

## ON-LINE DETERMINATION OF THE MEASUREMENT UNCERTAINTY OF THE STOCHASTIC MEASUREMENT METHOD

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**Abstract** – The paper presents a stochastic sampling method applied to a measurement of the mean value of the product of two arbitrary signals. This method is especially convenient for measuring electric power, electric energy consumption, as well as root-mean-square of voltage and/or current. The method features a very simple hardware and the possibility of selecting the measurement accuracy by choosing a suitable measurement interval. When measuring electric energy, accuracy better than 0.01 % is achieved.

The measurement uncertainty of the stochastic sampling method is analysed and the influence of dominant factors is quantitatively derived. Special consideration is given to the influence of dispersion of A/D converter outputs and the waveforms of the two input signals as contributors to the measurement uncertainty. As the exact shapes of the waveforms are not known in advance, it is not possible to determine their influence on the measurement uncertainty. To overcome this issue, a novel additional hardware is designed, which provides the estimate of measurement uncertainty during the experiment - simultaneously with the measurement result, its measurement uncertainty is available on-line.

**Keywords:** stochastic measurement, sampling method, measurement uncertainty.

### 1. INTRODUCTION

The common way to measure constant and variable signals is the application of sampling methods – A/D converters, usually of sufficient resolutions and speeds, as well as powerful DSPs, are readily available. The fast progress of digital hardware, as well as falling prices of these components, make this measurement concept to be dominant nowadays.

However, in some areas of signal measurement this concept is not superior. Following the ideas of [1], methods for stochastic sampling and signal processing have been developed [2,3,4]. A number of stochastic sampling instruments [3,4] has been developed for either generic or very specific applications. These achieved higher accuracy, lower device complexity, higher measurement speeds, efficient multi-channel measurements and/or lower instrument price.

The stochastic instruments are based on utilisation of very-fast low-resolution A/D converters (flash converters).

Their quite simple hardware produces a small number of systematic errors (mainly due to comparator offsets) that can be easily kept in check. Discretization error, inherently large due to a low resolution, is suppressed by superimposing random dithering signal onto the measured input. This signal sum is converted by the flash A/D converter and then very quickly processed by simple logic circuitry into the measurement result.

The same instrument can be used either for a high accuracy measurement over a longer time interval or for moderate accuracy for fast measurement. The realised instruments, employing low-resolution A/D converters, for 50 Hz r.m.s. voltage measurements [2,3] achieve the relative accuracy better than 0.1 % for twenty milliseconds and approximately 0.01 % for one second time interval. When the measurement result is an integral of the input signal (for instance, electric energy measurement), the time interval is not a limiting factor and the relative accuracy of a even 2-bit instrument can be below 0.01 % when the time interval is greater than half an hour [3].

The estimation of the measurement uncertainty of these stochastic methods should be in accordance with [5] and should point out the contribution of individual influencing factors. As most factors (sampling frequency, sampling resolution, measurement time interval, etc.) are known in advance, their contribution can be theoretically quantified. Numerous detailed computer simulations confirm the theoretically predicted uncertainties. However, some factors, like the exact waveforms of the measured signals, are not fully predictable for every individual measurement. Hence their influence on the uncertainty estimate escapes from the theoretical considerations – the only tool left is the computer simulation of a particular waveform, but this is only available in the post-processing manner.

The utilised stochastic methods already have sufficient information for estimating the influence of such factors onto the measurement uncertainty. This has been the motivation to develop theoretical methods and experimental devices for on-line estimation of such influences. One such procedure for estimation of the measurement uncertainty when the signal waveform is not known in advance and an additional hardware that executes it, is presented in this paper. The instrument is primarily developed for accurate measurement of electric energy, but the novel method and the proposed additional hardware can be applied to a whole class of similar tasks.

