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OPTIMISATION OF ORTHOGONAL POLYNOMIAL SIGNALS FOR DIRECT IDENTIFICATION OF EQUIVALENT CIRCUIT PARAMETERS

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Abstract − The equivalent electrical circuits (mostly multielement two-terminals) are the common used method of modelling many technical and biological objects. The parameter identification of this kind of circuits is important in testing and diagnosis of many objects. The paper presents a time-domain identification method dedicated mainly for monitoring and diagnosis of anticorrosion coating. The method is based on applying a sequence of polynomials and measuring the object's response at the end of every signal in the sequence. Equivalent circuit parameters are calculated directly from responses using analytical equations, determined by impedance circuit topology. In the paper the optimisation of non-conventional Gegenbauer and Jacobi polynomial signals against the criteria of stationary error and uncertainty propagation is presented.

Keywords: anticorrosion coating diagnosis, parameter identification, non-conventional signals

1. INTRODUCTION

The modelling of many technical and biological objects with electrical circuits has recently become very popular. The reason is that such modelling allows simulating performance evaluating, monitoring and diagnosing of their state with the aid of well-developed tools and methods designed for electrical circuits. Particularly popular is impedance parameters measurement of objects modelled by multi-element, two-terminal impedance circuits (e.g. anticorrosion coatings [1], materials [2], sensors [3], reinforced concrete constructions [4], and biological objects - physiological fluids and tissues [5]). The conventional method of equivalent circuit parameter identification is an impedance spectroscopy. The process consists of two stages. First, the impedance spectrum is measured, secondly the parameter-dependant function is fitted to the spectrum, e.g. by Complex-Non-linear Least-Squares (CNLS) method.

The main disadvantage of the conventional method is the spectrum measurement starting from very low frequencies (order of mHz and μHz), resulting in a very long measurement time (order of hours). Moreover, the fitting algorithms require large computational power, thus making the realization of low-cost field anticorrosion testers difficult. To circumvent these disadvantages, the alternative method of circuit parameter identification via shape designed polynomial signals has been proposed.

The idea of the method is explained in Fig.1. The Object Under Test (DUT) is stimulated by a sequence of polynomials $u_i(t)$, $t\in[0,T]$, synthesized by an arbitrary waveform generator with an input circuitry (impedance probe). The DUT response $i_i(t)$ for every signal is sampled at time instance T. The method allows simplifying the measurement system hardware structure – S&H is used instead of sampling input channel. The DUT parameters are calculated analytically from set of samples i_i(T), named observables. The shape of the signals, their number (equal to the number of identified parameters) and time T are chosen in the pre-testing stage, considering the assumed equivalent circuit topology. The early results presented in [6], have confirmed the possibility of measurement time reduction Then, some classic polynomials have been compared in terms of propagation of systematic error from set of measured values to set of identified parameters [7].

In this paper, the non-conventional Gegenbauer and Jacobi orthogonal polynomials are examined as potential stimuli signals in the polynomial identification method. The 4-elements Beaunier's equivalent circuit modelling the anticorrosion layer in its early stage of degradation has

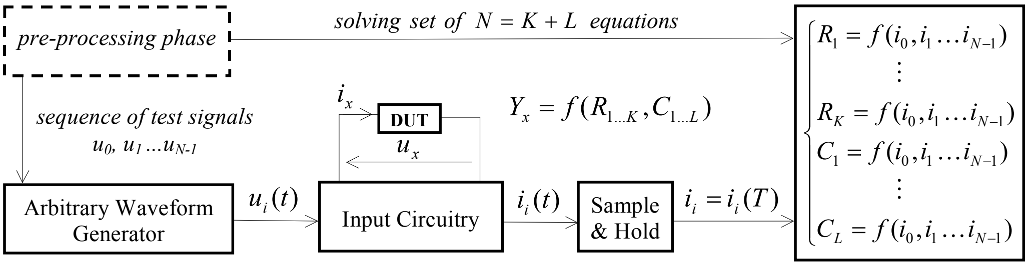

Fig. 1. The idea of equivalent circuit direct parameter identification via polynomial signals.

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been chosen as a test object. The propagation of the stationary measurement errors and propagation of measurement uncertainty, both from the set of observables to the set of moments of impulse response and the set of identified circuit parameters are investigated as polynomial comparison criteria.

2. THEORETICAL BASICS OF THE METHOD

The detailed analysis of theoretical background was already presented in [6][7]. In this paper, only the most important facts are reminded.

