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THE CONSTRUCTION AND ACCURACY ANALYSIS OF THE MULTIREFERENCE EQUIPMENT FOR CALIBRATION OF ANGLE MEASURING INSTRUMENTS

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Abstract – Precise geodetic instruments are very widely used in geodesy, surveying, machine engineering, etc. Testing and calibration of these instruments is quite complicated task, although it is evidently needed. In this paper we present the creation and research of accuracy of the multireference (incorporating photoelectrical angular encoder, polygon/autocollimator and the circular scale/microscope(s)) test bench designed to perform the testing and calibration of such geodetic instruments.

Keywords: calibration, angle measurements, bias

1. INTRODUCTION

There are instruments allowing precise planar angle measurements widely used in geodesy, surveying, machine engineering and other branches of industry. Such instruments are theodolites, digital theodolites, tacheometers, total stations, etc. Since such are instruments often used for very precise and important measurements, like all the other they must be tested and calibrated.

For the determination of both biases and random errors produced by the geodetic measuring instrument a special device must be used, also the calibration of the geodetic measuring instruments requires a large number of angular values to be compared with the reference values. Such procedure due to its technical complexity and expensiveness of the testing device is not regulated by any standard at all. Devices capable of performing such procedures are usually operated by companies – manufacturers of measurement equipment and are not available for the wide public and the users of these instruments [4].

Main geodetic devices testing principle implements the collimation to several fixed points and the accuracy calculation of the results [5]. Such method does not allow random and systematic errors (biases) determination in the entire circle of the measurements at a small pitch. To determine the errors of geodetic instruments measurements in full horizontal circle some kind of a devices (comparator) comparing the angular measurements performed by the geodetic instrument with the reference ones was needed. Obviously the rotary table of such comparator has to be positioned or at least its position determined with higher precision then the precision of instruments it has been designed to test (or calibrate). It was decided to create such test bench at Vilnius Gediminas Technical University. The test bench was supposed to comply with the need of testing or calibration of geodetic angle measuring instruments and additionally be suitable for testing of the angle measuring devices used in other branches of industry.

Generally there are several groups of plane angle measurement principles (methods) [11]:

1. Solid angular gauge method:
   - polygons (multiangular prisms);
   - angular prisms;
   - angle gauges, etc.
2. Trigonometric method (angle determination by means of linear measurements);
3. Goniometric method (plane angle determination by means of a circular scale):
   - full circle (limb, circular code scales etc.);
   - non-full circle (sector scales).

Several technical decisions for the precise angle determination can be implemented. The most significant means used for this purpose are:

1. Polygon/autocollimator.
2. Moore's Precision Index table .
3. Visual scale/microscope(s).
4. Angular encoders.
5. Laser interferometer.
6. Ring laser (laser gyro).

Assuming technical specifications of the most commonly used geodetic instruments it can be considered that all the means of angle measuring listed in could be used for their calibration and testing – the difference being only in technical suitability for this task. In case of some means it is too technically complicated (as in case of *Moore's Precision Index table and ring laser*) to automate the measuring process or due to its principle of action (*Ring laser*) it is too difficult to adopt the means for the specific need of geodetic instruments testing/calibration [10]. Similarly the angular pitch of measurement of the multiangular prism/autocollimator is too large for calibration.

2. CONSTRUCTION OF THE TEST BENCH

A test bench capable of testing and calibration of planar angles measuring geodetic devices has been constructed.

The machine consists of the stabile base where the high precision pure rolling bearings were mounted, on them the rotary disk with the circular scale on it was fitted [2].