## 2. STOCHASTIC MEASUREMENT METHOD

The block-diagram of one stochastic sampling two-channel instrument is shown in Fig. 1 [2,3]. The two input signals,  $y_1$  and  $y_2$ , which are arbitrary time-varying voltages, are brought to the two instrument inputs. Two random, uniform, uncorrelated dithers  $h_1$  and  $h_2$ , with probability density distributions  $p(h_1)$  and  $p(h_2)$ , are superimposed onto the two inputs. The DC voltage levels  $g$  and  $-g$  serve as threshold levels for comparators  $C_1$ ,  $C_2$ ,  $C_3$  and  $C_4$ . For proper operation, the following conditions must be satisfied:

$$\begin{aligned} |y_i| &\leq 2g \\ |h_i| &\leq g \quad (i=1,2) \\ p(h_i) &= 1/2g \end{aligned} \quad (1)$$

The level limiter (LL) blocks adjust the comparator outputs to digital TTL level. Comparators  $C_1$  and  $C_2$  with their LL blocks make up one A/D 2-bit flash converter with the resolution of  $\Delta = 2g$ . The conversion result is a 2-bit numerical variable  $\Psi_1$ , defined as  $\Psi_1 = b_{11} - b_{-11}$ , which can assume a value from the following set  $\{-1, 0, 1\}$ . In the same manner, comparators  $C_3$  and  $C_4$  and their LL blocks make up the second flash A/D converter, whose output result is the numeric variable  $\Psi_2$ ,  $\Psi_2 = b_{12} - b_{-12}$ .

Logic signals  $b_{11}$ ,  $b_{-11}$ ,  $b_{12}$  and  $b_{-12}$  are the inputs to the logic network made up of AND and OR circuits which acts as a hardware multiplier of variables  $\Psi_1$  and  $\Psi_2$ . The result is the stochastic variable  $\Psi$ ,  $\Psi = \Psi_1 \cdot \Psi_2$  which also assumes the value from the set  $\{-1, 0, 1\}$ . When the product is  $\Psi = 1$ , the counter is incremented; when  $\Psi = -1$ , the counter is decremented and when  $\Psi = 0$ , the counter state remains unchanged.

The average value measurement result,  $\bar{\Psi}$ , is [2]:

$$\Delta^2 \bar{\Psi} = \frac{\Delta^2}{N} \sum_{i=1}^N \Psi_1(i) \cdot \Psi_2(i) \approx \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} f_1(t) f_2(t) dt \quad (2)$$

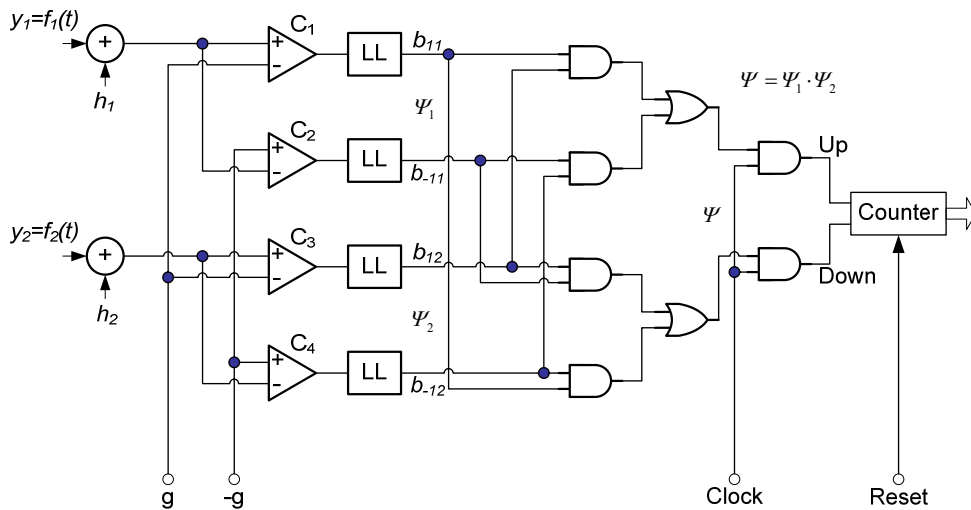


Fig. 1. Two-channel 2-bit stochastic sampling instrument

where  $\Psi_1(i)$  and  $\Psi_2(i)$  are the consecutive results of A/D conversions while  $N$  is the number of samples within the measurement interval  $(t_2 - t_1)$ .

