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FAST AND ACCURATE MEASUREMENT OF THE RMS VALUE OF A NONCOHERENT SAMPLED SINE-WAVE

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Abstract – In this paper a new estimator for the measurement of the rms value of a noncoherent sampled sine-wave is proposed. The key feature of this estimator is that the sine-wave offset, *a priori* estimated, is removed from the sine-wave. A formula used to estimate the sine-wave offset is also given in the paper. It has been proved by means of computer simulations and experimental results that by the proposed estimator accurate rms measurements are obtained if the total harmonic distortion (*THD*) of the sine-wave is smaller than or equal to −30 dB.

Keywords: sine-wave rms estimation, noncoherent sampling mode, windowing.

1. INTRODUCTION

In many engineering applications, it is very important to know with high accuracy the rms value of a sine-wave because it related directly to its power. In digital measurements, the sine-wave is often sampled in noncoherent way (noncoherent sampling mode). Different methods have been proposed in scientific literature to measure the rms value of a sine-wave in this case. These methods can be classified either in time-domain methods [1]-[2] or in frequency-domain methods [3]-[8]. In [2] the rms value of a noncoherent sampled sine-wave is calculated by the formula used in an AC analog electronic voltmeter. For this purpose first the rectified mean value of the sine-wave is calculated and then the result obtained is multiplied by the factor form for a sine-wave. To increase the measurement accuracy the sine-wave is *a priori* multiplied by the sequence of a cosine window. The estimator proposed in [2] is based on a low complexity algorithm and it is the most simple to implement and so, the faster. In [2] it has been proved by means of computer simulations and experimental results that for a free offset and high spectral purity sine-wave the measurements obtained using this estimator are relatively high accurate. The sine-waves often encountered in practice have an offset and a spectral purity not so high. Unfortunately, these situations have not been analyzed in [2].

In this paper first the influence of the offset and harmonic components of a sine-wave on the accuracy of the rms value measurement obtained using the estimator proposed in [2] is analyzed. From this analysis a new estimator for the measurement of the rms value of a sine-wave with offset and harmonic components is proposed. The efficiency of this estimator is analyzed by means of computer simulations and experimental results as well.

2. PROPOSED ESTIMATOR FOR RMS VALUE MEASUREMENT

Let us consider a sine-wave $x(t)$ of amplitude A, frequency f (equal to $1/T$), phase φ, and offset d, with K harmonic components. The kth harmonic component is characterized by their amplitude A_k, frequency f_k (equal to kf), and phase φ_k. It is assumed that the noise of $x(t)$ is smaller than the offset and harmonic components. The signal $x(t)$ is sampled at the frequency f_s (equal to $1/T_s$) and M samples are acquired. Thus, the following discrete-time signal is obtained:

$$x(m) = A\sin\left(2\pi\frac{f}{f_s}m + \varphi\right) + d + \sum_{k=2}^{K} A_k \sin\left(2\pi\frac{f_k}{f_s}m + \varphi_k\right),\quad m = 0,1,\ldots,M-1 \tag{1}$$

The ratio between the frequencies f and f_s is:

$$\frac{f}{f_s} = \frac{J+\delta}{M} \tag{2}$$

where J and δ are respectively the integer part and the fractional part of the number of acquired sine-wave cycles, and $-0.5 \le \delta < 0.5$. When $\delta = 0$ the sampling process is coherent with the input sine-wave (coherent sampling mode). Conversely, the noncoherent sampling mode is characterized by $\delta \neq 0$. The latter mode is very common in practical applications.

The estimator proposed in [2] to measure the rms value of $x(\cdot)$, X_{rms}, is given by:

$$\hat{X}_{rms} = \frac{K_f \sum_{m=0}^{M-1} \left| x(m)\cdot w(m) \right|}{NPSG\cdot M} = \frac{K_f \sum_{m=0}^{M-1} \left| x_w(m) \right|}{NPSG\cdot M} \tag{3}$$

where $w(\cdot)$ is a window sequence, K_f is the form factor ($K_f = \pi / 2\sqrt{2}$), and $NPSG$ is the window normalized peak signal gain $\left(NPSG = \sum_{m=0}^{M-1} w(m)/M \right)$.

