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# VOLUME RATIO DETERMINATION IN STATIC EXPANSION SYSTEMS BY MEANS OF TWO PRESSURE BALANCES

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**Abstract** – Static expansion systems are used to generate known pressures in the high and medium vacuum range. In these systems the volume ratios are the crucial parameters and have to be determined very accurately. For this task different methods can be used. We have used the gas accumulation method with a dead weight pressure balance, a force-balanced piston gauge (FPG) and a differential pressure transducer (CDG 133.32 Pa). The final uncertainty of the expansion ratios obtained with this method is similar to other institutes uncertainties.

**Keywords**: vacuum, expansion.

## 1. INTRODUCTION

Static expansion systems or series expansion systems are used as primary standards for generating pressures in the high and medium vacuum range [1-2].

These systems are usually a set of different vessels connected via pipes and valves. In order to generate different pressures, a known gas enclosed in a small volume is expanded into a much larger evacuated volume by opening a valve between both volumes with the resulting decrease in the final pressure.

Under isothermal conditions, the gas pressure is reduced by the ratio of the small volume to the sum of the small and the large volumes. This ratio is called expansion ratio and its inverse is called volume ratio ($V_0$ is the small volume and ($V_1$ is the large volume).

$$f = \frac{V_0}{V_0 + V_1} \qquad \frac{1}{f} = \frac{V_0 + V_1}{V_0} \tag{1}$$

In most static expansion systems several different expansion ratios are available and expansions are carried out in series to generate very low pressures. The expansion ratios are the crucial parameters in all static expansion systems and have to be determined very accurately.

There are several techniques for the accurate determination of expansion ratios. The main ones are the gravimetric technique and the expansion technique.

In the gravimetric technique the unknown volume is weighed both empty and filled with a suitable liquid of known density, usually distilled water. The unknown volume is calculated from the weighing results and the liquid density, which depends basically on temperature.

In the expansion technique the pressures before and after the expansion are measured. It is possible to use two calibrated gauges [3] or a single uncalibrated gauge with strictly linear pressure response [4].

This paper shows a practical procedure for the determination of the volume ratio by means of a force-balanced piston gauge (FPG), a dead weight pressure balance and a differential pressure transducer (CDG 133.32 Pa).

## 2. EXPERIMENTAL PROCEDURE

Static expansion system at Spanish Metrology Centre (CEM) consists of five vessels: two 100 litre-volumes, two1 litre-volumes and one 0.5 litre-volume. Fig. 1 shows schematically the static expansion system.

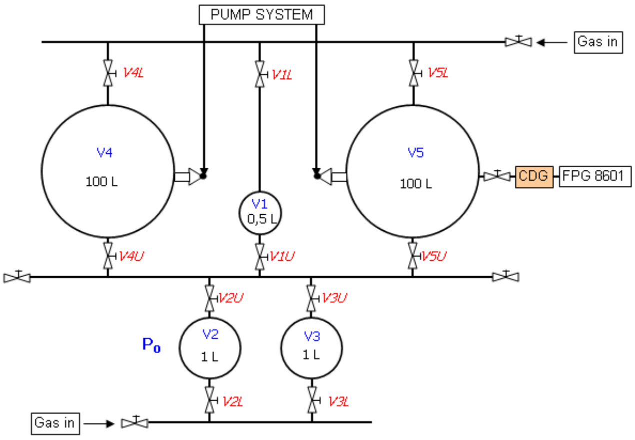

Fig. 1. Scheme of the static expansion system at CEM, which is used as primary standard to generate pressures from $10^{-4}$ Pa to 1000 Pa. Volumes are denoted with normal letters and numbers and valves with slanted letters.

In order to generate different pressures several expansion ratios are used. There are 12 expansion routes available.

Table 1 shows the static expansion routes and theoretical volume ratios.

