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MEASUREMENT OF PARAMETERS TO VALUE HUMAN LIFE EXTENSION

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Abstract – In safety analysis as in engineering, the development of a satisfactory mathematical model is required to identify the parameters that need to be measured and calculated. The establishment of a proper calculus of safety begins with the recognition that the fundamental concept is life expectancy, and then, by extension, the increase in life expectancy that a safety measure brings about. J-value analysis, which rests on this concept, is a method of estimating how much should be spent on a new safety system to improve health and/or safety, with the amount depending on both actuarial and economic data. Measurements made to quantify the first J-value trade-off between the average person's free-time fraction and his income result in an inferential estimate of the elasticity of marginal utility. This is an important economic parameter in its own right, and moreover feeds into a second trade-off between life expectancy and income, which J-value analysis shows to be the balance that must be struck in evaluating a new safety scheme.

Keywords J-value, calculus of safety, elasticity of marginal utility

1. A CALCULUS OF SAFETY

In physics and engineering, a whole body of learning on the governing laws is available and indeed widely accepted, but this fortunate situation does not pertain to the same extent when it comes to socio-economy theory. This was noted by Von Neumann and Morgenstern in 1944 [1], and, while undoubtedly there has been progress since then, the basic premise still holds true today. But as in engineering, the development of a satisfactory mathematical model is the first priority, since it is only in this way that we can identify the parameters that need to be measured and calculated.

In the case of safety analysis, it is necessary to start with a clarification of principles, since the topic is highly emotive, and confusion can be introduced, intentionally or unintentionally, by an appeal to plausible but imprecise and imperfectly representative concepts. In particular, we need to clarify the concepts of "saving a life" and "preventing a fatality". While superficially attractive and in frequent use, these concepts contain a fundamental flaw: both are impossibilities. Since we are all going to die, no-one's life can be saved and no-one's death can be prevented. We need to realise that the best we can do is to rescue someone from a position of hazard, and restore that person's life expectancy to what it was in the absence of the hazard. To be sure we could avoid the logical flaw by adding the qualifier "temporarily", but this is not done in normal practice even by scientists and certainly not by politicians and the media, who have, quite properly, a large influence over safety debates. Largely people are unaware of the need to do so. But the logical flaw inherent in the two concepts means that they are unsuitable as foundations for a rigorous calculus of safety.

Having established that we cannot save a life, it is clearly not sensible to ask how much should be spent to save a human life. Moreover, "the value of a human life" is not an entirely obvious concept. If a child of 10 is rescued from a threat to his life, that child will have about 69 years of life expectancy left, whereas if a person of, say 60, is rescued, that person will have about 22 years left. Discounting will have a part to play, so that the worth to each person now of the later years gained will be worth relatively less because of the delay before they can be enjoyed. But this will still leave the child having a greater number of expected life-years restored to him. So one can speak loosely of the child's life being worth more, but only on the understanding that what we really mean is that the life expectancy of the child when the threat is removed is greater than the unthreatened life expectancy of the 60 year old, and so the restoration of life expectancy for the child is more. This is not an entirely new concept to the man in the street, and may be the basis for the cry in times of danger: "women and children first!" – it is the children's life expectancy that is the prize, with women perhaps being valued for their greater role in sustaining the child on the early parts of road to that life expectancy.

This philosophical and ethical review prompts the important realisation that a safety system cannot prevent death, but it can postpone it, ideally by restoring life expectancy to what it was in the unthreatened state. As realised in a pioneering work by Lord Marshall [2], the calculus of safety needs to be founded on the mathematically precise concept of life expectancy.

This viewpoint transfers our focus onto life expectancy and its associated variables such as mortal hazard rate as the important parameters to be measured and/or calculated in order to make progress with safety analysis.

The J-value method [3] (J for judgment) is developed from this standpoint, and is a fully objective technique for estimating the maximum amount that should be spent on a new system to improve health and/or safety. It depends on measurements of both actuarial and economic data to quantify two trade-offs. The first trade-off is between the average person's free-time fraction and his income, while the second draws on the outcome of the first to establish a balance between income and life expectancy. The first trade-off, carried out at a societal level, will be modelled in this paper, and the model will indicate which economic parameters need to be measured. Ways of measuring those parameters will be discussed, and values provided for the UK. A brief discussion will then be given of the way in which the data gathered can be used to value life extension in the context of a safety system.