The sine-wave $x(\cdot)$ is multiplied by $w(\cdot)$ in order to reduce the bias of the rms value determined by the part of signal period δT at the end of $(J + \delta)T$. By this multiplication the signal $x_w(m) = x(m)\cdot w(m)$ is obtained.

The cosine windows are used in the estimator $\hat{X}_{rms}$. It should be noted that for these windows $NPSG$ is equal to the first window coefficient, a_0.

In [2] it has been proved by means of computer simulations and experimental results that for $d = 0$ and small total harmonic distortion (THD), X_{rms} is estimated with relative high accuracy by $\hat{X}_{rms}$.

In the following the case in which $d \neq 0$ and THD has an important value is investigated. Since the expression of $\hat{X}_{rms}$ contains the modulus of $x_w(\cdot)$ the relative error of X_{rms}, γ, is smaller than a upper limit, $\gamma_{\lim}$. For $\delta = 0$ this upper limit is given by (see (A.13) from Appendix):

$$\gamma_{\lim} = \frac{K_f |d|}{X_{rmsi}} + \sum_{k=2}^{K} \frac{X_{rmski}}{X_{rmsi}} \qquad (4)$$

where X_{rmsi} is the theoretical value of X_{rms} ($X_{rmsi} = A/\sqrt{2}$) and X_{rmski} is the theoretical value of rms value of the kth harmonic component ($X_{rmski} = A_k/\sqrt{2}$).

For $\delta \neq 0$ the expression of the upper limit it is very difficult to obtain and it will therefore not be derived.

From (4) it is obvious that the accuracy of the X_{rms} measurement increases as d and A_k (k = 2, 3,..., K) decrease. Thus, the best solution to obtain an accurate X_{rms} measurement is to remove the offset and the harmonic components from $x(\cdot)$. In order to remove the harmonic components it is necessarily to estimate each harmonic component parameters. This task is too complex and increases the complexity of the measurement of X_{rms} and therefore, it will not be made. On the other hand, d can be estimated by:

$$\hat{d} = \frac{X_w(0)}{M \cdot NPSG} = \frac{\sum_{m=0}^{M-1} x_w(m)}{M \cdot NPSG} \qquad (5)$$

where $X_w(0)$ is the first component (DC component) of the discrete Fourier transform (DFT) of $x_w(\cdot)$.

Thus, to increases the accuracy of the X_{rms} measurement, $\hat{d}$ is *a priori* removed from $x(\cdot)$ and X_{rms} is estimated by:

$$\tilde{X}_{rms} = \frac{K_f \sum_{m=0}^{M-1} \left| \left(x(m) - \hat{d} \right) \cdot w(m) \right|}{NPSG \cdot M}. \qquad (6)$$

3. SIMULATION AND EXPERIMENTAL RESULTS

The aim of this section is to determine the efficiency of the estimator $\tilde{X}_{rms}$ by means of computer simulation and experimental results.

3.1. Simulation results

Fig. 1 shows the maximum of the modulus of γ, $|\gamma|_{\max}$, as a function of δ and d (Fig. 1a), δ and THD (Fig. 1b), and d and THD (Fig. 1c).