Operation of the test bench for the calibration of geodetic instruments is based on the comparator principle [2, 4, 9]. The readings from the tested device (tacheometer, theodolite) are to be compared to the readings from the reference measure (horizontal angle etalon in this case). The tribrach of the instrument under testing (tacheometer, theodolite) (14, Fig. 1) is attached to the rotary table (4) of the machine. During the calibration process the rotary table together with the tribrach is being rotated to a certain angle position by means of step motor with the worm-gear (8) controlled by PC (1) via the control unit (7). Angle of rotation is determined by the photoelectrical rotary encoder (6). The final angular position of the housing of the geodetic instrument is measured by pointing autocollimator (10) at the mirror (12) attached to the housing of the instrument. The instrument tested operates conversely to the way it should work during the normal geodetic measurements when the tribrach of the measuring device is stable and the device itself is being turned to the desired direction. After rotating the rotary table the readings from the geodetic instrument are taken, they are compared to the readings from the angle standard used in the test bench. Having the difference in angle readings it is possible to determine the errors of the tested device.

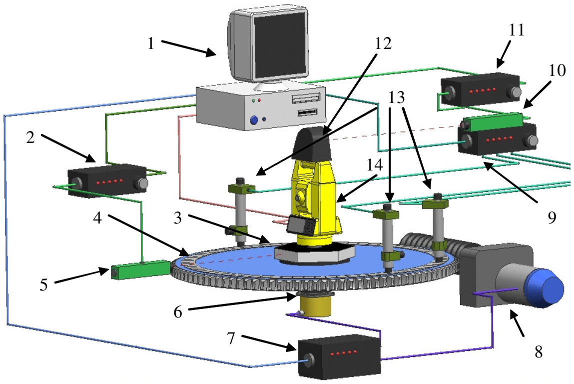

Fig. 1. A basic composition of the test rig: 1 – PC, 2, 11 – autocollimator control unit, 3 – multiangular prism (polygon), 4 – rotary table, 5 – autocollimator, 6 – photoelectric rotary encoder, 7 – rotation control unit, 8 – step motor drive, 9 – microscope's control unit, 10 – autocollimator, 12 – reflecting mirror, 13 – photoelectric microscopes (optional number), 14 – geodetic instrument

The flat angle calibration test bench is designed with the possibility for angle calibration using several angle measurement facilities, such as: multiangle prism (3, Fig. 1) - autocollimator (5); rotary encoder (6); circular scale (on the surface of the disc (4) - photoelectric microscope(s) (13). They can be chosen arbitrary according to their accuracy parameters and an accuracy of the instrument to be calibrated. Output of these measurement instruments is input into the PC.

The measuring methods using a rotary encoder, autocollimator and polygon, circular scale and microscopes combined in one device can perform the measurements both independently and in the united system. Such composition of measurement methods allows monitoring constantly the performance of each measuring device and performing the cross calibration or self calibration of each component of the system. Multiangular prism/autocollimator system is considered as the reference means of angle determination and all other measuring systems are being compared to it, although for the regular geodetic instrument calibration the photoelectric encoder or the circular scale/microscopes are used.

3. MAIN ANGLE REFERENCE

As it was mentioned before, the multiangular prism/autocollimator measuring system is considered as the main angle reference in the described test rig. The multiangular prism used is the 12 faces polygon with the pitch of the angle measurements (due to the angle between faces) of 30°. The multiangular prism was calibrated at the PTB Braunshweig.

For obtaining of the measurements data the custom made autocollimators with the modern CCD cameras were implemented. Since the autocollimators were modernised and systematic errors neither of autocollimators optics nor of the CCD cameras were clearly known, collimators had to be calibrated.

For calibration of the autocollimators a rotary table constructed by *Wild Heerbrugg* company in Switzerland and transferred to Vilnius Gediminas Technical University by Swiss Federal Institute of Technology was used. Mirror (3, Fig 2) was placed on the rotary table (2) and autocollimator (1 or 4) was pointed to the mirror. Therefore, the rotary table in this case acts as a small angle generator and the reference mean of angle measurement (and angular positioning).

