XIX IMEKO World Congress
Fundamental and Applied Metrology
September 6–11, 2009, Lisbon, Portugal

WATER SURFACE PROFILES IN DIVIDED CHANNELS VERIFIED EXPERIMENTALLY

Maurizio Leopardi [1] , Maria Teresa Todisco [2]

[1] Structures, Waters and Land Engineering Department
Faculty of Engineering, L'Aquila, Italy, maurizio.leopardi@univaq.it

[2] Structures, Waters and Land Engineering Department
Faculty of Engineering , L'Aquila, Italy, terry@ing.univaq.it

Abstract – In the laboratory of Hydraulics and Hydraulic Construction of the Faculty of Engineering of L'Aquila, a physical model was made for the study of steady-state, gradually varied motion flow in open channel networks. For this study, a calculus algorithm has been used proposed by [1] which gives, in the case of slow water flow into gradually declined channels, in the direction of motion, the volumes and water levels respectively, in all the nodes and sides of the network.

Keywords: open-channel, networks steady-state.

1. INTRODUCTION

The design and verification of open channel networks is usually based on the assumption that the discharge of water flowing out in every element of the network runs in uniform flow. This hypothesis is very difficult to comply with the conditions for effective movement of the liquid, a more real evaluation of hydraulic phenomena that have been established in networks should be determined by the use of equations of unsteady flow.

For incompressible fluids they are reduced to equations of motion and continuity. Taken together they give rise to a system of differential equation to partial derivative, however except for a few cases, defined by drastic simplification compared to actual phenomena, they do not give finite term solutions.

The difficulties involved in the study of unsteady hydric flow, on the other hand, resides not only in the problem of reaching the analytical solutions of the equations in word, but in particular to give, the limits conditions and the initial ones. For the definition of the initial conditions, defined by the situations of steady flow before the various arrangements that is wanted to study, it is necessary to identify the discharge and location of water surface position in all elements of the network.

The definition of the last one was obtained from [1] which proposed an algorithm that defines them using the analytical solutions from the flow profile equation obtained from [2] and [3]. The physical model consists of a network of open channels with known flow (as measured by instruments), and where the limits condition will be allocated from the readings of the depth of flow to the terminal end of the channel.

2. METHOD OF CALCULATION

The solution procedures that lead to tracking the flow profiles of steady and gradually-varied flow, flowing with constant flow Q in cylindrical beds, are reported in the integration of differential equation [4]:

$$\frac{dh}{ds} = \frac{i_f - J(h)}{i - \Omega(h)} \quad (1)$$

The hydraulic calculations to verify and design open channel networks are usually based on (1) being: *h* the depth of flow in the generic section, i_f the slope (constant) of bottom channel; *J(h)* dissipation of energy (per unit of weight and path) due solely to friction resistance; $\Omega(h) = \alpha \frac{Q^2 l}{g\sigma^3}$; α coefficient of Coriolis (hereafter considered constant and equal to 1), *l* top width of the water section of height *h*, *g* acceleration gravity; *s* the distance measured along the axis of the section of the channel taken as the origin (positive in the direction of motion).

The solution in finished form of the equation (1) can be obtained only for channels in which the cross-section is such a shape that the areas σ and corresponding heights h among monomial formula

$$\sigma = bh^n \quad (2)$$

To the practical effects (2) results compliant by almost all sections used in the open channels within those accuracy limits normally allowed in these kinds of problems: The only exceptions were the sections closed contour used in underground pipes [3] & [4]. The law of resistance may be specialised in monomial formula :

$$Q = kh^p J^q \qquad (3)$$

where k is with a given size parameter dependent mainly on the nature of the walls and parameters that define the shape of the cross-section channel. In these conditions, the differential equation (1) may be closely integrated [2], [4]. Given the fact (2) and (3), (1) translates into:

$$\frac{dh}{ds} = i_f \frac{1-\left(\frac{Q}{i_f^q k}\right)^{1/q} \frac{1}{h^{p/q}}}{1-\frac{nQ^2}{gb^2}\frac{1}{h^{2n+1}}} \qquad (4)$$

