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PROBE RADIUS COMPENSATION AND FITTING ERRORS IN CAD-BASED MEASUREMENTS OF FREE-FORM SURFACE: A CASE STUDY

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Abstract the present paper will present problems connected with accuracy inspection of free-form surfaces, performed with coordinate measuring machines equipped with touch measurement probes. The most important among them are, apart from the number and distribution of measurement points on a surface, errors caused by the probe radius compensation and determining the coordinate system. A theoretical analysis as well as the results of tests on the influence of compensations errors and errors in fitting the data to a CAD model on the results of computation of the points on the milled free-form surface will be presented. Considering any of these factors separately makes it impossible to obtain the complete picture of their mutual dependence. It turns out that applying compensation, leading to errors in determining measurement points, influences the uncertainty of the position and orientation of the coordinate system determined by fitting the compensated data to the CAD model.

Keywords: CAD-based measurements, free-form surface, accuracy inspection

1. INTRODUCTION

Numerically controlled measurement machines (CMM) equipped with touch trigger or scanning probes with ball-end styluses are widely used in inspecting the surface accuracy. Such measurements aim at determining the workpiece coordinates of the points taken by the centre of the ball moving from point to point along the surface of the workpiece in the machine coordinate system. The coordinates of their corresponding points on the workpiece surface are determined by the system software through taking into consideration the correction vector whose length equals this of the ball radius, and the direction is perpendicular to the surface while the orientation is towards this surface.

Growing demands concerning product functionality, ergonomics, and aesthetics, force creating machine parts composed of 3D curvilinear surfaces. The accuracy inspection involves digitalising the measured object (coordinate measuring with the use of the scanning method) and later comparing the obtained measurement points to their corresponding points on a CAD design (model). At each measurement point, geometric deviations, or the distances of these points from their projections on the nominal surface, are determined [1]. Different sampling strategies (number and location of measurement points) provide different measurement results for the same surface. This is connected to the fact of measuring a finite number of discrete points on the measured surface described actually with an infinite number of points. Since geometric deviations are different at each point, measurement results depend on the number and location of these points [2]. The processing accuracy inspection results may be presented in the form of a three-dimensional plot or a deviation map.

Software for numerically controlled machines makes it possible to generate a path along which the end moves on the surface on the basis of a CAD model. A typical solution is to measure some specified number of points with automatic probe radius compensation. Theoretically, the direction of the correction vector at each measurement point (in the case of measuring with trigger probes, also the direction of the probe approach to the surface) is normal to that of the model. Due to inevitable machining errors, the actual contact points are not identical to the nominal ones, and the directions of the correction vectors are not normal to the actual surface. This fact leads to obvious errors in determining measurement points on the surface of the measured workpiece [2,3]. The errors do not exclusively depend on the values of geometric deviations; the greater the measuring probe radius is, the greater the errors are. In order to avoid probe radius compensation errors, measurements without such compensation are performed. The coordinates of the indicated points (coordinates of the probe end centre) are compared to their corresponding points on the off-set theoretical surface which is shifted from the one of the model by the probe radius in the direction normal to the surface, towards the ball centre [3].

At the first stage of measurements, it is necessary to establish the relationship between the coordinate system of the model superimposed on the workpiece and that of the machine. To achieve this, the workpiece coordinate system is defined in the manual mode, and later the coordinate systems of the model and the workpiece are superimposed virtually. The relationship between these coordinate systems is described by the transformation (rotation and translation) matrix. This common procedure makes it possible for the CMM software to generate theoretical measurement points on the workpiece (through the virtual model). Next, to obtain a more accurate mutual location of the workpiece and the model, after performing automatic scanning of a specified number of points (usually a few dozen points because of time limits), the obtained data should be fitted to the model. The least square method provides an optimal solution [4]. The fitting accuracy increases with the number of measurement points. The existence of different geometric deviations at each surface point results in a fitting uncertainty dependent on the number and location of measurement points, and thus uncertainty of the relationship between the workpiece coordinate system (and, obviously, each point indicated in this system) and the coordinate system [5-8]. The uncertainty area is limited with a hyperellipsoid whose dimensions depend on the transformation parameters scatter and the adopted confidence level [6,9].

It is not possible to avoid errors in determining geometric deviations, even if measurements without compensation are taken, since fitting uncertainty is transferred to the uncertainty of the correction vector direction.

All the factors listed above influence the measurement results at each point at the same time. The influence of the direction error of the correction vector can be minimised without applying any probe radius compensation as it turns out that compensation leads, above all, to increasing the received data scatter and then the uncertainty of determining the coordinate system on the basis of fitting a few dozen points to the CAD model (which also means the uncertainty of determining the measurement points) exceeds the value of the probe radius compensation errors.