In order to obtain a deeper insight into the measurement uncertainty, let us introduce the instantaneous error  $e$ , which is the difference between the sampled and the actual values of the product of the two signals:

$$e = \Delta^2 \Psi - y_1 y_2 \quad (3)$$

It can be shown that the two terms in (3) are statistically independent, hence the average value of the multiplier is:

$$\Delta^2 \bar{\Psi} = \overline{y_1 y_2} + \bar{e} \quad (4)$$

and the variance of  $\Psi$  is given as:

$$\Delta^2 \sigma_{\Psi}^2 = \sigma_{y_1 y_2}^2 + \sigma_e^2 \quad (5)$$

It can also be shown that the third central moment  $M_3$  of the measurement error  $e$  is limited, i.e.

$$M_3 = \sqrt{|(e - \bar{e})^3|} \leq K \quad (6)$$

and hence both the Central limit theorem and the Theory of samples apply to the instantaneous error  $e$ . Consequently, the variance of its average value is:

$$\sigma_e^2 = \frac{\sigma_e^2}{N} \quad (7)$$

From (3) and (7), the variance of the average value of the measurement error  $\bar{e}$  can be obtained as:

$$\begin{aligned} \sigma_{\bar{e}}^2 &= \frac{\Delta^2 \sigma_{\Psi}^2 - \sigma_{y_1 y_2}^2}{N} \\ &= \frac{1}{N} \left\{ \frac{\Delta^2}{t_2 - t_1} \int_{t_1}^{t_2} |f_1(t) f_2(t)| dt - \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} f_1^2(t) f_2^2(t) dt \right\} \quad (8) \end{aligned}$$

To sum up, the mathematical expectation of  $e$  is zero (from (2) and (4)), its variance is given by (8) and its probability density distribution is normal.

If the waveforms  $f_1(t)$  and  $f_2(t)$  are known, then the variance given by (8) can be exactly calculated. Then the standard measurement uncertainty,  $u$ , of type A, according to [5], defined as:

$$u = \sigma_e \quad (9)$$

can be exactly calculated as well.

If the waveforms  $f_1(t)$  and  $f_2(t)$  are not known, the variance (8) cannot be calculated. However, it can be shown that the measurement method carries sufficient information to enable experimental on-line determination of the variance.

This extended abstract analyses only the contribution of A/D conversion to the total measurement uncertainty, as it is the dominant factor.

### 3. ON-LINE DETERMINATION OF MEASUREMENT UNCERTAINTY

The continuous integration in (8) can be substituted by a digital integration, hence (8) becomes:

$$\sigma_e^2 \approx \frac{\Delta^4}{N} \sum_{i=1}^N |\Psi_1(i) \cdot \Psi_2(i)| - \frac{\Delta^4}{N} \sum_{i=1}^N \Psi_1^2(i) \cdot \Psi_2^2(i) \quad (10)$$

If the two sums in (10) are denoted as  $I_1$  and  $I_2$ , the variance can be rewritten as:

$$\sigma_e^2 = \frac{\Delta^4}{N} I_1 - \frac{\Delta^4}{N} I_2 \quad (11)$$

To determine the standard deviation  $\sigma_e$  for the unknown waveforms of measured input signals, (11) can be utilised in a dedicated additional hardware. With the known parameters  $\Delta$  and  $N$ , it only remains to determine the sums  $I_1$  and  $I_2$ . For that purpose, circuits of Figs. 2 and 3, are proposed.