2.1. Moment-based alternative object description

The native transfer function of the voltage-stimulated object with current response being measured, is the admittance, which can be described in the form of a rational function with a_i and b_i coefficients dependent on parameters of the equivalent circuit:

$$Y(s)=\frac{a_m s^m+\ldots+a_2 s^2+a_1 s+a_o}{b_n s^n+\ldots+b_2 s^2+b_1 s+b_o}\,. \tag{1}$$

The system transfer function can also be represented in the form of Taylor's expansion, which coefficients are in simple relation with so-called moments of impulse response *h(t)* defined as a functional *h(t)* with power function kernel t^i :

$$\mu_i=\int_0^\infty t^i h(t)\,dt\quad,\qquad i=0,1,2,\ldots,n\,. \tag{2}$$

If, for the given equivalent circuit topology, the set of moments is measured, it will allow to calculate model parameters directly from measurement results. Thus, the set of moments of impulse response is an alternative method of describing object's dynamic response.

In practice, it is more convenient to use approximants of normalized moments of impulse response measured at time-instant *T*, where *T* is the length of stimulus signal:

$$m_i(T)=\int_0^T\left(\frac{t}{T}\right)^i h(t)\,dt=\frac{1}{T^i}\,\mu_i(T)\cdot \tag{3}$$

2.2. Moment measurement via polynomial signals

The moments (3) can be measured with *T*-normalized polynomials given by formula:

$$\begin{aligned}x(t)=P_k(t/T)&=a_{kk}(t/T)^k+a_{kk-1}(t/T)^{k-1}+\ldots+\\&+a_{k2}(t/T)^2+a_{k1}(t/T)+a_{k0}\end{aligned}\,. \tag{4}$$

The object response for such signal is:

$$y(t)=P_k\left(\frac{t}{T}\right)*h(t)=\int_0^t P_k\left(\frac{(t-\tau)}{T}\right)h(\tau)\,d\tau\cdot \tag{5}$$

The sample of response at a given time *T*, named polynomial observable is:

$$o_k=P_k\left(\frac{t}{T}\right)*h(t)\bigg|_{t=T}=\int_0^T P_k\left(1-\frac{\tau}{T}\right)h(\tau)\,d\tau\cdot \tag{6}$$

The relation between observables and moments can be found according to the algebraic theorem, that every polynomial of order *N* can be expressed as a linear combination of *N*+1 polynomials of order *0* to *N*. In this case, exponential kernel of the moment m_i (3) (the special case of polynomial), is substituted by the weighted sum of *N*+1 polynomials mirror to P_i of order *0* to *N*:

$$\left(\frac{t}{T}\right)^i=\sum_{k=o}^{i} w_{ik}P_k\left(1-\frac{t}{T}\right)\cdot \tag{7}$$

Consequently, the moment (3) can be written as:

$$\begin{aligned}m_i(T)&=\int_0^T\left(\frac{t}{T}\right)^i h(t)\,dt=\int_0^T\sum_{k=o}^{i} w_{ik}P_k\left(1-\frac{t}{T}\right)h(t)\,dt=\\&=\sum_{k=o}^{i} w_{ik}\int_0^T P_k\left(1-\frac{t}{T}\right)h(t)\,dt\cdot\end{aligned} \tag{8}$$

By comparing (8) and (3) it can be seen, that an *i*-th moment of impulse response can be calculated as a weighted sum of *i*+1 responses to polynomial stimuli:

$$m_i(T)=\sum_{k=o}^{i} w_{ik}o_k(T)\,\cdot \tag{9}$$

The weights depend only on the polynomials being used (independent on identified circuit topology). The methods to calculate these weights are beyond the scope of this paper.

Evaluation of moment value from several measured observables has the possibility to decrease the propagation of measurement error from the set of observables to the set of moments, and as a consequence, to the set of identified object parameters.

3. ERROR PROPAGATION CRITERIA

The polynomial based method consists of 2 stages. Firstly, moments are calculated from polynomial observables. Secondly, the parameters are calculated from moments of impulse response. The error propagation in the second stage depends only on equivalent circuit topology and is not dependant on the chosen polynomials.

On the contrary, the propagation of error in the first stage is dependent only on a chosen family of polynomials, and independent on circuit's topology. Thus, it can be treated as a good a criterion to compare various families of polynomials or as an optimisation criterion for designing the polynomials.