$$h_u = \left(\frac{Q}{i_j^q k}\right)^{\frac{1}{p}} \quad \text{and} \quad h_c = \left(\frac{nQ^2}{gb^2}\right)^{\frac{1}{2n+1}}$$

represent, respectively, the height h_u of uniform flow and the height of h_c critical state, both related to the discharge Q. The algorithm proposed by [1] transpires when the discharge Q, flowing on the stretch of canal in question is given, the distance separating the two sections at what are known values of the connecting depth of flow h_1 and h_2 (connected by a flow profile continuous), which is given by:

$$l = \frac{h_u}{i_f} \Gamma[z_1(h_u), z_2(h_u)] \qquad (5)$$

with $\Gamma = F(z_2(h_u)) - F(z_1(h_u))$ and $z = h/h_u$. **3.1 3.**

3. EXPERIMENTAL INSTALLATION

The experimental model installed consists of a three channel system with open rectangular section confluent in a single node, Fig.1.

The first of the three, the main one, is 4.55 m long and has an internal width of 38.2 cm, while the other two are both 1.70 m long and 19.2 cm wide.

Given the modest heights of water surface reached by the currents that drains in them it was enough to achieve the shores of about 10 cm.

The two secondary channels (B and C), placed downstream from the main (A), forms an angle between them of about 30 degrees.

The described support system is made up of steel tubes whose height was calibrated in such a way as to achieve the desired slopes and to vary it when necessary.

Initially, the section A has been assigned a slope i_A = 1.44‰ or, at a B i_B = 1.76‰ or C and a i_C = 2.24‰. Subsequently, groped to step in sub critical flow in channels B and C, were increased i_B and i_C respectively to 2.94‰ and 5.88‰, unfortunately without success because practical reasons didn't allow to obtain higher slopes.

The model is completed by a feed tank (upstream), two Thomson weirs, each made from a series of 3 calm tanks, located downstream at channel B and C, and from an additional retention tank downstream from channel B and C.

Moreover the necessary measurement instruments of discharge and water surface heights. The model is powered by a discharge Q_A, known because it's detected by an electromagnetic flow-meter located upstream of the entire system.

The flow into the tank upstream may be regulated by a gate valve. The overflow into the main channel in regime conditions will be exactly equal to the discharge Q_A.

This discharge downstream on the node will split in Q_B &, obvious reason of continuity imposed necessarily that $Q_A = Q_B + Q_C$.

The flow entered into the system was varied between 0.80 and 3.60 l/s; technical reasons have not allowed higher values.

The connecting depth of flow in the three branches have been detected by hydrometers places along the canals and at the two weirs.

Their number, initially equal to 6 +2, was increased to 11 +2 in order to obtain a greater amount of information on the profiles.

At the same time, a system of linear obstructions was installed, made from 4 flat metal elements all placed on edge between the Channel A and the upstream tank in order to limit the initial turbulence.

After, a reduction of channels' A and C section was made, by affixing a plate in perspex which reduced their width respectively at 29 cm and 10 cm , Fig.2.