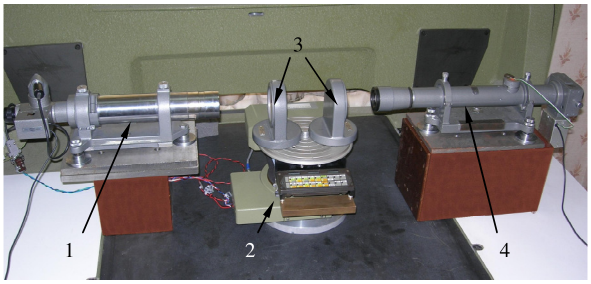

Fig. 2. Example layout (optional) of measuring equipment for experiment: 1 – autocollimator *II*, 2 – *Wild* rotary table, 3 – mirrors, 4 – autocollimator *I*

The rotary table used implements the dynamic encoder for angular position determination, it has the rotation step of 4.5″ and measuring sensibility of 0.0324″; theoretical repeatability of the system is in a range of 0.03″, and the experimental standard deviation stated by the manufacturer in no case exceeded 0.32″ [7]. Systematic errors of the particular rotary table used was not clearly estimated (since there were no available devices of higher accuracy to use as the reference), but the standard deviation of measurements (of particular table used for experiment) was experimentally determined and did not exceed 0.166″.

Placement of the mirrors which is shown in Fig. 2 is optional and not very typical since moving the mirror reflecting surface from the centre of rotation of the table may produce some additional errors (caused by the flatness deviation of the mirror). Such layout allows comparison of the results (with interchanging the mirrors and autocollimators) and therefore performing the correlation analysis to determine the influence of each mirror on the measurements. During the normal autocollimator calibration mirror was placed so that its reflecting face was on the centre of rotation of the table [1, 8].

According to the results of the calibration of autocollimators the best fit 3rd order polynomial curves were drawn for each autocollimator. These determined polynomial curves could be stated as the typical curves of the tested autocollimators. Mean measurement deviations (with main linear constituent removed) for each series with the calculated polynomial curves are shown in Fig. 3.

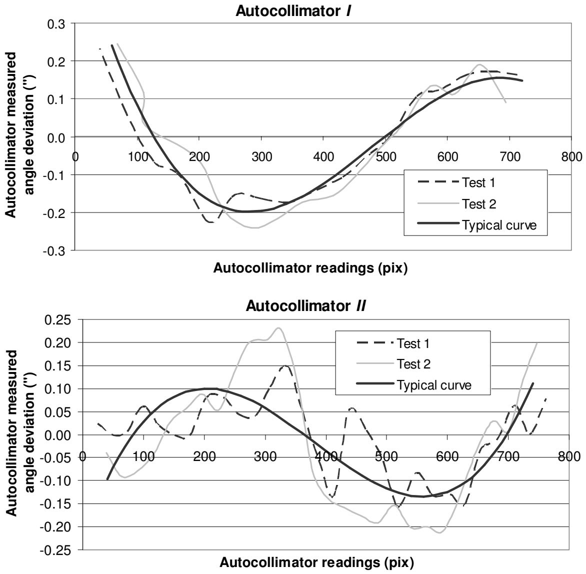

Fig. 3. Mean measurement deviations of autocollimators for each measurement series with the best-fit polynomial curves

As can be seen, the autocollimator measurement deviations have clearly random character and any other kind of typical curve (4th order polynomial etc.) could hardly be implemented. The standard deviations of the best-fit 3rd order polynomial curve (typical curve) for Autocollimator *I* and *II* are respectively 0.113″ and 0.142″. Having the stated practical standard deviation of the rotary table (0.32″), a general standard deviation of determined typical curves for Autocollimator *I* and *II* is respectively 0.339″ and 0.350″ [6].