The two approaches to error propagation have been applied, resulting in synthesis of two criteria.

3.1. Propagation of systematic errors

The first criterion was created from the conventional indirect measurement error propagation analysis. The moment m_i, calculated with systematic error can be written:

$$(m_i+\Delta m_i)=\sum_{k=o}^{i} w_{ik}(o_k+\Delta o_k)=\sum_{k=o}^{i} w_{ik}o_k+\sum_{k=o}^{i} w_{ik}\Delta o_k\,, \tag{10}$$

thus:

$$\Delta m_i=\sum_{k=o}^{i} w_{ik}\Delta o_k\,\cdot \tag{11}$$

The systematic multiplicative error represents the system parameters affecting proportionally on the measured values, the absolute and relative values are denoted as ^{m} and ^{m}. The error is assumed stationary for the time of measurement; its relative value is the same for all observables:

$$\Delta^m m_i = \sum_{k=o}^{i} w_{ik}\Delta^m o_k = \sum_{k=o}^{i} w_{ik}\left(\delta_{o_k}^m o_k\right) = \sum_{k=o}^{i} w_{ik}\left(\delta_{o}^m o_k\right) = \\ = \delta_o^m \sum_{k=o}^{i} w_{ik} o_k = \delta_o^m m_i \quad . \quad (12)$$

$$\delta_o^m = \Delta^m m_i / m_i = \delta_m^m \ . \quad (13)$$

Equation (13) shows, that the relative value of the systematic multiplicative error for moments and observables is the same (propagation index 1) – independently on the polynomial used.

The systematic additive error is usually created by voltage or current offsets in the measurement set-up. The error is assumed stationary – its absolute value is the same for all observables:

$$\Delta^a m_i = \sum_{k=o}^{i} w_{ik}\Delta^a o_k = \sum_{k=o}^{i} w_{ik}\Delta^a o = \Delta^a o \sum_{k=o}^{i} w_{ik} \quad (14)$$

$$\Delta^a m_i / \Delta^a o = \sum_{k=o}^{i} w_{ik} \quad (15)$$

Equation (15) shows, that propagation of stationary systematic error for the i-th moment is dependent on sum of coefficients w_{ik} (9) describing the relation between i-th moment and observables $0\ldots k$.

In order to compare various polynomials, the propagation (15) for all measured moments should be considered. Thus, concerning identification of N-element circuit, the first criterion η_1 is formulated as:

$$\eta_1 = \sum_{i=0}^{N}\sum_{k=o}^{i} w_{ik} \ . \quad (16)$$

3.2. Propagation of measurement uncertainty

The second criterion is a result of measurement uncertainty propagation analysis [8], according to Law of Uncertainty Propagation:

$$u_c(y) = \sqrt{\sum_{i=1}^{N}\left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i)} , \quad (17)$$

where $u_c(y)$ – complex standard uncertainty of estimate y of measured quantity Y, $u(x_i)$ – standard uncertainties of input quantities, $f(\cdot)$– the relation between estimate of measured quantity and estimates of input quantities.

In this case, it can be assumed that all the input quantities are measured in the same set-up with the same uncertainty $u(x_i) = u(x)$, so the (17) can be rewritten as:

$$u_c(y) = \sqrt{\sum_{i=1}^{N}\left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x)} = u(x)\sqrt{\sum_{i=1}^{N}\left(\frac{\partial f}{\partial x_i}\right)^2} \ . \quad (18)$$

The propagation of uncertainty is:

$$\frac{u_c(y)}{u(x)} = \sqrt{\sum_{i=1}^{N}\left(\frac{\partial f}{\partial x_i}\right)^2} , \quad (19)$$

and for the polynomial identification method:

$$\frac{u_c(m_i)}{u(o)} = \sqrt{\sum_{k=0}^{i}\left(\frac{\partial}{\partial o_k}\sum_{k=0}^{i} w_{ik} o_k\right)^2} = \sqrt{\sum_{k=0}^{i}\left(w_{ik}\right)^2} \ . \quad (20)$$

The second criterion η_2 concerning identification of N-element circuit can be assumed as:

$$\eta_2 = \sum_{i=0}^{N}\sqrt{\sum_{k=o}^{i}\left(w_{ik}\right)^2} \ . \quad (21)$$

The established criteria η_1 (16) and η_2 (21) will be used to compare and optimise polynomials.