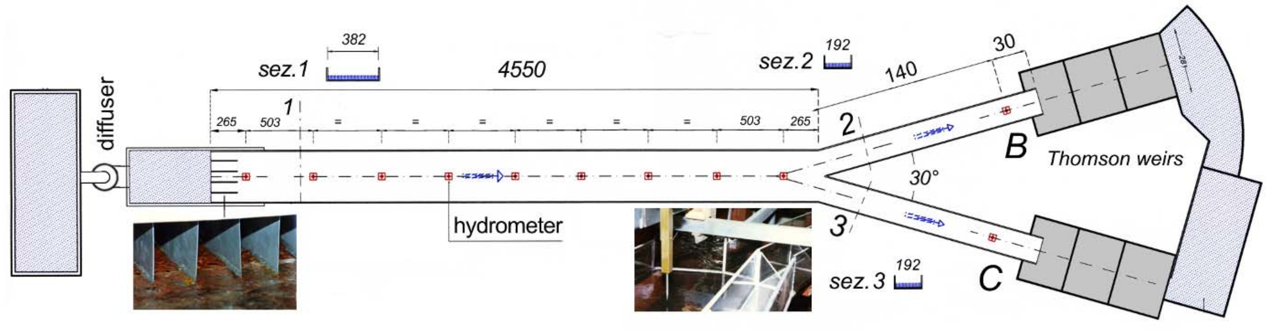

Fig.1 Experimental model

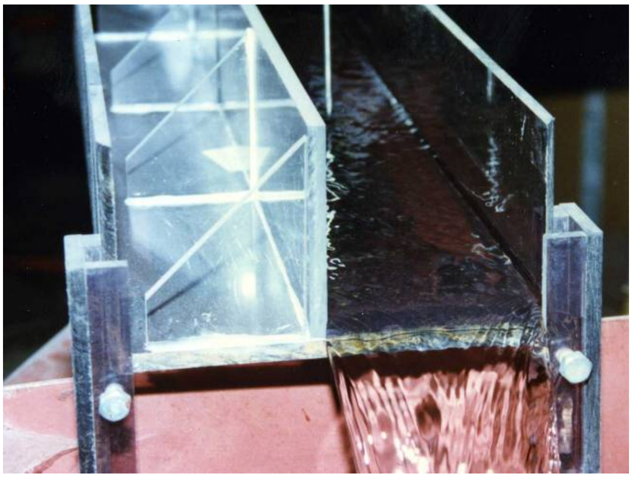

Fig.2 .

Further experimental variations were the introduction of shutter downstream channel B and C, of variable height between 20 and 40 mm.

The first readings, made in the main channel, made to get a Strickler's coefficient K of 120 $m^{1/3}/s$, which value is in perfect agreement with the deductible one for perspex models in existing literature [5].

Following, eight other readings were carried out with the floodgates, and a further ten without them.

4. RULES 'OF EXPERIENCE

4.1 Determination of roughness coefficient (K)

After adjusting the flow through a shutter, readings of the water levels have been made for each discharge value placed in the main channel (A) and such readings were performed at the nine hydrometric rods (Fig.3), a distance of 50 cm between each other.

Fig.3

Later an initial value of roughness coefficient of Strickler (K) was deduced by calculating an average of the values of those readings. The results of one of theses tests is reported in Table 1:

Table 1

Test A			Q = 2,40 l/s K = 121,16		
	lA = 0,29 m		I = 0,00144 L = 4,024m		
	ds	dc	Yexp	Ycalc.	Δ
hydrom.	m	m	mm	mm	mm
1		0,503	27,00	26,70	0,30
1 - 2	0,505	0,505	27,00	26,24	0,76
2 - 3	0,515	0,515	26,50	27,33	-0,83
3 - 4	0,503	0,503	25,35	26,78	-1,43
4 - 5	0,499	0,499	23,75	25,52	-1,77
5 - 6	0,496	0,496	23,75	23,12	0,63
6 - 7	0,503	0,503	23,30	23,63	-0,33
7 - 8	0,501	0,501	22,60	23,05	-0,45
8 - 9	0,502	0,502	21,75	22,03	-0,28
	Weir discharge			Qs = 0,64 l/s	
	hc = 19,11 mm			hu =24,27 mm	

Data was derived from the same tests which allowed the verification of the similarity between the model and the prototype, and then evaluate the correctness of the roughness coefficients previously found, Table 2.