Table 1. Static expansion system theoretical ratios at CEM (*Vc* is a volume of pipes between vessels)

| Nº | Expansion route | | Volume ratios | |
|---|---|---|---|---|
| | Inicial Volume | Final Volume | Theoretical ratio | |
| | | | f | 1/f |
| 1 | V1 | V1+Vc+V5 | 0,00497 | 201,20724 |
| 2 | V1 | V1+Vc+V4 | 0,00497 | 201,20724 |
| 3 | V1+Vc | V1+Vc+V5 | 0,00616 | 162,33766 |
| 4 | V1+Vc | V1+Vc+V4 | 0,00616 | 162,33766 |
| 5 | V2 | V2+Vc+V5 | 0,00989 | 101,11223 |
| 6 | V2 | V2+Vc+V4 | 0,00989 | 101,11223 |
| 7 | V2+Vc | V2+Vc+V5 | 0,01108 | 90,25271 |
| 8 | V2+Vc | V2+Vc+V4 | 0,01108 | 90,25271 |
| 9 | V3 | V3+Vc+V5 | 0,00989 | 101,11223 |
| 10 | V3 | V3+Vc+V4 | 0,00989 | 101,11223 |
| 11 | V3+Vc | V3+Vc+V5 | 0,01108 | 90,25271 |
| 12 | V3+Vc | V3+Vc+V4 | 0,01108 | 90,25271 |

### *2.1. Determination of volume ratios*

Using the ideal gas law the volume ratio may be determined with the initial pressure, the final pressure after expansion and the initial and final temperature. However, the resulting pressure is often too low to be measured with good accuracy using a pressure gauge after a single expansion.

A good solution to solve this problem is the succesive accumulation of higher pressures in the large vessel. This is achieved by repeatedly refilling the small vessel and expanding the gas into the large vessel, hence increasing its pressure. This procedure is repeated until the resultant pressure in the large vessel is high enough to be measured with sufficiently low uncertainty.

Under isothermal conditions, the final pressure after *n* expansions is (2) [5]

$$p_n = p_0[1-(1-f)^n] \qquad (2)$$

where $p_0$ is the initial pressure and *f* is the expansion ratio.

The number of expansions varies among laboratories. The expansions performed at NPL are 30 and the expansions performed at PTB are 25. At CEM we have fixed 25 expansions as standard method.

Equation (2) is valid for expansion ratio only if ideal gas and isothermal conditions are assumed. However, correction because temperature differences between vessels and temperature drift have to performed.

As in the PTB report [5], we assume that the initial pressure $p_0$ is constant and we can calculate the expansion ratio with temperature corrected as (3)

$$f = 1-\left(1+f\alpha-\frac{(p_1)_n}{p_0}\right)^{\frac{1}{n}} \qquad (3)$$

where $\alpha$ << 1 is given by

$$\alpha = \left[\begin{array}{l}\left(\dfrac{\Delta T}{T_0}\right)+\dfrac{V_1}{V_0+V_1}\dfrac{(\Delta T)_n}{(T_0)_{n-1}}+\left(\dfrac{V_1}{V_0+V_1}\right)^2\dfrac{(\Delta T)_n}{(T_0)_{n-1}}\\ +\cdots+\left(\dfrac{V_1}{V_0+V_1}\right)^{n-1}\left(\dfrac{(\Delta T)_n}{(T_0)_1}\right)\end{array}\right] \qquad (4)$$

and

$$\frac{V_1}{V_0+V_1} = 1-f \qquad (5)$$

All quantities indexed with 0 refer to the initial small volume and all quantities indexed with 1 to the large volume. $(T_0)_1$ denotes the temperature of the small vessel before expansion 1 when $p_0$ is determinated, $(T_1)_1$ the temperature of the large vessel after the first expansion when $p_1$ is measured as $(p_1)_1$.

To solve (3) for *f* we assume the algorithm described in the PTB report [5]:

1. Calculation of *f* without the temperature correction term ($\alpha = 0$) according to Eq. (2).
2. Use this approximate value of *f* to calculate $\alpha$ according to (4) and (5).
3. Insert $\alpha$ and the uncorrected *f* in the right-hand side of (3) to obtain the temperature-corrected *f*.