The general determined typical curves of the autocollimators (for transformation of pixel measurements to arc seconds) are for Autocollimator *I*:

$$y = -1.084 \cdot 10^{-8} x^3 + 1.57 \cdot 10^{-5} x^2 + 0.205x, \qquad (1)$$

and for Autocollimator *II*:

$$y = 1.048 \cdot 10^{-8} x^3 - 1.2 \cdot 10^{-5} x^2 - 0.330x, \qquad (2)$$

where: x – autocollimators measure in pixels, y – true measured angle value in arc seconds.

Therefore equations (1) and (2) are general equations for calculating the measures of autocollimators used. These equations represent the typical curves for Autocollimator *I* and Autocollimator *II*.

Generally the sources of systematic errors of the measurements performed by autocollimators are such:

- Influence of the non-parallelism of beams (autocollimator is not focused to the infinity);
- Flatness deviations of the mirror;
- Systematic errors of the CCD matrix;
- Errors caused by the optical system of autocollimator;
- Errors caused by the CCD orientation (CCD matrix not perpendicular to the beams).

The calibration of autocollimators allows compensating (in most of cases) mentioned systematic errors and therefore equations (1) and (2) can be used for (automated) determination of true angle values measured by one of the autocollimators.

4. OPERATIONAL ANGLE REFERENCE OF THE TEST BENCH

As was mentioned before the implementation of multiangular prism/autocollimator angle determination principle despite its high accuracy is quite limited due to the high pitch and small measurement range. Therefore principle of measurement having small pitch at wide angle range is needed. There were two measurement principles satisfying these requirements implemented in the test bench – these are photoelectric angle encoder and circular scale/microscope. Since at the time of construction of the test bench photoelectric angle encoder with only a limited accuracy (±3 arc sec) was available it was decided to use the circular scale/microscope measuring principle for the operational reference rotary disc angle determination.

Similarly as with the autocollimators microscopes used were modified by fitting the CCD cameras to the optical microscopes and calibrating them. The scale originally imprinted to the rotary disc was used. Such arrangement allowed using the scale/microscope in an automated mode of measurements very similar to the one of the photoelectric angle encoder, but with simple rearrangement of the measuring equipment, its recalibration, control of accuracy and modification (which is almost impossible in case of the industrial angle encoder).

To use the circular scale as the reference for measurements its calibration was essential. There is vast number of circular scales calibration methods like *Approximation, Opposite Matrix, Yeliseyev* (or *Heuvelink*) and *Wild.* Most of these methods are quite complicated, require a large number of microscopes used and often do not give the unambiguous value of the systematic error of particular scale stroke [11].

One of the simplest classical scale calibration methods, based on the fact that a total sum of the angles between the investigated strokes in full circle is equal to 360°, is the *constant angle setting in full circle method* (Fig. 4) [11]. The *advantages* of the mentioned method are:

- simplicity of method;
- it is possible to determine the total bias of the scale;
- only two microscopes are needed for the calibration.

Despite the advantages this *classical* method also has some serious *disadvantages*:

- method is mostly intended for the general evaluation of scale quality without the possibility of evaluation of each specific scale stroke;
- difficulty of receiving the data at small angles caused by the necessity of placing two microscopes very close to each other. at a small angles (5° and less) is almost impossible due to the dimensions of microscopes;
- need to position one of the strokes precisely at the centre of the positioning microscope;
- random errors of calibration accumulate during the calculation due to the sequential strokes biases at every step of measurement.

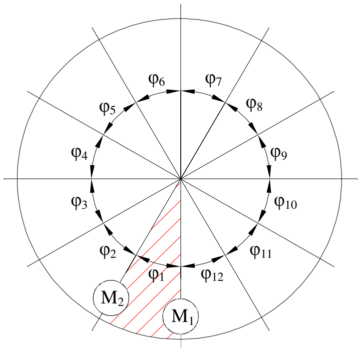

Fig. 4. Circular measurement sequence using two microscopes, M$_1$, M$_2$ – microscopes

To avoid some of shortcomings of the method and apply it for the use with modern equipment (photoelectrical microscopes, etc.) it has been modified, the essence of the modification is further briefly described.