Table 2

Test A	Lr = 26		
	Kr = 0,581	K =70,39	
	Qr = 3447	Q = 8,27 m^3/s	
	Yexp.rid.	Ycalc.rid.	Δ
hydrom.	mm	mm	(Ysr-Ycr)
1	702,0	698,4	3,6
1 - 2	702,0	679,5	22,5
2 - 3	689,0	705,7	-16,7
3 - 4	659,1	692,6	-33,5
4 - 5	617,5	662,6	-45,1
5 - 6	617,5	610,6	6,9
6 - 7	605,8	621,0	-15,2
7 - 8	587,6	609,4	-21,8
8 - 9	565,5	591,2	-25,7

A corresponding flow profile was made with the derived values and it resulted in the typical profile of subcritical flow .

4.2 Readings with sluice gates and weirs

Readings were made of the water levels for different values of discharge introduced in the main channel (A), after the installation of sluice gates with heights variable between 20mm and 40mm, situated at terminal extremity of channels B and C; the flow from these measures in these channels were made by two type Thomson weirs, Fig.4 .

The presence of sluice gates influence from downstream, in the three sides, the flow profiles which are defined by direct readings with hydrometers.

Here are some results obtained, assuming, in a first series of tests, H = constant (Table. 3a - 3b) and, in a second one, Y = node constant (Table 4a- 4b).

Fig.4

Table 3a

Section A	**Section B**	**Section C**
ds_A=3.86 m	ds_B=1.4 m	ds_C=1.4 m
i_A = 0.00144	i_B = 0.00176	i_C = 0.00244
l_A = 0.382 m	l_B = 0.192 m	l_C = 0.192 m
1°test: H = cost K= 121.16	**Sluice Gate: h = 20 mm**	**Sluice Gate h = 20 mm**
Q_A = 1.22 l/s	Q_B = 0.76 l/s	Q_C = 0.46 l/s
Y_A = 24 mm	Y_B = 30.5 mm	Y_C = 31.7 mm
dsc = 3.859 m	dsc = 1.406m	dsc =1.4 m
Y_{AN} = 29.25 mm	Y_{BN} = 28.17 mm	Y_{CN} = 28.64 mm
H* = 3.0*10^{-2}	H* = 3.0*10^{-2}	H* = 3.0*10^{-2}
Q_{LA}=1.22 l/s	Q_{LB}=0,76 l/s	Q_{Lc}=0,46 l/s
Δ(Q_{LA} - Q_A)= 0 l/s	Δ(Q_{LB} - Q_B)= 0 l/s	Δ(Q_{LC} -Q_C)= 0 l/s
dsl = 3.86 m	dsl = 1.400 m	dsl = 1.400 m
Δ(dsl - dsc)= 0.001 m	Δ(dsl - dsc)= -0.006 m	Δ(dsl - dsc)= 0.001 m
hu =12.94 mm hc = 10.13 mm	hu =14.30 mm hc = 11.68 mm	hu =9.40 mm hc = 8.36 mm
Delayed Slow Flow	*Delayed Slow Flow*	*Delayed Slow Flow*

Table 3b

2°test: H = cost K= 121.16	**Sluice gate: h = 40 mm**	**Sluice Gate h = 40 mm**
Q_A = 1.23 l/s	Q_B = 0.75 l/s	Q_C = 0.48 l/s
Y_A = 44 mm	Y_B = 51 mm	Y_C = 52 mm
dsc = 3.859 m	dsc = 1.4 m	dsc =1.399 m
Y_{AN} = 49.51 mm	Y_{BN} = 48.56 mm	Y_{CN} = 48.86 mm
H* = 4.9*10^{-2}	H* = 4.9*10^{-2}	H* = 4.9*10^{-2}
Q_{LA}=1.23 l/s	Q_{LB}=0,76 l/s	Q_{Lc}=0,47 l/s
Δ(Q_{LA} - Q_A)= 0 l/s	Δ(Q_{LB} - Q_B)= 0.01 l/s	Δ(Q_{LC} -Q_C)= -0,01 l/s
dsl = 3.86 m	dsl = 1.400 m	dsl = 1.400 m
Δ(dsl - dsc)= 0.001 m	Δ(dsl - dsc)= -0.000 m	Δ(dsl - dsc)= 0.001 m
hu =12.94 mm hc = 10.13 mm	hu =14.18 mm hc = 11.58 mm	hu =9.68 mm hc = 8.60 mm
Delayed Slow Flow	*Delayed Slow Flow*	*Delayed Slow Flow*

