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INFLUENCE OF THE MUTUAL GRAVITATIONAL ATTRACTION IN A SET OF MASSES OF DEADWEIGHT MACHINES

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Abstract – Reciprocal gravitational attraction among the masses of deadweight machines could be significant in certain conditions. A simulation with finite elements method was applied on two different types of deadweight machines: INRIM force standard machines Amsler 100 kN and Galdabini 1 MN. Results are reported in comparison with the simplified method that considers the masses concentrated on theirs barycentres.

Keywords: gravitational force, deadweight machine, masses attraction.

1. INTRODUCTION

A deadweight machine roughly consists of a main frame, which supports the whole structure, a loading frame and a set of masses. For generating the required force, some masses are weighed down on the loading frame. So the generated force is due to the gravitational attraction among the masses and the earth.

In several cases, the masses of a deadweight machine are affected by a reciprocal gravitational attraction which could be significant.

Some studies have been carried out considering the influence on the variation of the gravity field due to geometry of deadweight machines and the corresponding positioning of the masses [1]. Since studies and analysis on the influence of the reciprocal attraction among the masses of the deadweight machines are not in our knowledge, we have investigated this topic.

Aim of this paper is the estimation of the effect of masses attraction in two different type of deadweight machines, in order to take it into account if it is relevant for the uncertainty budget.

2. DEADWEIGHT FORCE STANDARD MACHINES

The masses in deadweight machines are usually plates of steel, stainless steel, cast iron, etc. Apart their form and material, they are stacked one on the other according to an increasing or decreasing weight sequence. Thus a certain mass is affected by an attractive force exerted by all the others. We expect this effect to be larger for that masses that are stacked at a closer distance.

Two examples are presented with respect to the INRIM deadweight Force Standard Machines (FSM), namely the Amsler 100 kN (briefly called MCF100) and the Galdabini 1 MN (MCF1000), with different design [2]. In our analysis, we will consider the entire attractive force in the worst case.

2.1. Description of INRIM force standard machines

MCF100 has two baskets: the first with 35 masses of 50 kg stacked upon the other with 17 masses of 500 kg. The masses are loaded in a sequential method. A photograph is shown in Fig. 1.

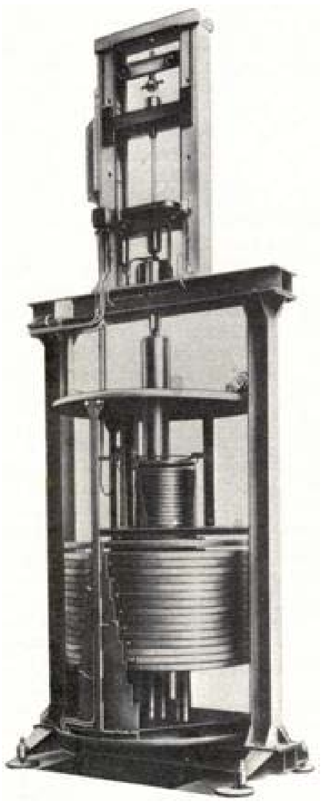

Fig. 1. Picture of Amsler 100 kN (MCF100). The two groups of masses are visible.

In this design, the large number of masses induces to very small distances among them.

MCF1000 adopts a binary method for upload the masses, each of them can be applied to the loading frame independently from the others. So it is possible to generate a large number of force values with only 10 groups of masses. Consequently, the distance among the masses was kept reasonably large (Fig. 2 and Fig. 3).

Fig. 2. Picture of small masses of Galdabini 1 MN (MCF1000).

Fig. 3. Picture of large masses of Galdabini 1 MN (MCF1000).

3. MUTUAL GRAVITATIONAL ATTRACTION

In a deadweight force standard machine, the maximum mutual attraction occurs when only one mass is used to generate the force (worst case). The remaining unloaded masses, in fact, disturb the local gravitational field and, as a result, the generated force.

The well-known law of gravitational force is expressed as

$$\vec{f} = -G \frac{m_1 m_2}{|\vec{r}|^2} \frac{\vec{r}}{|\vec{r}|} \qquad (1)$$

with G: gravitational constant, m_1 and m_2: two related masses, $\vec{r}$: position vector between the barycentres of the two masses.

Harshly applying this formula the contribution of the mutual attraction leads, in MCF100, to a relative error in the generated force slightly below 10^{-6}. A value that could be comparable with other influence factors in the uncertainty calculation.

The same procedure applied to the MCF1000 masses gives a relative error that is almost one order of magnitude lower than the previous one.

2.3. Simulation with Finite Element Method (FEM)

Equation (1) is applicable only when the ratio between the dimension of the masses and their relative distances is negligible.