One of the main difficulties in application of *constant angle setting in full circle* method is the fact that for precise measurements one of the scale strokes must be necessarily coincident with the bisector of the first microscope (M$_1$, Fig 4). It is not difficult to accomplish such task in a manual calibration of scale but in a completely automated mode of action of the measuring device (using some kind of approximate positioning of the rotary table) it is quite complicated. For that reason the method, or rather obtaining of deviation principle must be modified and suited for the particular automated task.

Modification of method is shown in Fig. 5. As can be seen from the Fig. 5 the microscopes (M$_1$ and M$_2$) are placed at an angle (φ_p) which is close to the reference angle or calibration pitch (φ_r – reference angle, at which the microscopes are supposed to be placed, multiple to 2π) established using the strokes of the scale to be calibrated (i and $i+1$) with an unknown deviation ($\delta\varphi_p$). The real angle (unknown) between the investigated scale strokes is marked as (φ_i).Both microscopes are used to measure the deviation of the strokes position regarding the optical axis of the microscopes ($\Delta M_{1.i}$ and $\Delta M_{2.(i+1)}$).

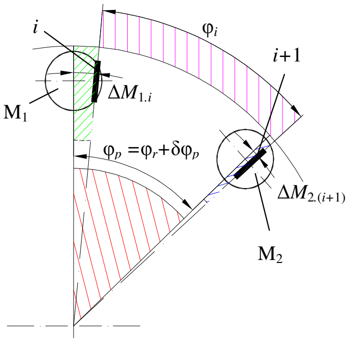

Fig. 5. Measurement step using two microscopes, M$_1$, M$_2$ – microscopes, i - number of scale stroke ($i = 1, 2, 3, \ldots\ n$)

Considering that first stroke of the scale has no systematic errors, biases of each particular scale stroke can be calculated [3]:

$$\delta\varphi_1 = 0\,,$$

$$\delta\varphi_2 = \delta\varphi_1 + \left(\Delta M_{2.2} - \Delta M_{1.1}\right) - \frac{\sum_{i=1}^{n}\left(\Delta M_{2.(i+1)} - \Delta M_{1.i}\right)}{n}\,,$$

$$\delta\varphi_3 = \delta\varphi_2 + \left(\Delta M_{2.3} - \Delta M_{1.2}\right) - \frac{\sum_{i=1}^{n}\left(\Delta M_{2.(i+1)} - \Delta M_{1.i}\right)}{n}\,,$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..,$$

$$\delta\varphi_n = \delta\varphi_{n-1} + \left(\Delta M_{2.n} - \Delta M_{1.n-1}\right) - \frac{\sum_{i=1}^{n}\left(\Delta M_{2.(i+1)} - \Delta M_{1.i}\right)}{n}\,. \quad (3)$$

Thus the biases (regarding the first investigated stroke) can be determined, stored in the memory of computer and used for determination of rotary table position. The biases determination using equations (3) is based on the subsequent calculation of them and does not require special software (*MathCAD* or *Microsoft Excel* can deal with the task).

Using described calibration method all of the biases of the scale strokes, including the ones caused by the eccentricity, non-perpendicularity of the scale regarding the rotation axis or any short period biases are determined and included in the value calculated (and can later be segregated).

Despite the advantages, described *modified circular scale calibration method* has the disadvantage common to the *constant angle setting in full circle* method – limited calibration pitch due to necessity to place two microscopes very close to each other (the pitch is limited by the size of microscopes). The least complicated way of achieving good results in small angle calibration is implement new *multiple scale turn* method in which to microscopes are placed at an angle multiple of $2\pi \cdot k$ (where k – integer).

The *multiple scale turn* method of calibration of the circular scale at the pitch of 30° using the microscopes placed at an angle of 150° to each other is presented (Fig 6).