This paper presents a theoretical analysis as well as the results of tests on the influence of compensation errors and errors in fitting the data to a CAD model (at the same sampling strategy) on the results of the accuracy inspection of machining a free-form surface with the milling method. The influence of applying compensation to determining the coordinate system will also be described.

The experiments were carried out with the use of a MISTRAL STANDARD 070705 coordinate measuring machine equipped with a Renishaw TP200 touch trigger probe with a stylus of 20mm in length, with a ball tip of 2mm in diameter, MPE_E=2.5+L/250 (PC DMIS software).

2. MAJOR SAMPLING PROBLEMS

2.1. Contact error

In CAD-based measurements, the direction in which the probe approaches the surface (the correction vector), determined by software, is normal to the model. Geometric deviations are the reason why this direction is not perpendicular to the surface. As the result of converting the indicated measured point, a corrected measured point, not a point on the actual surface, is obtained. Sampling errors occur, which is illustrated in Fig.1.

Not applying the radius compensation means minimising the contact error (Fig. 2). In this case, the indicated measured points shall be compared to the points obtained from off-setting the corresponding points from the CAD model by the value of the probe radius. However, this does not mean that the exact points are known. The unavoidable influence of the uncertainty of fitting data to the model, resulting from the surface geometric deviations, still remains.

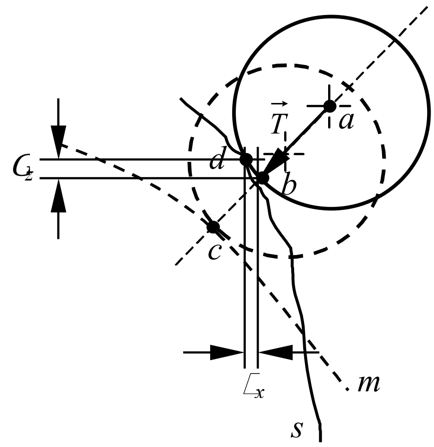

Fig. 1. Contact error: m – CAD model, *s* – actual surface, *a* – indicated measured point, *b* – corrected measured point, *c* – target contact point, *d* – actual contact point, $\vec{T}$ - tip correction vector, G_x – error component in *X* direction, G_y – error component in *Y* direction.

2.2. Fitting uncertainty

If surface geometric deviations *H* are random in character and have normal probability distribution, for a big enough number of measurement points assumed as the transformation base, it can be assumed that the transformation parameters are random variables of normal probability distribution. In a border situation, for an infinite number of measurement points, the expected values of transformation parameters describing the location of the coordinate system of the specific measured surface will be obtained. Consequently, the probability distributions of transformation parameters deviations from the expected values are also normal.

For the case of analysing the joint probability distribution $f(\Delta t)$ of the vector of transformation parameters deviations centred around the expected values (P = 0), dependence can be illustrated as follows:

$$f(\Delta t) = \frac{1}{\sqrt{(2S)^n \det[O]}} \exp\left(-0.5\, \Delta t^T O^{-1} \Delta t\right) \quad (1)$$

where:

O – 6 x 6 covariance matrix,

Δt = *[dx,dy,dz,ax,ay,az]* – the vector of transformation parameters deviations from their expected values.

Variability of the parameters deviations vector is connected with equal probability (probability concentration) surfaces described by the equation (2)[9]:

$$\Delta t^T O^{-1} \Delta t = K^2 \quad (2)$$

where:

K – the constant dependent on the assumed probability.

These surfaces have the shapes of hyperellipsoids whose centres are determined by the expected values vector [6,9]. The directions of the hyperellipsoids axes determine eigen (unit) vectors of the covariance matrix, and the squared lengths of the semi-axes – the corresponding eigen values of the covariance matrix.

The eigen vectors and values of a covariance matrix might be obtained by decomposing this matrix (3) (matrix properties allow for this) [9].

$$\Sigma = U \Lambda U^T \tag{3}$$

where:

U – matrix whose columns are the covariance matrix eigen vectors

Λ – diagonal matrix of the covariance matrix eigen values.

The hyperellipsoid size is dependent on the assumed probability, and the constant K value is determined from the chi-square distribution, in this case for six degrees of freedom [6,9].

3. EXPERIMENTAL RESEARCH

3.1. Measured surface characteristics

The experiments were performed on a free-form surface obtained in a three-stage milling process. In the last stage (profiling), the following parameters were applied: a ball-end mill of 6 mm in diameter, rotational speed equal to 7500 rev/min, working feed 300 mm/min and zig-zag cutting path in the XY plane (Fig. 2).