4. OPTIMISATION OF POLYNOMIALS

The number of various polynomial families described in the literature encourages to check and compare, by means of criteria (16) and (21) their applicability for the polynomial identification method.

The method has already been tested on some arbitrary chosen Chebyschev and Legendre orthogonal polynomials [8]. These classic orthogonal polynomials are special cases of the generalized class of Gegenbauer polynomials, which in turn, are the special case of Jacobi class of polynomials.

In this paper, the optimal (in sense of developed criteria) polynomials will be chosen from these classes. All the polynomials are normalized in order to fulfil the relation:

$$x \in [0,1] \Rightarrow -1 \le P_k(x) \le 1 , \quad (22)$$

resulting from the range of t/T expression in (4) and the normalized bipolar stimulus range [-1;1].

4.1. Gegenbauer class of polynomials

The class of Gegenbauer polynomials is a family of λ parameterised polynomials described by equation:

$$C_k^{\lambda^*}(x) = \frac{1}{\Gamma(\lambda)}\sum_{i=0}^{\frac{k}{2}}(-1)^i \frac{\Gamma(\lambda+k-i)}{i!(k-2i)!}(4x-2)^{n-2k} , \quad (23)$$

where k is the degree of polynomial and parameter $\lambda > -0{,}5$. The special cases of Gegenbauer polynomials are: Chebyschev of 1st kind ($\lambda = 0$), Legendre ($\lambda = 0{,}5$)and Chebyschev of 2nd kind ($\lambda = 1$). The Gegenbauer polynomials need to be normalized, in order to fulfil the right side of (22).

The analytical form of normalized Gegenbauer polynomials will not be presented for the sake of clarity. Instead, the influence of parameter λ on the 5th degree polynomial is presented in Fig. 2.

It can be seen, that for negative λ, the shape of $\left\|C_5^{\lambda}(x)\right\|$ does not differ much from Chebyschev polynomials of 1st kind. For $\lambda \in [0, 1]$, the shape takes the intermediate form between Chebyschev of 1st and 2nd kind. For $\lambda>1$, the shape does not change much – it aims towards zero in middle part of the range.

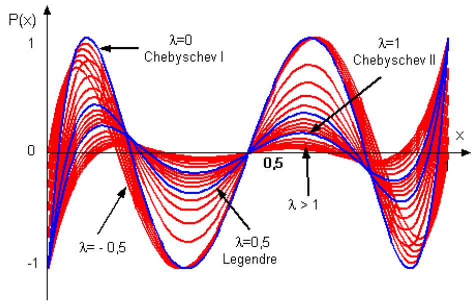

Fig. 2. Gegenbauer polynomials of 5th degree, $\lambda\in(-\frac{1}{2}, 3]$.

The optimisation of Gegenbauer polynomials has been conducted according to the operational scheme in Fig. 3.

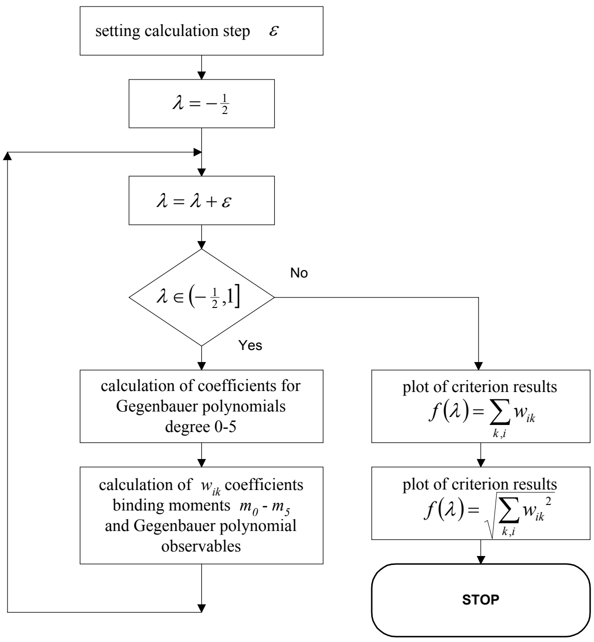

Fig. 3. Scheme for optimisation of Gegenbauer polynomials.

The optimal value of λ has been chosen on the basis of graphical presentation of criteria values, calculated for the case of measuring of 6 moments: $m_0 \ldots m_5$ (6-elements circuit identification). The results for criterion η_1 (systematic additive error propagation) are presented in Fig. 4, whereas results for criterion η_2 (propagation of measurement uncertainty) are presented in Fig. 5. Both figures present the plot of criteria value against parameter λ, with pointed-out special cases of Gegenabauer polynomial.