In the case of distributed systems, like the masses of deadweight machines, it is necessary to take into account the geometry of masses and their reciprocal placement.

In order to carry out this analysis, a FEM simulation was conducted on the MCF100 and MCF1000 schemes.

The tool used for the analysis is a commercial FEM software (COMSOL® 3.5). In particular, the gravitational field was modelled by means of the electrostatic module replacing the density of charge and the permittivity in vacuum with the density of mass and the gravitational constant, respectively (duality between electricity and gravity) [3].

The validation consisted in comparing the results obtained with the FEM software and the results obtained with an analytical solution for a sphere of known diameter and density of mass [4].

The comparison shows that the error can be kept less than $4{\times}10^{-10}$ g by adjusting the mesh density (Fig. 4).

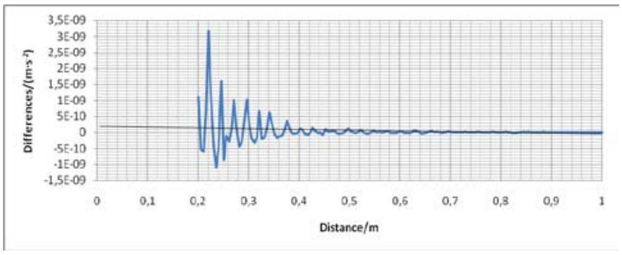

Fig. 4. Comparison between the results obtained using analytical and simulated solutions.

The gravity field generated by the unloaded masses and acting on the mass under observation is shown in Fig. 5, for both the MCF100 and the MCF1000 FSMs. The FEM simulation was simplified by the axial symmetric shape of the masses. The gravity disturbance is therefore given with respect to the radial distance r from the mass axis.

The magnitude of the gravity attraction is similar for both the machines. Significant differences are shown concerning the radial distribution of the gravity field. The external diameter of the masses under observation are similar in both the MCF100 and MCF1000 (about 1.5 m). Nevertheless, the bigger diameter of the central aperture and the closer distance of the masses of the MCF100 with respect to the MCF1000, result in a decrease of the gravity field near the axis.

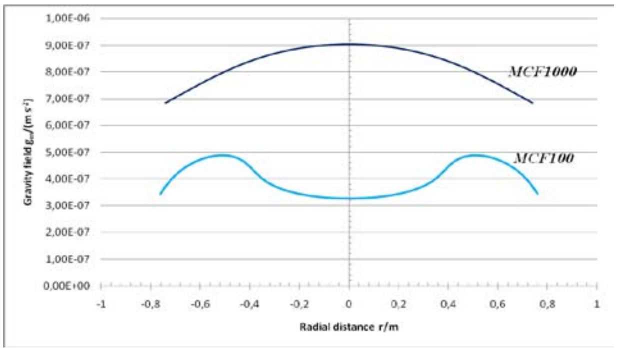

Fig. 5. Gravity field on mass under observation for both Amsler 100 kN (MCF100) and Galdabini 1 MN (MCF1000).

The attraction force is derived considering the variation of the gravity field along the radial distance r and neglecting the vertical gradient along the mass under analysis.

The results obtained are reported in Table 1 for the MCF100 and in Table 2 for the MCF1000. The mass under observation is equal to 500 kg for the MCF100 and about 1000 kg for the MCF1000. Actually, the relative error computed for the MCF100 considers mass under observation of 750 kg, including the mass of the loading frame.

Table 1. Attraction forces ΔF and relative error E_r for the MCF 100 (Model and FEM results).

Calculation	ΔF /mN	E_r ($\times 10^{-6}$)
Model	5	0,66
FEM	0,3	0,04

Table 2. Attraction forces ΔF and relative error E_r for the MCF 1000 (Model and FEM results).

Calculation	ΔF /mN	E_r ($\times 10^{-6}$)
Model	1,8	0,18
FEM	0,9	0,09

4. CONCLUSIONS

The analysis treated on the two examples reported shows that the expected results are not confirmed in the simulation. Due to the closeness of the masses in the MCF100, their geometry influences the mutual attraction significantly. In fact, the large central aperture of the MCF100 masses concentrates the attraction effects on the edge of the mass under study. Instead, in MCF1000, the smaller central aperture and the larger distance between the masses, yields to a lower difference between the approaches.

The FEM simulation shows that the influence of mutual masses attraction is negligible in both the examples. Nevertheless, it worth simulating this phenomenon in order to prevent rough errors.

Furthermore, usually the masses dimensioning doesn't consider this effect. Taking into account the results presented in this paper, the conducted analysis should be carried out, above all, in the design of new deadweight machines.

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