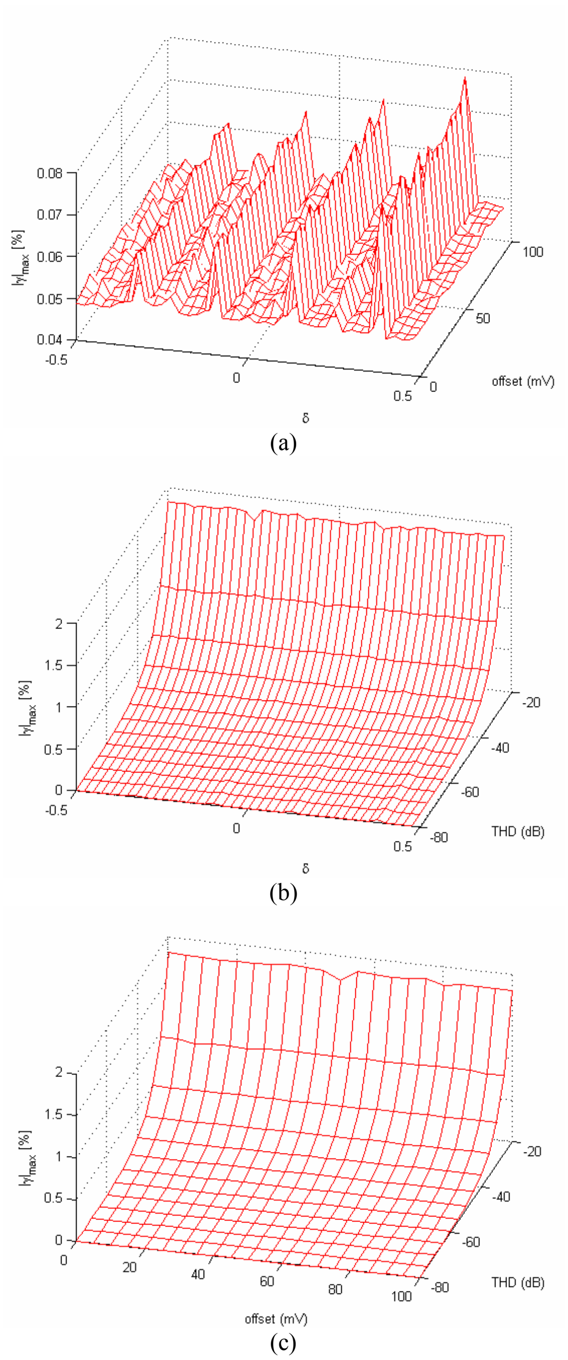


Fig. 1. $|\gamma|_{\max}$ as a function of: (a) δ and d, (b) δ and THD, (c) d and THD.

The modelled $x(\cdot)$ is characterized by: A = 1 V, J = 11 and M = 1024. The sine-wave has the 2nd and 3rd harmonic components of amplitude A_2 and $A_3 = A_2/2$. The phases of the sine-wave and harmonic components are uniformly distributed on [0, 2π) rad. The Hann window is used.

In Figs. 1a and 1b, δ varies in the range [–0.5, 0.5) with a step of 1/40 and in Fig. 1c, δ = 0.2. In Figs. 1b and 1c, *THD* varies in the range [–80, –20] dB with a step of –5 dB and in Fig. 1a, *THD* = –50 dB. In Figs. 1a and 1c, d varies in the range [0, 100] mV with a step of 5 mV, and in Fig. 1b, d = 50 mV. The signal is applied to an ideal digitizer with a 14-bit bipolar analog-to-digital converter (ADC). The full-scale range (*FSR*) of the ADC is equal to 5 V. Thus, the quantization noise of the ADC is the only noise which affects the sine-wave. This is modelled by a uniform additive noise. For each δ and d (Fig. 1a), δ and *THD* (Fig. 1b), and d and *THD* (Fig. 1c), $|\gamma|_{max}$ occurring during the phases variation is retained. Each time 100 runs are done.