According to criteria η_1 (Fig. 4), all polynomials with non-negative λ have the same compensation of systematic additive error – the sum of propagation indexes is equal to 1. The investigation has shown, that the 0th moment is calculated with error propagation index equal 1 (without any change), and higher moments are calculated with full compensation of systematic additive error. For negative λ, instead of compensation, the amplification of measurement error occurs.

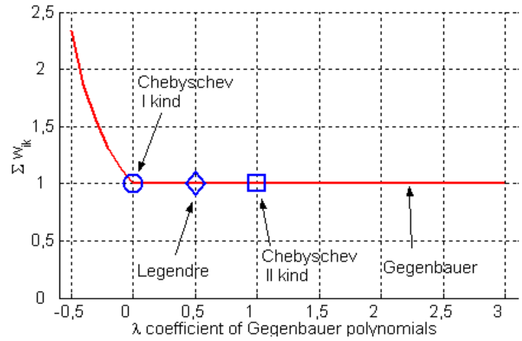

Fig. 4. Graphical presentation of criterion η_1 for Gegenbauer polynomials.

Considering the second criterion, for all non-negative λ Gegenbauer polynomials the propagation of measurement uncertainty is quite similar. The higher the λ is, the compensation is better. However, the difference is so small, that in authors' opinion the further criteria should be considered, e.g. the shape of polynomials, which for high values of λ is not correct in metrological sense.

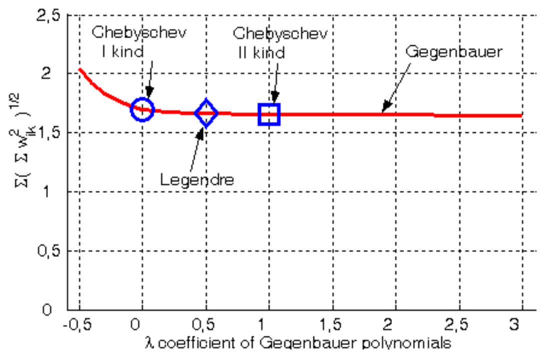

Fig. 5. Graphical presentation of criterion η_2 for Gegenbauer polynomials

4.2. Jacobi class of polynomials

The Gegenbauer polynomials are sub-class of wider class of hypergeometrical (Jacobi) polynomials, parameterised by 2 coefficients α and β, where $\alpha,\beta>-1$. The Jacobi polynomials are described by equation:

$$G_k^{(\alpha,\beta)*}(x)=\frac{\Gamma(\beta+k)}{\Gamma(\alpha+2k)}\sum_{i=0}^{k}(-1)^i\binom{k}{i}\frac{\Gamma(\alpha+2k-i)}{\Gamma(\beta+k-i)}(2x-1)^{k-i} \quad (24)$$

The relation between Jacobi and Gegenbauer polynomials is defined by:

$$\left\|G_k^{(\alpha,\beta)*}(x)\right\|=\left\|C_k^{\lambda^*}(x)\right\|\Leftrightarrow\alpha=\beta=\lambda-0{,}5\,. \quad (25)$$

The Jacobi polynomials are two-parametric, so in order to show the influence of α, β parameters to shape of exemplary polynomial $\left\|G_5^{(\alpha,\beta)}(x)\right\|$ of 5th degree, 4 plots are presented in Fig. 6-9.

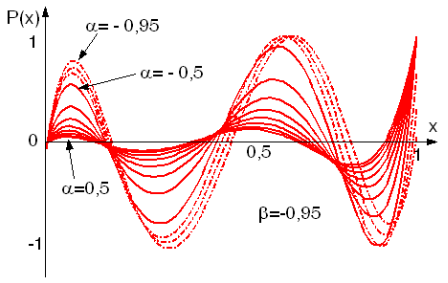

Fig. 6. Jacobi polynomials of 5th degree $\alpha \in (-1,\ 0{,}5]$, $\beta \approx -1$.

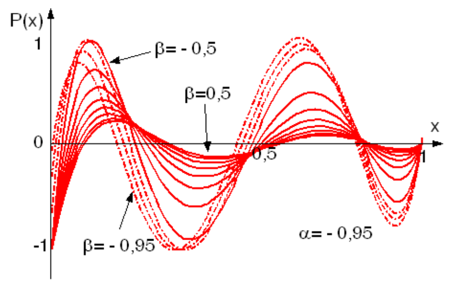

Fig. 7. Jacobi polynomials of 5th degree $\alpha \approx -1$, $\beta \in (-1,\ 0{,}5]$.