Table 4a

Section A	**Section B**	**Section C**
ds_A=3.86 m	ds_B=1.4 m	ds_C=1.4 m
i_A = 0.00144	i_B = 0.00176	i_C = 0.00244
l_A = 0.382 m	l_B = 0.192 m	l_C = 0.192 m
1°test: Y= cost K= 121.16	**Sluice Gate: h = 20 mm**	**Sluice Gate h = 20 mm**
Q_A = 1.22 l/s	Q_B = 0.76 l/s	Q_C = 0.46 l/s
Y_A = 24,31 mm	Y_B = 31.5 mm	Y_C = 32.3 mm
dsc = 3.859 m	dsc = 1.406m	dsc =1.4 m
Y_{AN} = 29.57 mm	Y_{BN} = 29.57 mm	Y_{CN} = 29.57 mm
Q_{LA}=1.23 l/s	Q_{LB}=0,78 l/s	Q_{Lc}=0,45 l/s
Δ(Q_{LA} - Q_A)= 0,01 l/s	Δ(Q_{LB} - Q_B)= 0,02 l/s	Δ(Q_{LC} -Q_C)= 1 l/s
dsl = 3.859 m	dsl = 1.406 m	dsl = 1.400 m
Δ(dsl - dsc)= 0.001 m	Δ(dsl- dsc)= - 0.006 m	Δ(dsl - dsc)= 0.00 m
hu =15.94 mm hc = 12.57 mm	hu =14.30 mm hc = 11.68 mm	hu =9.40 mm hc = 8.36 mm
Delayed Slow Flow	*Delayed Slow Flow*	*Delayed Slow Flow*

Table 4b

2°test: Y = cost K= 121.16	**Sluice Gate: h = 40 mm**	**Sluice Gate h = 40 mm**
Q_A = 1.23 l/s	Q_B = 0.75 l/s	Q_C = 0.48 l/s
Y_A = 44 mm	Y_B = 51 mm	Y_C = 52 mm
dsc = 3.859 m	dsc = 1.4 m	dsc =1.399 m
Y_{AN} = 49.51 mm	**Y_{BN} = 49.51 mm**	**Y_{CN} = 49.51 mm**
Q_{LA}=1.25 l/s	Q_{LB}=0,77 l/s	Q_{Lc}=0,48 l/s
Δ(Q_{LA} - Q_A)= 0.02 l/s	Δ(Q_{LB} - Q_B)= 0.02 l/s	Δ(Q_{LC} -Q_C)= 0,00 l/s
dsl = 3.86 m	dsl = 1.400 m	dsl = 1.400 m
Δ(dsl - dsc)= 0.001 m hu =15.69 mm hc = 12.24 mm	Δ(dsl - dsc)= -0.00 m hu =14.18 mm hc = 11.58 mm	Δ(dsl - dsc)= 0.001 m hu =9.68 mm hc = 8.60 mm
Delayed Slow Flow	*Delayed Slow Flow*	*Delayed Slow Flow*

4.3 Readings without sluice gates with restriction of section A and C and change of slope of the slopes B and C

In this phase we operated a narrowing of the section of the two sections A and C and we evaluated the effects that this causes on profile performance, at the same time we changed the slope for the two terminal tracts B and C to cause the transition from a state of slow flow to a fast flow; at both ends we always found the two weirs to read the heights of water (Fig.5).