As expected, the variation of d does not affect very much the estimation of X_{rms} - the greater $|\gamma|_{max}$ obtained from the results shown in Fig. 1 is equal to 0.08%, which is a small error. On the other hand, $|\gamma|_{max}$ increases as the *THD* increases. Thus, the worst case is obtained for *THD* = –20 dB, when $|\gamma|_{max}$ is equal to 1.84% (in both Figs. 1b and 1c), which is an acceptable error. However, it should be noted that for a smaller *THD* more accurate measurements of X_{rms} are obtained – for example for *THD* = –30 dB, the greater $|\gamma|_{max}$ is equal to 0.3% (Fig. 1b), which is a relative small error.

Many other simulations were performed for different values of A (smaller than *FSR*/2) and J (higher than 5) and in all cases a behaviour like the one depicted in Fig. 1 was always observed.

3.2. Experimental results

The results obtained using the estimator $\tilde{X}_{rms}$ are compared with those obtained using the interpolated DFT (IpDFT) method [5]-[8]. The IpDFT method provides very accurate measurements of the amplitude of a sine-wave (and also of the rms value). In both cases the Hann window is employed. For this purpose a graphical interface has been also implemented using MATLAB. In this graphical interface the acquired sine-wave and its spectrum are shown. To compute the spectrum the 4-term minimum error energy window is employed [4]. The rms values obtained using the estimator $\tilde{X}_{rms}$ and the IpDFT method are also given.

The sine-waves are obtained from different signal generators. The acquisition system has a 14-bit bipolar ADC with *FSR* equal to 6 V. The sampling frequency is equal to 48.077 kHz.

The sine-waves are first obtained from the Agilent 33220A signal generator. The sine-waves parameters are: amplitude 1.5 V, frequency 1.2 kHz and offset 100 mV. The user's manual specifies a *THD* smaller than 0.04% at this frequency. A number of 25 records are collected of M = 1024 samples each. Fig. 2(a) shows, for each record, the results obtained using the estimator $\tilde{X}_{rms}$ and the IpDFT method. Fig. 2(b) shows by means of the graphical interface the results obtained for the 13th acquired sine-wave.

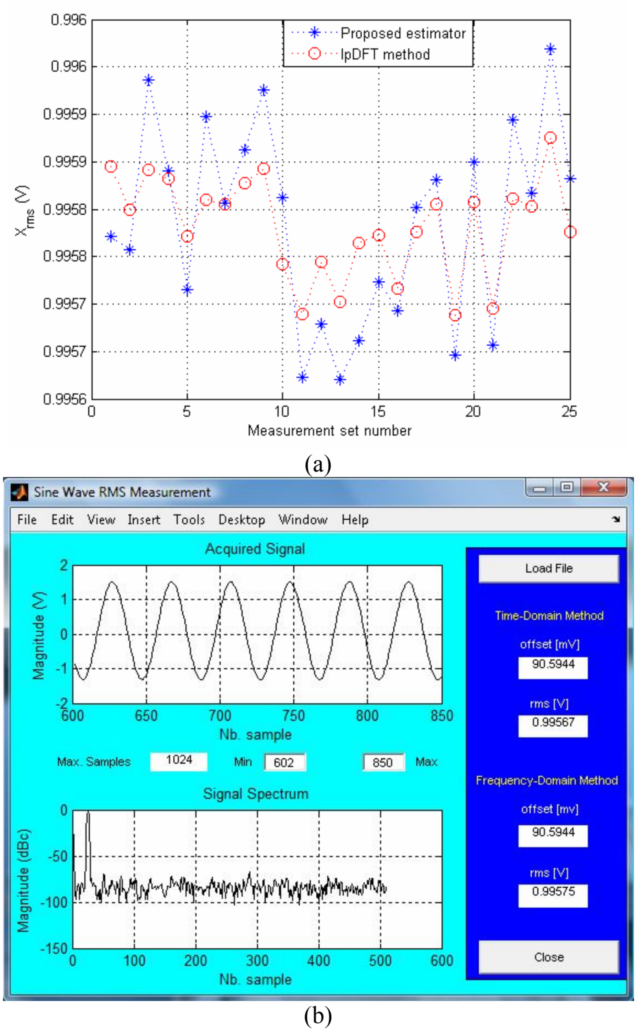
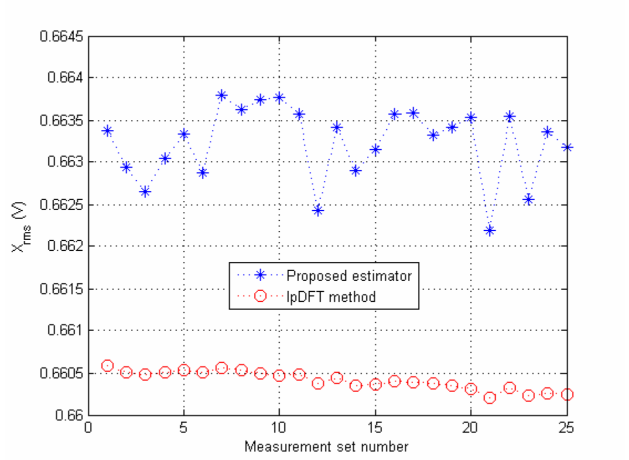