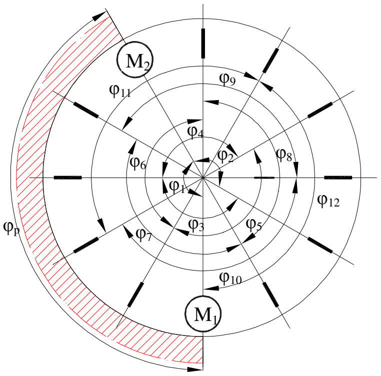

Fig. 6. Circular measurement using the angle no multiple to 2π (150° in this case), M$_1$, M$_2$ – microscopes

As can be seen from the picture using this method at a rotation pitch of 30° the equation system can be written:

$$\begin{cases} \varphi_1 = \varphi_p + \Delta_1, \\ \varphi_6 = \varphi_p + \Delta_6, \\ \varphi_{11} = \varphi_p + \Delta_{11}, \\ \varphi_4 = \varphi_p + \Delta_4, \\ \varphi_9 = \varphi_p + \Delta_9, \\ \varphi_2 = \varphi_p + \Delta_2, \\ \varphi_8 = \varphi_p + \Delta_8, \\ \varphi_{12} = \varphi_p + \Delta_{12}, \\ \varphi_5 = \varphi_p + \Delta_5, \\ \varphi_{10} = \varphi_p + \Delta_{10}, \\ \varphi_3 = \varphi_p + \Delta_3, \\ \varphi_7 = \varphi_p + \Delta_7, \end{cases} \tag{4}$$

In case of the described microscopes placement (when the angle between them is not multiple to 2π; for example 150° as in Fig 6) the number of measurements for full circle (during the full scale calibration procedure) can be determined:

$$k = n \cdot \frac{\varphi_r}{2\pi}, \tag{5}$$

where: φ_r - reference angle between the microscopes ($\varphi_p = \varphi_r + \delta\varphi_p$); n – total number of measurements performed during full circle calibration.

In the equation (5) number of measurements n (being an integer) should be selected so that k also is an integer. To simplify the calculations measurements number n:

$$n = \frac{2\pi}{|\varphi_r - \varphi_{mult}|}, \tag{6}$$

where: φ_{mult} – nearest to φ_r angle multiple of 2π.

After the number of measurements has been determined the scale calibration pitch can be calculated:

$$\varphi_{cal} = \frac{2\pi}{n}. \tag{7}$$

In Fig 6 such calibration pitch being $\varphi_{cal} = 30°$.

That way performing the calibration of the circular scale at a pitch of 1° (number of calibration steps $n = 360$) two microscopes can be placed at a variety of angles (φ_r) – 29°, 31°, 44°, 46°, 59°, 61°, etc. Similarly at a pitch of 5° ($n = 72$) the angles could be – 25°, 35°, 40°, 50°, 55°, etc. Considering the marginal cases – placement of microscopes at an angle of 180° to each other will result in the pitch of calibration of 180° ($n = 2$) etc., such cases represent classical *constant angle setting in full circle method.*

As can be seen without the serious calibration method modifications the biases of the strokes at any angular pitch value can be determined by means of *multiple scale turn*.

Finally several *main features* of the *constant angle setting in full circle method* with *multiple scale turn* could be highlighted. The *advantages* of the method are:

- bias of each scale stroke can be evaluated;
- only two microscopes can be used for calibration and only one for the further work;
- no need for precise mechanical positioning of the scale, the microscopes should be only capable of measuring the particular stroke;
- calibration can be performed at any pitch of the circular scale (the only limitation is a pitch of scale strokes);
- simplicity of method and the bias calculation process;

Disadvantages of the method are:

- random errors (of calibration) accumulating during the calculation due to the sequential strokes biases determination – the biases of the last strokes calculated may become quite high, nonetheless in case of repeated calibration process the random errors should be eliminated.

The calibration of circular scale of test rig was performed at the pitch of 5°. Average results of the calibration (from several test series) are shown in Fig 7.