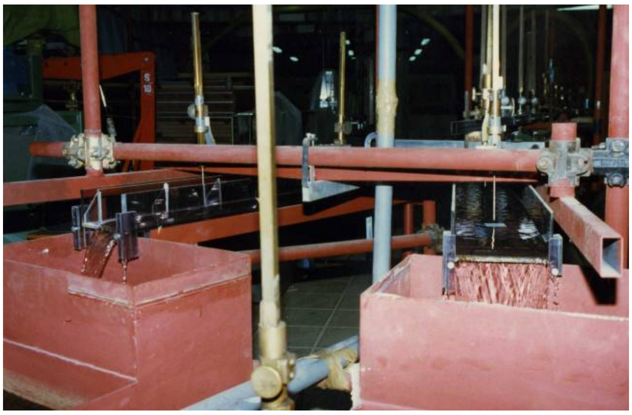

Fig. 5

We provided simultaneously to install a dissipation device placed at the beginning of the main section A, because the absence of sluice gates makes it difficult to stabilize the system.

So, relative readings were made on the hydrometric rods for both the main section (A) and the minor tracts B and C, also in this case the gap Δ between the values of physical model and the theoretical one was analysed.
The following tables shows the obtained results.

Reading	Q=	0,80 l/s			
n°1	K=	121,16	i_A=	0,00144	
Section	L=	4,55 m	l_A=	0,29 m	
A-N	d_s	d_c	Y_{sper}	Y_{calc}	Δ
Hydrometer	m	m	mm	mm	$Y_c - Y_s$
0 - 2		0,510	14,00	13,30	0,70
2 - 3	0,515	0,515	13,65	14,43	-0,78
3 - 4	0,503	0,503	12,75	14,03	-1,28
4 - 5	0,499	0,499	11,80	13,00	-1,20
5 - 6	0,496	0,496	11,75	11,83	-0,08
6 - 7	0,503	0,503	11,75	11,72	0,03
7 - 8	0,501	0,501	10,75	11,19	-0,44
8 - 9	0,502	0,502	10,75	11,03	-0,28
			Y=11,03 mm		
Profile tracement hu =11,84mm			hc = 9,18mm		

Reading	Q=	0,64 l/s			
n°1	K=	121,16	i_B=	0,00294	Q_s - Q_c
Section N-B	L=	1,70 m	l_B=	0,192 m	- 0,01
T=11,03mm	Q_{sper}	Q_{cal}	Y_{sper}	Y_{calc}	Δ
Hydrometer	l/s	l/s	mm	mm	$Y_c - Y_s$
10	0,63	0,64	10,95	15,20	-4,25
Intersects steady state					

Reading	Q=	0,16 l/s			
n°1	K=	121,16	i_C=	0,00588	Q_s - Q_c
Section **N-C**	L=	1,70 m	l_C=	0,10 m	0,01
T=11,03mm	Q_{sper}	Q_{cal}	Y_{sper}	Y_{calc}	Δ
Hydrometer	l/s	l/s	mm	mm	$Y_c - Y_s$
11	0,17	0,16	7,85	20,18	-12,33
Intersects steady state					

Reading	Q=	1,63 l/s			
n°2	K=	121,16	i_A=	0,00144	
Section	L=	4,55 m	l_A=	0,29 m	
A-N	d_s	d_c	Y_{sper}	Y_{calc}	Δ
Hydrometer	m	m	mm	mm	$Y_c - Y_s$
2		0,501	21,00	20,30	0,70
2 - 3	0,515	0,515	20,50	21,33	-0,83
3 - 4	0,503	0,503	19,35	20,78	-1,43
4 - 5	0,499	0,499	18,25	19,46	-1,21
5 - 6	0,496	0,496	18,25	18,06	0,19
6 - 7	0,503	0,503	17,70	18,06	-0,36
7 - 8	0,501	0,501	17,15	17,22	-0,07
8 - 9	0,502	0,502	16,45	16,03	0,42
			Y= 16,03 mm		
Profile tracement hu = 18,85mm			hc = 14,77 mm		

Reading	Q=	1,20 l/s			
n°2	K=	121,16	i_B=	0,00294	Q_s - Q_c
Section N-B	L=	1,70m	l_B=	0,192 m	0,03
T=16,03mm	Q_{sper}	Q_{cal}	Y_{sper}	Y_{calc}	Δ
Hydrometer	l/s	l/s	mm	mm	$Y_c - Y_s$
10	1,23	1,20	16,95	18,93	-1,98
Intersects steady state					