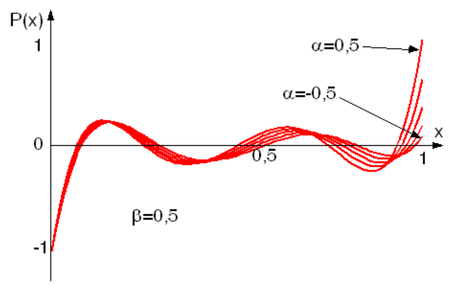

Fig. 8. Jacobi polynomials of 5th degree $\alpha \in [-0{,}5,\ 0{,}5]$, $\beta = 0{,}5$.

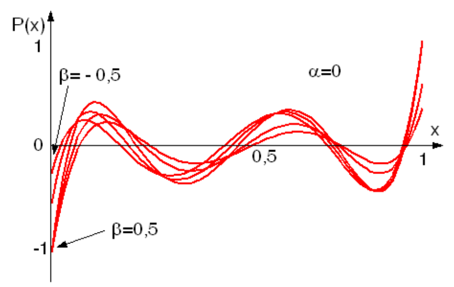

Fig. 9. Jacobi polynomials of 5th degree $\alpha = 1$, $\beta \in [-0{,}5,\ 0{,}5]$

The polynomials in Fig. 6 and Fig. 7 have one parameter close to its minimum value allowed and second parameter changes in a range (-1, 0,5]. The polynomials with parameter in a range (-1, -0,5) are plotted with dashed lines, whereas the polynomials with parameter in [-0,5, 0,5] range are plotted with solid lines. It can be seen, that the first group have a common value at start (Fig. 6) or at the end (Fig. 7).

This feature can lead to some interesting properties in terms of physical realisation of such stimulus, e.g. the polynomial with $\alpha = -0{,}95$, $\beta = -0{,}5$ (Fig. 6) starts with zero value and ends with full-scale value, which is a desirable in a real measurement system. The polynomials in Fig. 8 do not seem to have interesting properties – their values oscillate near zero. The polynomials in Fig. 9 are similar to Legendre polynomials.

The optimisation of Jacobi polynomials has been conducted as in the case of Gegenbauer polynomials – the operational scheme is presented in Fig. 10.

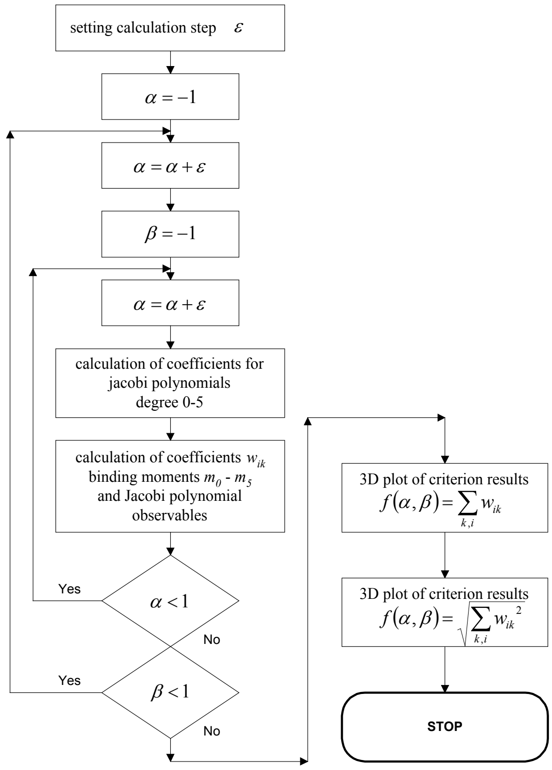

Fig. 10. Scheme for optimisation of Jacobi polynomials.

Due to 2-parametric form of Jacobi polynomials, the results for both considered criteria as a function of variable parameters are plot in 3D form, in Fig. 11 and Fig. 12. In both figures, the criteria values for Gegenbauer polynomials (being the Jacobi polynomial with equal values of α and β) are pointed-out with a thick solid line. Moreover, the special cases of Gegenbauer/Jacobi class – the classic Chebyschev, and Legendre polynomials are pointed-out with marks.