2. LIFE-QUALITY INDEX AND INCOME, LIFE EXPECTANCY AND FREE-TIME FRACTION

We follow [4] in postulating that the fundamental factors influencing the current quality of life for any given person are first how long he or she can expect to live from now on and secondly how much he or she will have available to spend, both on life's necessities and on its luxuries. The first factor corresponds to life expectancy, X (years), and may be developed further by noting that, in considering their quality of life, people will generally wish to increase the time that they are free to dispose of as they think fit, at the expense of the time that they are obliged to spend working. In fact, society as a whole and individuals within it will engage in a complex trade-off between free time and working time. In this, the benefit from the extra income derived from working longer is weighed against the disbenefit resulting from the loss of free time. Thus the average, expected, free time from now on, $F = fX$, in which f is the average free-time fraction from now on, is a better indicator of quality than pure life expectancy. Accordingly, we may advance a life-quality index, Q_1, for the average person as a Cobb-Douglas utility function [5], with arguments earnings per year and expected free time from now on:

$$Q_1 = \alpha_1 G^{\beta} F^{\gamma} \tag{1}$$

where α_1, β and γ are positive constants and G is the average earnings of a person as measured by the GDP per head (£y^{-1}). This figure includes both wages and return on capital, and is chosen for ethical reasons: everyone is then treated equally as regards income.

By the theory of utility [1], [6], we may manipulate (1) into the form

$$Q_2 = XG^q f \tag{2}$$

and still have an equivalent quality of life index, since the revised index is simply a scaled version of the former; in particular, it is still an increasing function of its three arguments, X, G and f:

Very low values of free-time fraction, $f \le f_h$, would undoubtedly have a detrimental effect on the population's health, and thus cause a reduction in life expectancy, X: "working to death". As f increases past the harmful limit, f_h , however, this effect will disappear, and it may be assumed that X and f will then be independent of each other. Thus there can be and will be no trade-off between X and f in the region where $f > f_h$. Two important, potential trade-offs remain, however. The first is between income and free-time fraction, *viz*. between G and f, while the second is between income and spending on a health and safety scheme that will prolong life expectancy, *viz*. between G and X.

These two trade-offs may be considered separately because of the independence of X and f (in the absence of "working to death"). In fact, this mirrors what happens in practice in advanced countries. Fixing the free-time fraction, f, involves a diffuse, population-wide bargaining process over working hours, retirement age, wages and pensions that involves Government, employers, trade unions and the judiciary. By contrast, spending on health and safety schemes is considered normally on a case-by-case basis, although it may still involve a number of players in practice, including the employer, the safety regulator, Government agencies, people living nearby, pressure groups and the media.

3. TRADE-OFF BETWEEN FREE-TIME FRACTION AND INCOME

Since in the free-time negotiation, we may regard X as a constant, without loss of generality, we may multiply both sides of (2) by the positive constant, $1/X$, to give a life-quality index in terms of G and f :

$$Q_f = G^q f \tag{3}$$

When (3) holds, it will be possible for the average person to give up some free time in return for an increase in income and retain the same value of Q_f , indicating the same level of satisfaction. In this case, the average person will have no preference as whether such a change occurs or not: he will be indifferent, hence the term, "indifference curve". Specifically, at any point, (f, G), on the indifference curve it is possible to give up a quantum of free-time fraction, Δf , in return for a compensating increase in income, ΔG . The ratio, $\Delta G/\Delta f$ as $\Delta f \to 0$ $= dG/df$ is then the marginal rate of substitution (MRS) of free-time fraction in the place of income. As (3) is convex to the origin, the well-established economic finding will hold that a plentiful supply will mean a lower price, so that the modulus of dG/df (the per-unit price) will be lower when f is high.

Meanwhile, following [7], we may use a Cobb-Douglas function as a simple but realistic model for the country's production. Then the Gross Domestic Product for the country, G_C (£y^{-1}), will be given by:

$$G_C = AK_C^{1-\theta} L_C^{\theta} \tag{4}$$

where K_C is a country's capital (£), L_C, is its labour flow (years per year and so dimensionless), θ is a constant in the range $0 \le \theta \le 1.0$, while A is a productivity constant, embodying notions such as the education-level of the workforce, general know-how and technical efficiency. The flow of labour, L_C, may be seen to be the number of people in the country, N_C, multiplied by the work-time fraction, $w = 1 - f$: $L_C = N_C w \times 1 = N_C(1-f)$. Substituting into (4) gives:

$$G_C = AK_C^{1-\theta} N_C^{\theta} (1-f)^{\theta} \quad (5)$$

Noting that the GDP per person, *G*, is $G = G_C / N_C$ and that *K* is the capital per person (£) allows:

$$G = AK^{1-\theta}(1-f)^{\theta} \quad (6)$$

Equation (6) represents a downward-sloping line in the plane of (*f*, *G*), which may be regarded as defining the collectively determined constraint linking the average person's possible earnings to his free time. A position on this line is selected by choosing an appropriate value of free-time fraction, *f*, provided only that it is located away from the low values of *f* where people's health would be impaired; obviously this would be recommended on both ethical and practical grounds, but importantly for this analysis, the condition is needed to ensure the independence of *f* and *X* is not compromised.