Reading	Q=	0,43 l/s			Δ
n°2	K=	121,16	i_C=	0,00588	Q_s - Q_c
Section N-C	L=	1,70 m	l_C=	0,10 m	-0,03
T=16,03mm	Q_{sper}	Q_{cal}	Y_{sper}	Y_{calc}	Δ
Hydrometer	l/s	l/s	mm	mm	$Y_c - Y_s$
11	0,40	0,43	12,30	14,12	-1,82
Intersects steady state					

5. CONCLUSION

For the tests performed with sluice gates and weirs the results of theoretical and experimental were quite coincidental.

In the Froude similitude the fluid characteristics are the same because the emulsion effects are limited.
The non-reproducibility of this phenomenon, however, does not alter the necessary indications for the phenomenon's perception in the study, providing useful elements for the design solution [10].

The introduced discharge varied between 0.80 l / s and 1.70 l / s and obtained profiles in each tract are typical of delayed slow current.

Regarding tests with both the lack of sluice gates and in presence of narrowing, the numerical process provides a typical profile of accelerated slow flow, in the tract A, but no consistent solution for B and C.

It is conceivable that in this last case the influence of pressure drop is such to compromise the convergence of the solution.

In the case of tests with unique water levels in the node, only in presence of sluice gates, the theoretical results were in agreement with experimental ones, but in their absence resulted in inconsistent solutions.

For these tests, the calculated differences between values obtained theoretically and measured experimentally, resulted low, although slightly higher than those determined in the hypothesis of constant load.

Ultimately, we can state that the calibration of the numerical process on physical model in question is reliable for delayed slow flow situations, while the test is more complex for situations in which there are gradual change of regime (from slow to fast flow). Indeed we noted that, when we tried the shift to slow flow conditions making a reduction of channel sections A and C and increasing the slopes of the lines B and C, that procedure provides a convergent solution only for the line A.

We can say that this is due to the loss of load localised in the node, disregarded in the theoretical model, and not reproducible in the actual model.

Further experimental investigations should be made on a larger dimension model (smaller scale) to explore the sufficient credibility of cinematic field of flow on the split, ,even in unexpected situations in real systems.

REFERENCES

[1] A. Russo Spena, M. T. Todisco and G. Vacca, " Profili di corrente in reti di canali aperti", *L'Acqua*, vol.3, pp 33-39, Maggio-Giugno,1996.

[2] G. Evangelisti, " Sui problemi di rigurgito in alvei cilindrici di sezione generica" , *Giuseppe Evangelisti: Scritti scelti*, Bologna, Settembre, 1982.

[3] G. Evangelisti, "Mathematical Models of Open-Channel Flow", *Proceedings of the NATO Advanced Study Institute on "Surface Hydrodynamics",* Bressanone, Aug.29-Sept 7, 1966.

[4] A. Russo Spena, "Sul tracciamento dei profili di corrente in alvei cilindrici", *L'Energia Elettrica*, vol XXVII, n.7,1950.

[5] M. Leopardi, "On roughness similarity of hydraulic models", *International Association of Hydraulic Engineering and Reserch - Journal of Hydraulic Research* n.3/2004.

[6] J.Allen, "*Scale models in hydraulic engineering* , Green and Co -1947 – Londra.

[7] H.L Langhaar, "*Dimensional Analysis and Theory of Models* ", Krieger Publishing Company - Huntington, New York , 1980.

[8] P.Novak, "Developpemnet in hydraulic engineering ". *Applied Science Publishers* - Barking , GB -1983.

[9] P. Novak, J. Cabelka, "*Models in Hydraulic Engineering",* Pitman Publishing Limited - Londra 1981.

[10] De Marchi G., "Studio delle questioni idrodinamiche con l'aiuto di prove di laboratorio su modelli in scala ridotta", *L'Energia Elettrica* n. 8 ,1931.