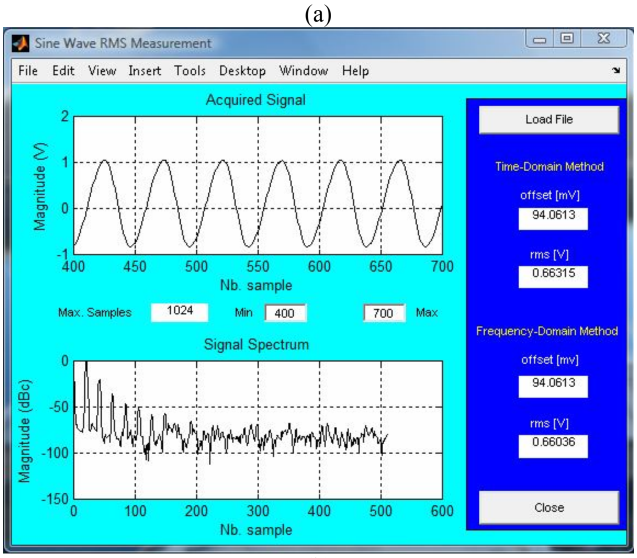
Fig. 2. For the sine-waves obtained from Agilent 33220A signal generator: (a) The values of the X_{rms} measurements obtained using the estimator $\tilde{X}_{rms}$ and the IpDFT method for each record, (b) The results obtained for the 13th acquired sine-wave.

For each record, the results obtained using the estimator $\tilde{X}_{rms}$ differs from those obtained using the IpDFT method beginning to the fourth digit after the decimal point. Thus, X_{rms} is accurately measured using the estimator $\tilde{X}_{rms}$. This behaviour is achieved because the acquired sine-waves have a high spectral purity (see Fig. 2(b)).

Moreover, the accuracy of the proposed estimator is investigated for sine-waves with important harmonic components. For this purpose asymmetric sine-waves are acquired from the TG315 signal generator. The amplitude of the signals is 1 V, the frequency is 1 kHz and the offset is 100 mV. The asymmetry ensures a high *THD*. A number of 25 records are collected of M = 1024 samples each. Fig. 3(a) shows, for each record, the results obtained using the estimator $\tilde{X}_{rms}$ and the IpDFT method. Fig. 3(b) shows by means of the graphical interface the results obtained for the 15th acquired signal.



(a)



(b)

Fig. 3. For asymmetric sine-waves obtained from the TG315 signal generator: (a) The values of the X_{rms} measurements obtained using the estimator $\widetilde{X}_{rms}$ and the IpDFT method for each record, (b) The results obtained for the 15th acquired signal.

From the spectrum of the 15th acquired signal presented in Fig. 3(b) it follows that after the fundamental the highest spectral line is equal to –21.07 dB, which is a high value. Even in this case the proposed estimator provides relative accurate rms measurements since they differs from those obtained using the IpDFT method beginning to the third digit after the decimal point.