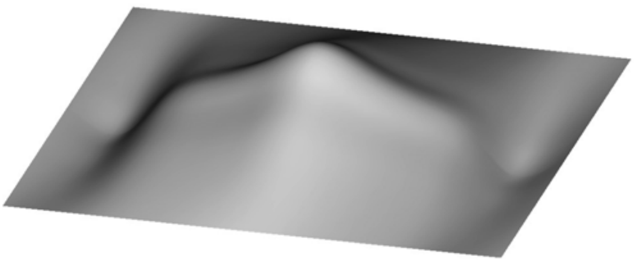

Fig. 2. CAD model of the surface

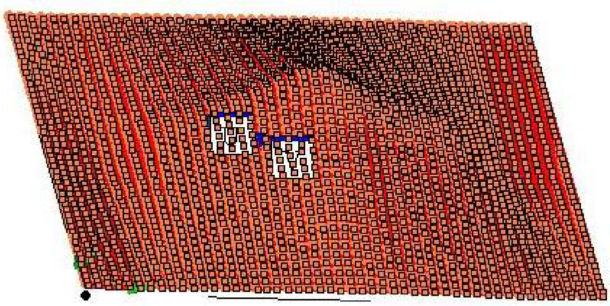

Fig. 3. Measurement points distribution on the CAD model (CMM software)

1. The surface was scanned (applying radius compensation) with the UV method, 2500 (50 rows and 50 columns) uniformly distributed measurement points were scanned on the surface (Fig. 3), and the process of fitting the data to the nominal surface was then carried out in which the least square method was applied and all the measurement points were used. The measurement process was repeated.

2. The described process was subsequently repeated without applying radius compensation (an option in PC DMIS software).

The surface was characterised by geometric deviations H whose 3D graph is illustrated in Fig. 4, probability distribution in Fig. 5. It can be assumed that the deviations values were of a quasi-normal (random) character.

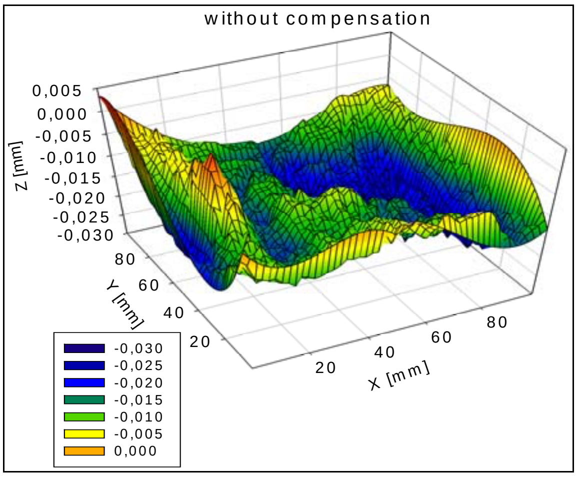

Fig. 4. Geometric deviations 3D graph (scanning without applying radius compensation)

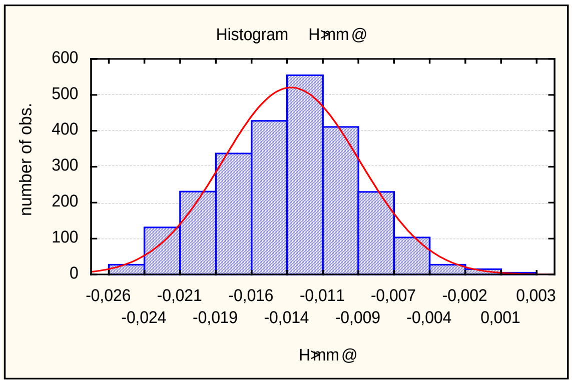

Fig. 5. Geometric deviations probability distribution (without applying radius compensation)

3.2. Impact of radius compensation on accuracy inspection results

The obtained data is presented graphically. The plot of H values for measurements applying radius compensation is illustrated in Fig. 6, without applying radius compensation in Fig. 7. Table 1 shows the statistical parameters of the H sample.

An analysis of the obtained graphs shows that applying correction caused systematic errors whose values were dependent on the surface direction gradient and curvature radius. The arithmetic mean of the determined surface deviation was greater by 0.004 mm, and the scatter increased more than threefold.

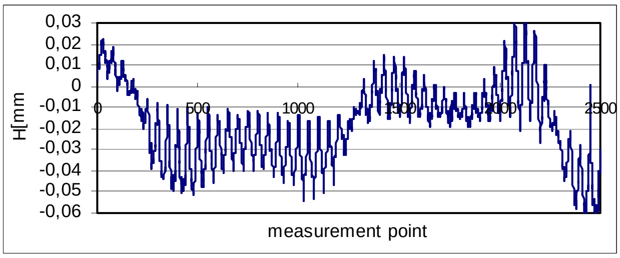

Fig. 6. Plot of geometric deviations; scanning with radius compensation

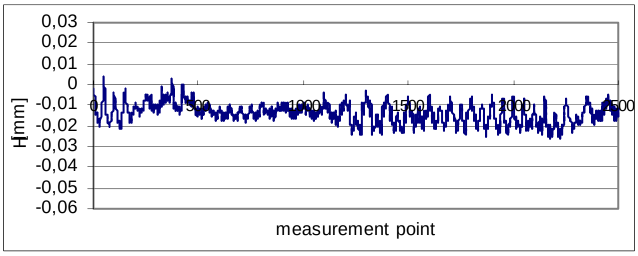

Fig. 7. Plot of geometric deviations H scanning without applying radius compensation.

