained when the value of lattice constant is bigger then 30 pixels. If the condition is true the localization can be better then half a pixel. The choose of lattice constant should be dictated by reaching a compromise between high density of VPs and accuracy of phase determination in each vortex point. The width range of optical vortex density is possible to use. Therefore is not required precision adjustment of optical setup. This is not required also because the number of fringes between nearest VPs is not very important. This parameter from 2 to 5 don't have influence on localization accuracy. The fringe width should be enough to registering intensity changing when phase was changed. If is smaller phase shift step then fringe width should be bigger.

3. WAVE-FRONT RECONSTRUCTION

For analyses was prepared wave-front deformation of one of waves shown in Fig. 3. The shape is described by Equation (3) where phase is defined as:

$$\varphi = 0,05 \cdot \sin\left(4 \cdot \pi \cdot \frac{x}{X}\right) + 0,05 \cdot \sin\left(4 \cdot \pi \cdot \frac{y}{Y}\right), \qquad (3)$$

where: φ – phase, [X,Y] – CCD size, [x,y] coordinate

All results of simulations in this article is related to this deformation. In simulation was generate OV lattice with about 900 OVs in 56 square millimetres area (CCD with 1344x1024 pixels). Deformation of wave-fronts of interfere waves change structure of the VPs lattice and can be analysed. To simplify analyses is assumed that only one wave is disturbed and rest of waves are plane.. In this case

only one lattice of OVs analyze is enough and relative phase between deformed wave and each plane wave is related to wave-front of deformed wave. In real case difference between results of two reconstructions should be calculate. Of course the vortex points localization before and after the object (e.g. parallel glass plate) insertion in the optical arrangement is required.
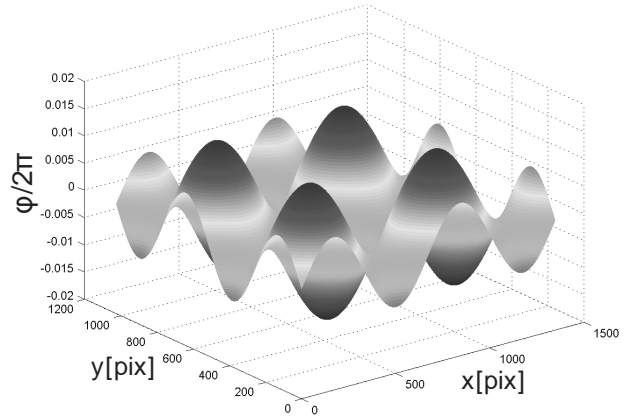

Fig. 6. Wave-front deformation used in simulations.

To calculate shape of wave-front from VPs coordinates after localization and sign of topological charge determination OVs must be split of into two groups according to the topological charge. And next the groups should be analyzed separately because of the phase difference between the VPs of vortices with positive and negative signs. If the amplitude of each wave is constant in the analyzed area then the relative phases between two waves in VP's from one group are the same (in 2π range). Then we obtain two values of the relative phase between the two chosen waves (for example A and B), $\varphi+$ and $\varphi-$ adequately to the sign of the topological charge. In the case then the amplitude of each wave is the same, the two values equal to $2\pi/3$ and $4\pi/3$. Coordinates of the VP's from two groups are presented in the Figure 7.
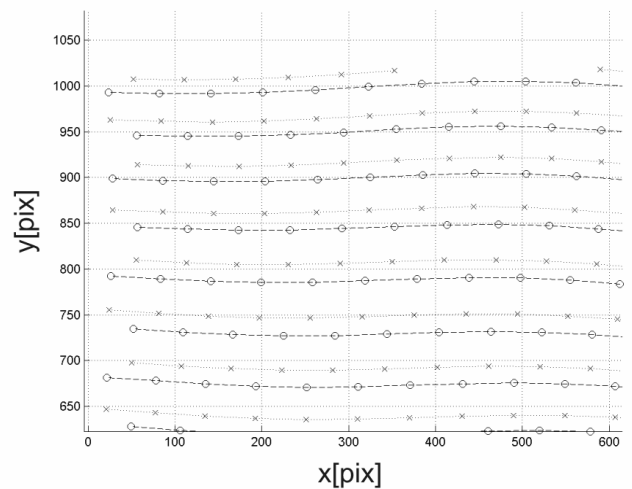

Fig. 7. VP's coordinates and lines which are used to analyze relative phase between waves A and B.