We may assume that the complex bargain being struck by society will find the position that yields the average person the greatest satisfaction. This will entail maximising the life quality index (3) subject to the constraint of (6). The optimal situation will occur when the line of equation (6) meets the convex curve of (3) at a tangent. This will occur when their derivatives, df/dG, are equal.

The total derivative of equation (3) is given by:

$$dQ_f = qfG^{q-1}dG + G^q df \quad (7)$$

Moreover, Q_f is constant on an indifference curve, implying that $dQ_f = 0$, and so:

$$\frac{df}{dG} = -\frac{f}{G}q \quad (8)$$

Meanwhile, differentiating equation (6) with respect to *G* gives:

$$1 = -\theta A\left(\frac{K_C}{N_C}\right)^{1-\theta}(1-f)^{\theta-1}\frac{df}{dG} = -\frac{\theta G}{1-f}\frac{df}{dG} \quad (9)$$

so that

$$\frac{df}{dG} = -\frac{1-f}{\theta G} \quad (10)$$

Assuming tangency at (f_0, G_0), we may equate (8) and (10) to give:

$$-\frac{f_0}{G_0}q = -\frac{1-f_0}{\theta G_0} \quad (11)$$

which gives a value of

$$q = \frac{1-f_0}{\theta f_0} = \frac{1}{\theta}\frac{w_0}{1-w_0} \quad (12)$$

[The same result may be found by substituting (6) into equation (3), then finding the value of *f* that maximises Q_f by differentiating the expression with respect to *f* and setting the differential to zero. This method is described in [7] and provides a check on the correctness of the outcome.]

Fig. 1 illustrates the situation for a set of parameters estimated for the UK in 2006, together with life-quality indices 10% above and below the actual. Also marked on the figure is a region where overwork will lead to a health hazard, where life expectancy and expected free time are not independent. The extent of this region does not need to be defined precisely: it is sufficient for our purposes that the negotiation over free-time fraction occurs outside it. Similarly, there will be a low-income region which will be infeasible, since the population would not be supported here. Again the upper level of this need not be defined precisely, since it is again sufficient that the negotiation over free-time fraction takes place outside this region.

Fig. 1 provides a mathematically precise yet intuitively meaningful representation of society's process of trading between free-time fraction and income. Society as a whole tests various positions on the average-income line for utility, and by a process of trial and error finds the point that yields the greatest overall satisfaction.

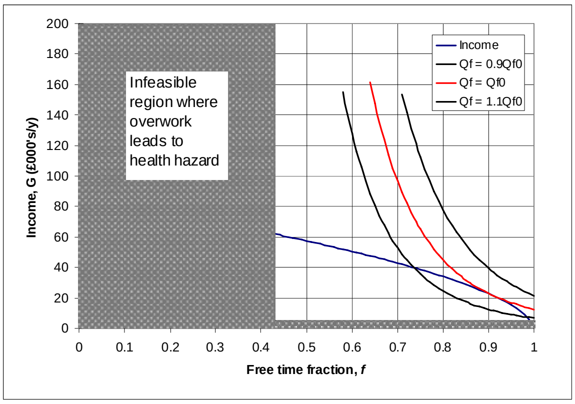

Fig. 1 Indifference curves and curve of average income

4. ECONOMIC MEASUREMENTS NEEDED

The model derived above requires the following economic measurements: the share of wages in the GDP, θ, the country's GDP, G_C, the number of people in the country, N_C, and the average work-time fraction, w_0 (from which f_0 can be found). w_0 is a composite measurement, requiring measurements of the following averages: hours worked per week, hours spent travelling to and from work per year, career length, life expectancy for workers and employment rate.

5. INFERENTIAL MEASUREMENT OF THE ELASTICITY OF MARGINAL UTILITY

The parameter, *q*, emerges as an inferred measurement, found from (12). It is a particularly important parameter, since the term in (3), G^q, is a power utility, used in the form $G^{1-\eta}$, where η = constant, by the UK Treasury [8] in deciding the advisability of publicly funded projects. The elasticity of marginal utility, η, has wide significance in welfare economics and also in insurance, where $\varepsilon = -\eta$ is known as the relative risk aversion coefficient. A number of ways of measuring this critical parameter have been explored, but the J-value approach offers a new and independent method.