Steps 2 and 3 are repeated until the difference in *f* is negligible. This procedure works well, because $f\alpha$ is much smaller than both 1 and the pressure ratio $p_1/p_0$ ($\alpha$ is typically of the order of $10^{-4}$ while $p_1/p_0$ is of the order of 0.01...0.1).

### *2.2. Expansion technique using gas accumulation method with two pressure balances*

The measurement procedure for determination of volume ratios with the gas accumulation method using two pressure balances is described as follows.

In order to explain the method we are going to use the following expansion as an example (6)

$$f_1 = \frac{V_1}{V_1+V_c+V_5} \qquad (6)$$

Firstly, pressure (10 040 Pa $N_2$) is established in $V_1$ with the dead weight pressure balance with valve *V1L* open and valve *V1U* closed. After closing *V1L* slowly, (the pressure is maintained constant using a variable volume), the temperature on $V_1$ is measured. Secondly, valve *V1U* is opened and the gas is expanded into the evacuated volumes $V_c$ and $V_5$. After a while the temperatures on $V_5$ and the differential pressure transducer (CDG) indication are read out. After that, the force piston gauge in mode absolute, which is connected to the other side of the differential pressure transducer (CDG), re-establishes the cero pressure in the CDG by increasing pressure in the CDG reference side.

For the following expansions valve *V1U* is closed, the pressure of 10 040 Pa is re-established, the temperature on $V_1$ is measured and the previous procedure is performed again. Since $V_1$ and $V_5$ are not evacuated between the expansions, the final pressure $p_f$ is the addition of the pressure contributions measured by the FPG (1261,31 Pa).

## 3. RESULTS

The results of the expansion ratios obtained with this method are shown in Table 2 for different expansions routes.

Table 2. Table of the static expansion system ratios obtained at Spanish Metrology Centre (CEM).

| | Expansion ratio | Volume ratio | Expanded uncertainty | Relative expanded uncertainty |
|---|---|---|---|---|
| Nº | f | 1/f | U(f) | W(f) |
| 1 | 0,005 353 3 | 186,801 | 1,2E-06 | 2,3E-04 |
| 2 | 0,005 419 0 | 184,536 | 1,3E-06 | 2,4E-04 |
| 3 | 0,007 787 0 | 128,419 | 2,0E-06 | 2,6E-04 |
| 4 | 0,007 801 0 | 128,188 | 2,1E-06 | 2,7E-04 |
| 5 | 0,010 702 9 | 93,433 | 2,6E-06 | 2,5E-04 |
| 6 | 0,010 835 3 | 92,291 | 2,6E-06 | 2,4E-04 |
| 7 | 0,013 086 0 | 76,418 | 3,4E-06 | 2,6E-04 |
| 8 | 0,013 081 5 | 76,444 | 3,8E-06 | 2,9E-04 |
| 9 | 0,010 840 5 | 92,247 | 2,7E-06 | 2,5E-04 |
| 10 | 0,010 720 2 | 93,282 | 3,5E-06 | 3,2E-04 |
| 11 | 0,013 322 2 | 79,546 | 3,0E-06 | 2,5E-04 |
| 12 | 0,013 343 2 | 74,945 | 3,1E-06 | 2,4E-04 |

The uncertainties of the expansion ratios which have been obtained obtained with the gas accumulation method with two pressure balances are similar to uncertainties which have been achieved by other institutes.

The temperature uncertainty contribution at expansion ratio is about $1,2 \times 10^{-4} \times f$, $k = 2$. This term is related to temperature difference between vessels and temperature drift in each expansion.

Although the uncertainties of pressures which have been measured by pressure balances are lower than the uncertainties which have been measured by other gauges there is not a final uncertainty improvement due to temperature effects.

## 4. CONCLUSIONS

A variation of the gas accumulation method has been presented. Although the uncertainties which have been obtained are similar to the ones obtained by other institutes, its main advantage is that this method gets direct traceability from piston cylinder assemblies.

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