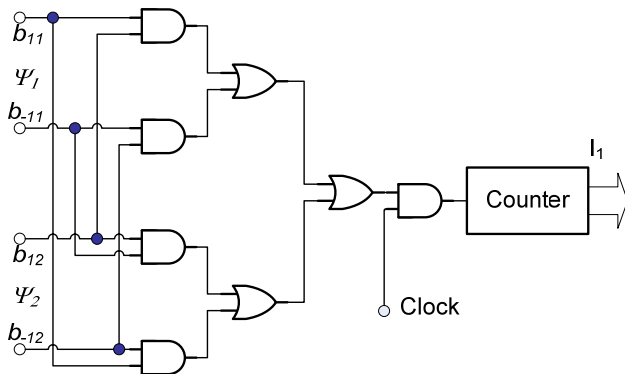


Fig. 2. The circuit for determination of  $I_1$  quantity

Combination of the basic instrument structure of Fig.1 with the additional circuits of Figs. 2 and 3, represents a

major improvement when measuring unknown input waveforms. After taking  $N$  samples of voltages  $y_1$  and  $y_2$  in the time interval  $(t_1, t_2)$  the main result is the mean value of the two signal product – the ratio of the content of the counter of Fig.1 and the number of samples  $N$ . At the same time, the measurement uncertainty is obtained using the contents of the two counters of Figs. 2 and 3 and applying equation (11).

### 4. INITIAL CASE STUDY

The described stochastic sampling method can be utilised for measurement of two arbitrary signals. It is useful to start the analyses of achieved measurement uncertainties for well-known and commonly met in practice signals. This paper describes the stochastic method variant aimed for measurements of electric power and electric energy. Hence signals  $y_1$  and  $y_2$  represent the AC voltage and AC current, with amplitudes  $V$  and  $I$ , respectively, and the mean power is measured.

To illustrate the contribution of A/D conversion process to the measurement uncertainty, some operating conditions have to be defined. The prototype instrument has 2-bit flash A/D converters, operating at 160 kHz sampling frequency. It is assumed that both voltage and current signals are purely sinusoidal 50 Hz waves and they are in phase.

For the above conditions, the variance due to A/D conversion process is obtained from (8) as:

$$\sigma_e^2 = \frac{V^2 I^2}{8N} \quad (12)$$

and its contribution to the relative standard measurement uncertainty is:

$$u_r = \frac{\sigma_e}{P} = \frac{\sigma_e}{VI/2} = \frac{1}{\sqrt{2N}} \quad (13)$$

Equation (13) indicates that the measurement uncertainty due to A/D conversion process decays with the increase in the number of samples, i.e. with the increase of the measurement time interval and/or sampling frequency. The illustration of absolute values, for the above conditions, is given in Fig. 4, showing the relative measurement uncertainty  $u_r$  as a function of measurement time interval. As an example, in order to achieve the relative (expanded) measurement uncertainty  $U = 0.01\%$  ( $U = k \cdot u$  where the coverage factor of  $k = 2$  is chosen), a time interval of 1250 s is necessary. For one day of measurements, the  $U = 0.00012\%$  is achieved.

It should be noted that measurement interval of few hours or few days are quite common in measurement of electric energy consumption.

The final version of the paper will present measurement uncertainty for typical cases of distorted voltages and/or currents, as well as for non-unity power factors.