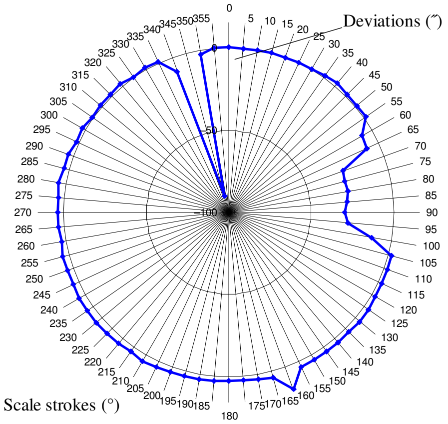

Fig. 7. Mean calculated bias of each scale stroke

According to Fig 7 several noticeable scale strokes biases are present, the largest of which being at the strokes 350° and 60°–100°. Knowing the numerical values of those strokes biases the precise rotary table angular position can be later determined by means of a single microscope.

Evaluate of standard deviation of each bias measurements is quite small (not exceeding $S_i = 0.423''$, including the evaluate of standard deviation of the microscopes measurements $S_m = 0.0125''$). Since the biases were calculated regarding the first stroke (bias is equal 0) in case of the described measurements (the angle of microscopes placement 95°) the maximal standard deviation evaluate will be reached at a scale stroke marked as 90° and $S_{\max} = 2{,}621''$. It should be noted that despite a quite high standard deviation of the last stroke bias, high amount of measurements should practically eliminate any kind of a random error in scale bias determination.

Since during the scale calibration the multiangular prism/autocollimator was used (Fig 1) and the measurements were taken simultaneously with those taken by the microscopes, the data received from both measurements could be compared, considering the multiangular prism/autocollimator data as the reference values [4, 8] (Fig 8).

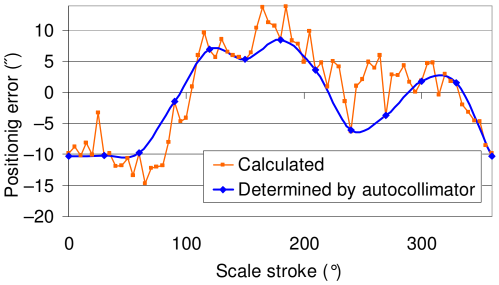

Fig. 8. Example of rotary table positioning error compared to the one determined by multiangular prism/autocollimator

Using these data, general (practical) standard deviation value of angle position determination by the suggested method taking the multiangular prism/autocollimator measurements as the reference, can be calculated $S_p = 0.172''$ [2].

As can be seen the operational accuracy of rotary disc angular position determination is quite sufficient for calibration of angle measuring instruments such as tacheometers or theodolites.

5. CONCLUSIONS

A test bench for angle measuring geodetic instruments testing and calibrations was created in Vilnius Gediminas Technical University, Institute of Geodesy.

At the test bench the angular positioning of rotary table can be determined by three independent methods – photoelectric angle encoder, polygon/autocollimator and scale/microscope(s), all these methods allow constant control and comparation of results so that the errors of one of the methods can be easily recognized.

Two autocollimators implemented in the constructed test rig were calibrated using a precise automated rotary table.

According to the calibration data, typical curves for Autocollimator *I* and Autocollimator *II* were determined with the standard deviations 0.339″ and 0.350″ respectively.

The circular scale calibration method suited for automation and based on *constant angle setting in full circle with multiple turn* enabling scale calibration practically at any possible pitch has been suggested and tested on the plane angle testing/calibration bench.

The circular scale was calibrated using two photoelectric microscopes placed at an angle of 95° to each other with a pitch of 5° and additionally controlled by the multiangular prism/autocollimator measurements.

Practical comparison of the data obtained by scale/microscope (after scale calibrations) to reference ones by the multiangular prism/autocollimator shows the practical standard deviation estimate low enough (S_p = 0.172″).

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