6. FURTHER MEASUREMENTS NEEDED BY THE J-VALUE

The J-value is the ratio of the actual spend on a safety system to maximum reasonable, so that J = 1 occurs when the actual spend matches the maximum sensible. This corresponds to the locus in the $G - X_d$ plane of the indifference curve given by

$$G^q X_d = Q_0 \tag{13}$$

where X_d is the average discounted life expectancy of the group potentially exposed to a hazard [3], and Q_0 is the life quality in the unthreatened state. It may be plotted only after the GDP per head has been measured, the average discounted life expectancy calculated and the value of *q* estimated based on the analysis of Sections 2 and 3 and the measurements detailed in Section 4. See Fig. 2.

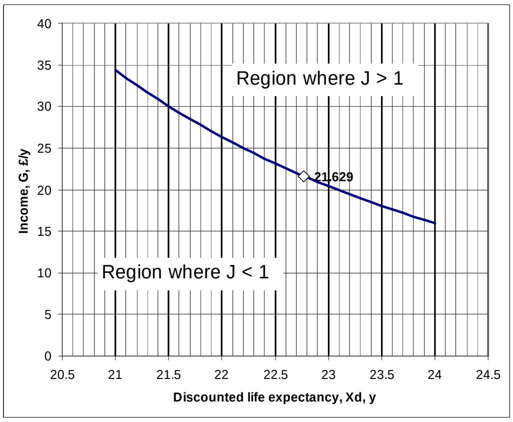

Fig. 2 Indifference curve, the locus of J = 1 (UK 2006 data)

The determination of the J-value requires additional measurements of mortality hazard rates. These allow the change in discounted life expectancy associated with a nuclear safety system to be calculated, for example [9].

7. MEASUREMENTS AVAILABLE

We are strongly indebted to successive generations of economists and statisticians who have measured and documented economic parameters over many years in the UK and other countries. Data may be sought from a range of sources, but particularly the Treasury [8], the Office for National Statistics, which subsumes the Government Actuary's Department [10], [11], [12], the Pension Service [13] and the Department for Transport [14]. The data are updated regularly (monthly, quarterly or annually), and data series are available from which appropriate averages may be drawn. For example, the share of wages in the GDP, θ, is given over a 60-year period in Fig.3, demonstrating its approximately constant nature, as predicted by the Cobb-Douglas equation (4). The fraction of GDP paid as wages

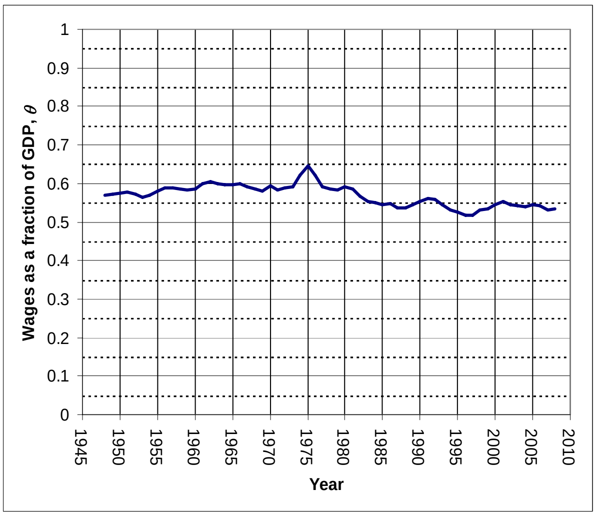

Fig. 3 Share of Wages, θ, in UK GDP from 1948 to present

rose during the 1960's and 1970's, reaching a peak in 1975, a time of great industrial unrest in the UK. It returned to approximately 1950's levels in about 1980, thereafter settling at roughly a constant level. The mean value for the whole data series (61 years in length) is $\theta = 0.569$ with a standard deviation of 0.028, but it has been decided to take the mean for the last 30 years as a better indicator for the future, namely $\theta = 0.546$, which has a smaller standard deviation of 0.018.

Meanwhile the other parameter needed for the determination of *q* (equation (12) is the average work-time fraction, w_0, which is itself a composite parameter, requiring a knowledge of the average retirement age (altering in the UK from 60 for women and 65 for men to a common figure of 68 by 2046), the probability of being in work, the probability of being of working age and the number of hours worked per week (including an allowance for travelling time). A detailed statistical model is needed to calculate the work-time fraction as a function of age, *w*(*a*), and thence a single, average figure for the population, w_0. The work-time fraction as a function of age is given in Fig. 4, and it can be shown that $w_0 \approx w(0)$, the fraction of his expected life from now on that a new born child may expect to devote to working. Thus $w_0 \approx 0.091$. Hence, from equation (12) we may estimate $q \approx 0.18$.