4. CONCLUSION

In this paper an estimator for measurement the rms value of a noncoherent sampled sine-wave is proposed. The signal used in the estimation is the sine-wave from which the offset, *a priori* estimated, is removed. The performed simulations and experimental results confirm that the proposed estimator has a relative high efficiency for sine-wave with THD smaller than or equal to –30 dB.

Moreover, the proposed estimator is very simple to implement. Therefore, this is well suited for real-time measurement of the rms value of a discrete-time sine-wave.

APPENDIX

Determination the limit of the relative error of X_{rms}, γ_{lim}

Let us assume that the sine-wave is coherently sampled, i.e. $\delta = 0$. The theoretical rms value of the continual-time signal $x_w(t)$ is given by:

$$X_{wrms} = \frac{K_f}{JT}\int_0^{JT}|x(t)\cdot w(t)|dt = \frac{K_f}{JT}\int_0^{JT}\left|A\sin(2\pi ft+\varphi)+\sum_{k=2}^{K}A_k\sin(2\pi kft+\varphi_k)+d\right|\cdot|w(t)|dt. \tag{A.1}$$

Assuming that $w(\cdot)$ is a H-term cosine window with the coefficients a_i, $i = 0, 1,\ldots, H-1$. In the time-domain $w(\cdot)$ is defined as:

$$w(t) = \sum_{h=0}^{H-1} a_h \cos\left(2\pi\frac{2ht}{JT}\right). \tag{A.2}$$

It should be noticed that for a cosine window $|w(t)| = w(t)$ and $NPSG = a_0$.

From (A.1) the following inequality can be established:

$$X_{wrms} \le \frac{K_f}{JT}\left[\int_0^{JT}|A\sin(2\pi ft+\varphi)|\cdot w(t)dt + \sum_{k=2}^{K}\int_0^{JT}|A_k\sin(2\pi kft+\varphi_k)|\cdot w(t)dt + |d|\int_0^{JT}w(t)dt\right]. \tag{A.3}$$

We have:

$$\begin{aligned}&\int_0^{JT}|A_k\sin(2\pi kft+\varphi_k)|\cdot w(t)dt\\ &= \int_0^{JT}|A_k\sin(2\pi kft+\varphi_k)|\sum_{h=0}^{H-1}a_h\cos\left(2\pi\frac{2ht}{JT}\right)dt\\ &= \sum_{h=0}^{H-1}a_h\cdot\int_0^{JT}|A_k\sin(2\pi kft+\varphi_k)|\cos\left(2\pi\frac{2ht}{JT}\right)dt\\ &= a_0\int_0^{JT}|A_k\sin(2\pi kft+\varphi_k)|dt\\ &+ \sum_{h=1}^{H-1}a_h\cdot\int_0^{JT}|A_k\sin(2\pi kft+\varphi_k)|\cos\left(2\pi\frac{2ht}{JT}\right)dt.\end{aligned} \tag{A.4}$$

The first integral from the last expression of (A.4) is given by:

$$\int_0^{JT} \left| A_k \sin(2\pi kft + \varphi_k) \right| dt = 2kJ \int_0^{T/2k} A_k \sin(2\pi kft) dt$$
$$= \frac{2A_k JT}{\pi}. \tag{A.5}$$