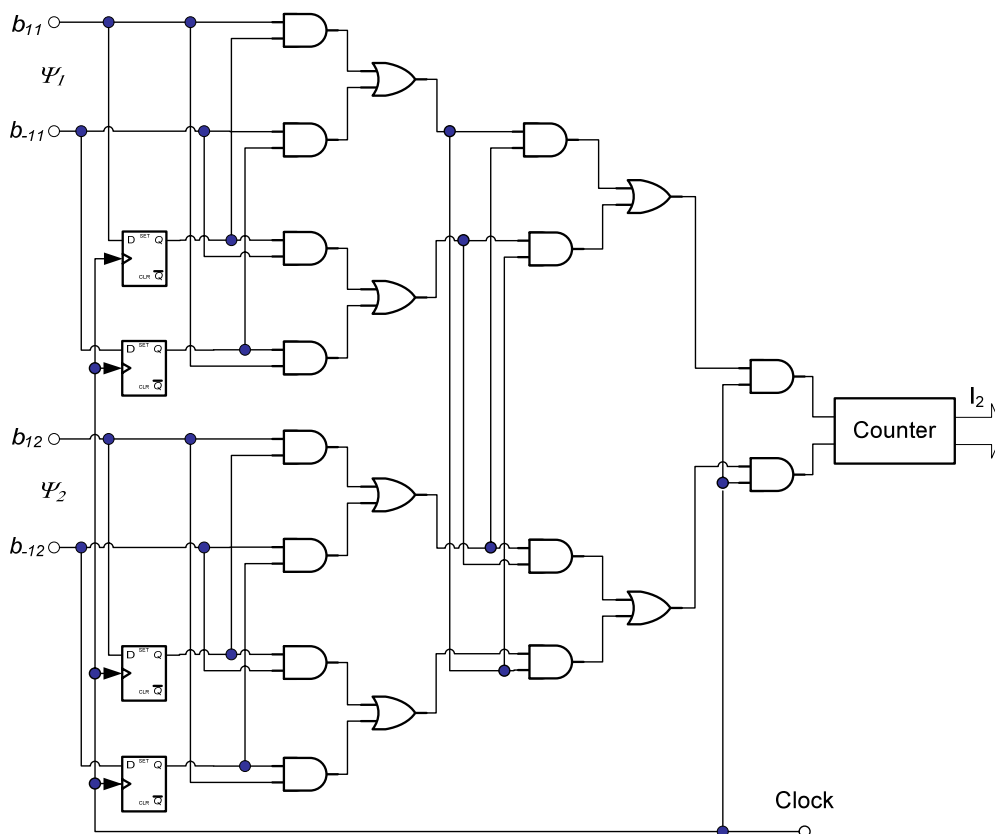


Fig. 3. The circuit for determination of  $I_2$  quantity

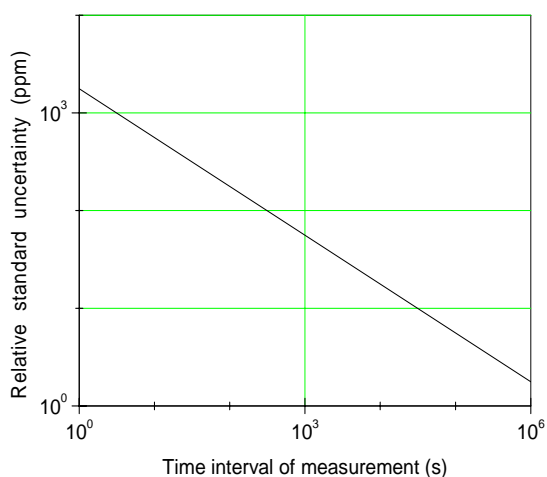


Figure 4: The contribution of A/D conversion to the relative standard measurement uncertainty as a function of measurement time interval, at 160 kHz sampling rate, sinusoidal waveforms.

## 5. CONCLUSION

The stochastic measurement method, described in the paper, utilises coarse but robust 2-bit flash A/D converters. The measurement uncertainty depends on the sampling frequency, chosen time interval of measurement and the unknown waveforms of the two input signals. On the basis

of the uncertainty formula, a simple hardware, that calculates the measurement uncertainty on-line, is proposed. This hardware calculates the uncertainty both partially and globally, thus the proposed device can at any instant give data both on measured variable and on its uncertainty.

The paper discusses application of the method for measurements of electric energy consumption. It is shown that, in typical circumstances, the relative extended ( $k = 2$ ) uncertainty due to the dispersion of A/D converter outputs becomes less than 0.01 % after an twenty minutes time interval and is negligible after a period of one day.

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