Fig. 4. Work-time fraction to the end of life against age

The only further structural parameter for the J-value techniques is the appropriate value of the discount rate. Guidance is available from the Treasury and from Pearce and Ulph [15], which suggests that a range of 1 to 4 % p.a. are likely to be appropriate. We have chosen a discount rate at the centre of this range, namely 2.5% p.a.

8. NUCLEAR INDUSTRY EXAMPLE

Concerns were raised over the discharge by British Nuclear Fuels Ltd (BNFL) of Tc-99 from its Sellafield complex into the Irish Sea, especially after the concentration in lobster exceeded limits recommended by the Food Standards Agency. Tc-99 is one of the constituents of the medium active concentrate (MAC) that is a by-product of Magnox fuel reprocessing and is a weak-gamma emitter. The Environment Agency review of Tc-99 disposal concluded that it would be desirable to reduce the discharge limit from 90 TBq/y current in 2001 to 10 TBq/y in 2006. The Agency set out options labelled A, B, C and D for consideration. Option C (the option chosen) was estimated to save each person in a critical group of 2663 members of the general public a dose of 30 μSv of radiation a year for 10 years.

The calculation of the change in life expectancy as a result of eliminating a low-level nuclear radiation dose is complex, since a radiation-cancer induced will have a latency of not less than about 10 years, and thereafter the effects are stochastic over a period of about 30 years. The method used by Lord Marshall et al. [2] to calculate the effects of a one-off exposure following an accident had to be extended to cover a prolonged exposure, where the effects on those unborn at the beginning of the exposure need to be accounted for [9].

Using the methods of [9], as well as further refinements added since then [16], [17], [18], an average gain in life expectancy of 3.20×10^{-4} years (approximately 3 hours) was calculated as the benefit of installing the protection system. Applying the J-value method with $q = 0.18$ to Option C, which has a cost in present day values of £12,600,000, a J-value of 116 is produced at a discount rate of zero, and 184 when an interest rate of 2.5% is assumed. Since the maximum sensible expenditure yields a J-value of unity, it is clear that there was an overspend of about two orders of magnitude on the Technetium-99 Removal Plant.

Such large overspends are not untypical of the nuclear industry, but they are not confined to that industry. One outstanding example with a similarly huge J-value was the spending by successive UK Governments of about £7 bn to save perhaps half a dozen lives at most from BSE/vCJD. This and other cases are examined in [19]. Although the regulators' stated targets are usually reasonably close to J = 1 (see Fig. 5), very often much more money is actually spent on safety schemes as a result of the lack, until very recently, of an objective comparative scale [20].

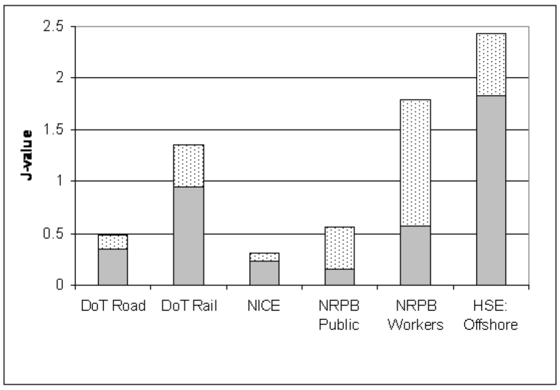

Fig. 5 Showing UK regulators' recommendations clustered around J = 1.

[DoT = Department for Transport; NICE = National Institute for Clinical Excellence, NRPB = National Radiological Protection Board (now part of the Health Protection Agency), HSE = Health and Safety Executive. DoT Rail was for multiple fatalities; it has now been reduced to DoT Road.]

While the Technetium example concerned a prolonged exposure, dealing with the output of a probabilistic risk assessment means, very often, evaluating cases where a nuclear safety measure reduces the frequency of a radioactive release rather than its magnitude. It is possible to calculate the change in life expectancy in such a case also and hence the J-value [21].

9. CONCLUSIONS

Making good safety decisions with an impact on human lives requires both an adequate prediction of the before and after effects and a method of weighting these. Knowing what parameters are important requires a rigorous calculus of safety, and this needs to be founded on the mathematically precise concept of life expectancy. Change in life expectancy is a much finer tool than death count. Only in this way can we gain a proper knowledge of what parameters should be measured and calculated.

J-value safety analysis starts from this firm foundation, and achieves an objective judgment on safety system expenditure by producing a mathematical model. The model is dependent on economic and actuarial data derived from measurements as explained in this paper. The method provides, for the first time, an objective scale by which to judge the efficacy of safety expenditure.

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