For $0 \le \varphi_k < \pi$ the second integral from the last expression of (A.4) is given by:

$$\sum_{h=1}^{H-1} a_h \cdot \int_0^{JT} \left| A_k \sin(2\pi kft + \varphi_k) \right| \cos\left(2\pi \frac{2ht}{JT}\right) dt$$
$$= \sum_{h=1}^{H-1} a_h \cdot \left[\int_0^{t_1} A_k \sin(2\pi kft + \varphi_k) \cos\left(2\pi \frac{2ht}{JT}\right) dt \right.$$
$$+ \sum_{i=1}^{2kJ-1} \int_{t_i}^{t_{i+1}} (-1)^i A_k \sin(2\pi kft + \varphi_k) \cos\left(2\pi \frac{2ht}{JT}\right) dt \tag{A.6}$$
$$\left. + \int_{t_{2kJ}}^{JT} A_k \sin(2\pi kft + \varphi_k) \cos\left(2\pi \frac{2ht}{JT}\right) dt \right]$$

where $t_i = \frac{i\pi - \varphi_k}{2\pi k} T,\ i = 1, 2, \ldots, 2kJ.$

After some algebra we obtain:

$$\sum_{h=1}^{H-1} a_h \cdot \int_0^{JT} \left| A_k \sin(2\pi kft + \varphi_k) \right| \cos\left(2\pi \frac{2ht}{JT}\right) dt$$
$$= \frac{A_k T}{2\pi} \cdot \left(\frac{1}{k + \frac{2h}{J}} + \frac{1}{k - \frac{2h}{J}} \right) \cdot \sum_{i=0}^{2kJ-1} \cos\left((i\pi - \varphi_k) \frac{2h}{Jk} \right) = 0. \tag{A.7}$$

It should be noticed that the same result is achieved for $\pi \le \varphi_k < 2\pi$. Thus, from (A.4) and (A.7) it can be established:

$$\sum_{k=2}^{K} \int_0^{JT} \left| A_k \sin(2\pi kft + \varphi_k) \right| \cdot w(t) dt = \sum_{k=2}^{K} \frac{2A_k JT}{\pi} a_0$$
$$= \sum_{k=2}^{K} \frac{2A_k JT}{\pi} NPSG. \tag{A.8}$$

By a similar demonstration the following equality is achieved:

$$\int_0^{JT} \left| A \sin(2\pi ft + \varphi) \right| \cdot w(t) dt = \frac{2AJT}{\pi} a_0 = \frac{2AJT}{\pi} NPSG. \tag{A.9}$$

The last integral from (A.3) is given by:

$$\int_0^{JT} w(t) dt = \int_0^{JT} \left(\sum_{h=0}^{H-1} a_h \cos\left(2\pi \frac{ht}{JT}\right) \right) dt$$
$$= \sum_{h=0}^{H-1} a_h \cdot \int_0^{JT} \cos\left(2\pi \frac{ht}{JT}\right) dt = JTa_0 \tag{A.10}$$
$$+ \sum_{h=1}^{H-1} a_h \cdot \int_0^{JT} \cos\left(2\pi \frac{ht}{JT}\right) dt = JTa_0 = JT \cdot NPSG.$$

Using (A.8) - (A.10), (A.3) becomes:

$$X_{wrms} \le \left(X_{rmsi} + K_f |d| + \sum_{k=2}^{K} X_{rmski} \right) \cdot NPSG \tag{A.11}$$

where X_{rmsi} is the theoretical value of X_{rms} ($X_{rmsi} = A/\sqrt{2}$) and X_{rmski} is the theoretical value of rms value of the kth harmonic component ($X_{rmski} = A_k/\sqrt{2}$).

From the above expression it follows that the limit of the X_{wrms}, $X_{wrms\lim}$, is given by:

$$X_{wrms\lim} = \left(X_{rmsi} + K_f |d| + \sum_{k=2}^{K} X_{rmski} \right) \cdot NPSG \tag{A.12}$$

Based on (A.12) the limit of the relative error of X_{ms}, $\gamma_{\lim}$, can be determined:

$$\gamma_{\lim} = \frac{\frac{X_{wrms\lim}}{NPSG} - X_{rmsi}}{X_{rmsi}} = \frac{K_f |d|}{X_{rmsi}} + \sum_{k=2}^{K} \frac{X_{rmski}}{X_{rmsi}}. \tag{A.13}$$

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