A disadvantageous feature can be seen, by comparing these two figures. If both criteria are considered, the α, β parameters given by a curve for Gegenbauer polynomials are optimal. On the left side of curve, the propagation of uncertainty criterion gives good results. However, from the criterion of systematic additive error propagation we know, that the amplification of error occurs instead of compensation. On the right side of curve, the first criterion shows that the systematic additive errors are compensated. However, the second criterion (propagation of uncertainty) gives poor results. If one should decide, which criterion is more important, the authors suggest considering the results of 2nd criterion – the uncertainty propagation, as systematic errors can be compensated in a real measurement setup.

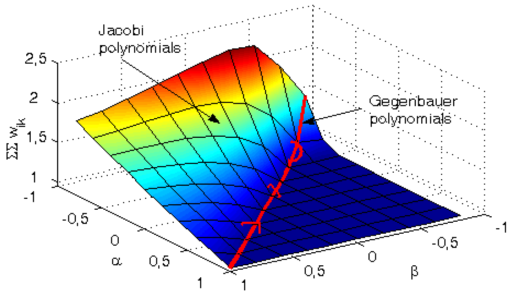

Fig. 11. Graphical presentation of criterion η_1 for Jacobi polynomials.

Thus, the values on the left of Gegenbauer curve can be acceptable. Unfortunately, the already mentioned Jacobi polynomials with interesting shape (Fig. 6) do not fall in that group.

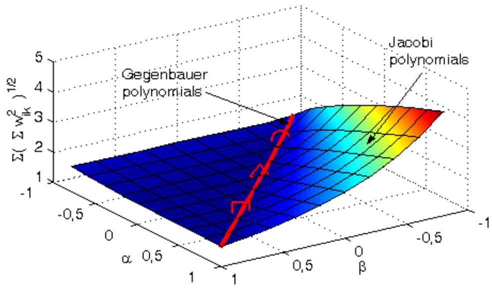

Fig. 12. Graphical presentation of criterion η_2 for Jacobi polynomials.

5. RESULTS AND CONCLUSIONS

To verify the results of analytical analysis, the Monte Carlo simulation experiment has been conducted in Matlab environment with a transient state simulator.

As the polynomial identification method is oriented for diagnosis of anticorrosion coatings, the Beaunier's model has been chosen as a test engine to compare propagation of errors for different stimuli. This model represents an anticorrosion coating in its early stage of degradation by a two terminal 4 elements network, shown in Fig. 13.

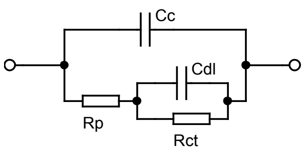

Fig. 13. Beaunier's equivalent circuit of anticorrosion coating.

The uncertainty of parameter identification with 5 different polynomials has been simulated with the same assumptions about measurement error values. The results are in Tab. 1. To summarize, it should be stated, that the generalized class of Jacobi polynomials contains many polynomials with interesting, in metrological sense, shapes.

However, considering the two independently defined criteria: systematic additive measurement error propagation and propagation of measurement uncertainty, the most optimal Jacobi polynomials are the Gegenbauer polynomials. Moreover, from the class of Gegenbauer polynomials, the classic orthogonal Chebyschev and Legendre polynomials have very good properties (similar to each other), confirmed by the simulation results.

Table 1. Uncertainty of Beaunier's model identification.

Polynomial	Relative uncertainty of parameter [%]			
	$u_c(R_p)$	$u_c(R_{ct})$	$u_c(C_c)$	$u_c(C_{dl})$
Chebyschev I	1,124%	1,125%	2,662%	1,196%
Chebyschev II	1,079%	1,080%	2,556%	1,146%
Legendre	1,089%	1,090%	2,579%	1,158%
Jacobi $G_k^{(-0,5,-0,99)}$	3,211%	3,210%	7,677%	3,384%
Jacobi $G_k^{(-0,99,-0,5)}$	1,395%	1,395%	3,294%	1,488%

The sophisticated analysis has shown, that the most interesting polynomials are the ones that were selected intuitively at the beginning of the research.

To sum up, the considered identification method can be treated as an interesting alternative to impedance spectroscopy approach, in cases similar to the example – e.g. monitoring and diagnosis of an object varying over time. The method makes use of a known circuit topology and expected range of parameter values. The results presented in previous papers [6][7] have shown that with decent uncertainty, the identification time is shorter than with conventional CNLS methods.

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