Vortex points from one group are marked by 'x' (positive sign) and from the second group are marked by 'o' (negative sign). In the figure 4 each line contains the coordinates of the VP's where the relative phases between two waves are the same. All neighboring lines of the same type (e.g. dashed) contain VP's where relative phase is changed by 2π. Fig. 7. shows lines which are used to analyze relative phase between waves A and B. Because the wave-front of wave B is not plane then the is not straight. In the same part of analyzed area is possible to draw analogous lines for phase between waves A and C and there are straight. If the lines from figure 7 are numerated from bottom to top the relative phase decrease for every next the same type line by 2π. Then can be assumed that at the first line phase is zero and result of recalculated phase in two groups is shown in Figure 8. Coordinates of VPs are change along the vertical axis. Additionally VPs from two groups are shifted each other.
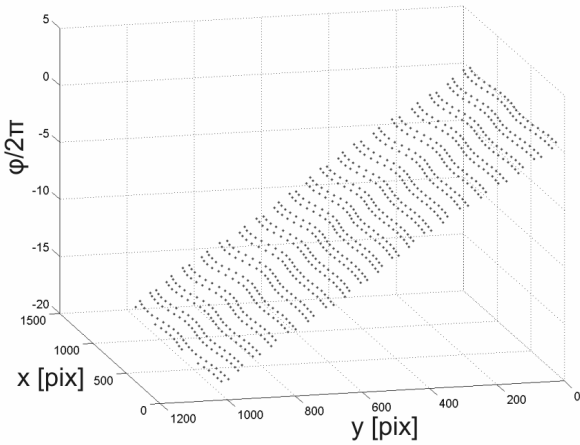

Fig. 8. Relative phase between waves A and B in vortex points.

As is easy to see in Figure 8. points create not plane surface. If we approximate the surface by the plane and the differences between the plane and the points from the non-plane surface are calculated then the shape of the wave-front is obtained. Applied the method presented by authors is way to obtain reconstruction result shown in Figure 9. In analogical way from the same data is possible to obtain relative phase between waves A and C or B and C. In this simulation there are not curious because phase between A and C is plane, and phase between B and C look likes shown in Figure 9.

In analyzed interferograms only the OVs with topological charge "+1" and "-1" can be found then vortices can be split of into two groups right or not. More important is how localization error influence to phase deformation reconstruction error. Theoretically we obtain reconstruction error $3*10^{-3}\lambda$ as is shown in Fig. 10. but in real measure it seems impossible. The reconstruction error is linear depended on vortex points localization error and lattice constant. This relation is described by Equation (4):

$$E2[\lambda] = \frac{1}{a[pix]} \cdot E1[pix], \qquad (4)$$

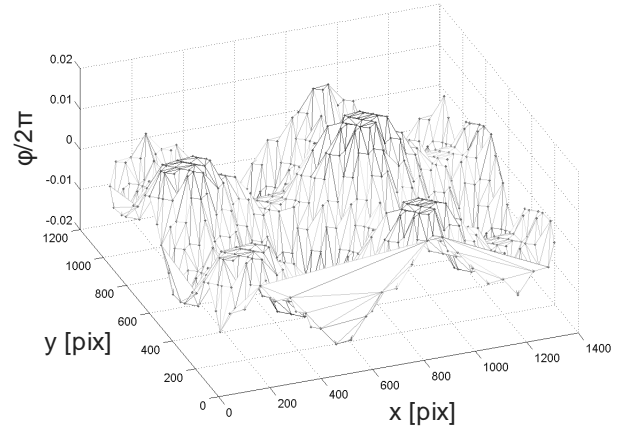where E2 – reconstruction error, E1 – localization error, a – lattice constant.

Fig. 9. Result of phase reconstruction.

The simulation described in this paper refer to lattice constant about 50pix. and localization error 0,1pix. than reconstruction error should be $2*10^{-3}$ λ. In simulation phase reconstruction error turned out to be bigger because of error caused by numerical algorithm. If be used localization method described in section 2 and VPs localization error below 0,5pix. then phase reconstruction error should be below $1,5*10^{-2}$ λ.
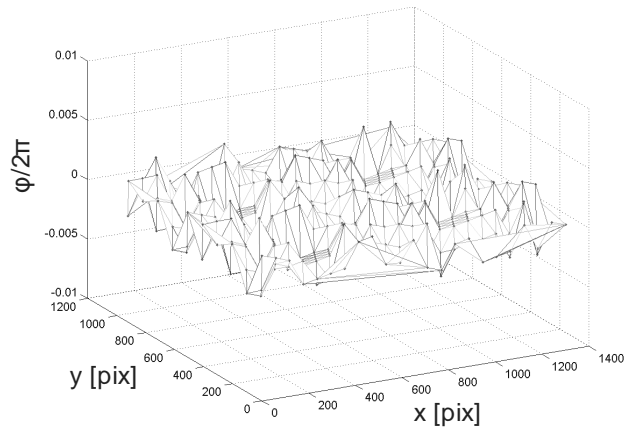

Fig. 10. Theoretically obtained reconstruction error.

Due to calculation difference between surface created by VPs. (Figure 8.) and approximated plane surface from the same points presented method have same limitations. Is not possible to get angle of beam deviation caused by tested object. Only shape of phase is possible to obtain without orientation relative to wave-vector.

4. CONCLUSIONS

Presented method with the use of the phase shift technique to localization of vortex points is simply to applied. It can be used in different optical vortex interferometers and with width range of optical vortex density. Interference patterns required for this method is also enough to determine sign of topological charge of optical vortices then may be utilized for many application. Vortex points localization topics is still open and method proposed in this paper may be competitive other exist. Phase reconstruction method described in section 3 is promising for future. However the accuracy is not confirm by experimental result but in theory it have potential. Applied two mentioned methods may be useful to test optical equipment in laboratory.

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