Table 1. Statistical parameters of *H* population (in mm)

	without radius compensation				with radius compensation			
	geom. dev. *H*	component x	component y	component z	geom. dev. *H*	component x	component y	component z
mean	-0.0137	-0.0006	-0.0001	-0.0141	-0.0178	0.0000	0.0000	0.0000
std. dev.	0.0047	0.0092	0.0055	0.0057	0.0181	0.0376	0.0247	0.0226
min.	-0.0263	-0.0316	-0.0167	-0.0108	-0.0642	-0.0919	-0.0585	-0.0893
max	0.0034	0.0299	0.0227	0.0330	0.0320	0.0526	0.0612	0.0420

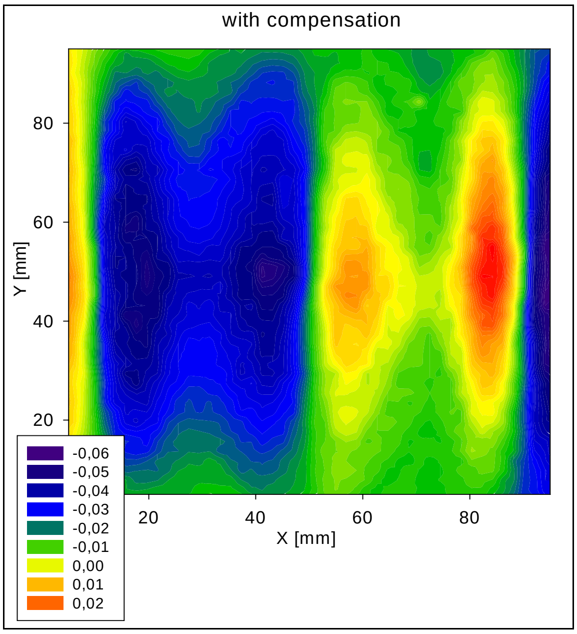

Fig. 8. Map of geometric deviations; scanning with applying radius compensation

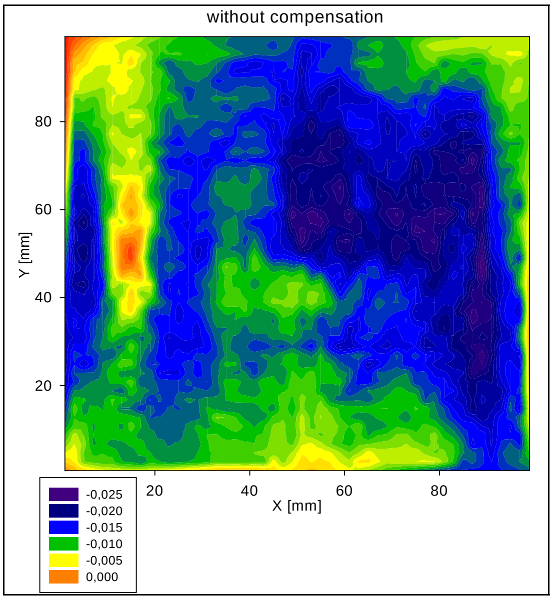

Fig. 9. Map of geometric deviations; scanning without applying radius compensation

In order to illustrate the differences in measurements results more precisely, maps of these differences in relation to the XOY plane were prepared. The maps are presented in Fig. 8 and Fig. 9. Analysing these maps, it can be seen that compensation errors have actually determined (regular) distribution on the surface, connected to the surface shape, while the distribution of the geometric deviations determined in measurements without compensation has a different character. A comparison of the graphs suggests that the contact errors resulting from geometric deviations are not the only source of the correction errors.

3.3. Determining fitting uncertainty

In the next stage, groups of 50 measurement points were randomly selected out of the scanned with radius compensation 2500 points fifty times in order to perform the fitting. 50 sets of transformation parameters deviations from their expected values, or the values obtained in the process of fitting on the basis of all the scanned points, were obtained. The described process was subsequently repeated for data obtained from measurement without applying radius compensation. The normalities of the transformation parameters deviations (*dx,dy,dz,ax,ay,az*) probability distributions were checked graphically (example in Fig. 10).

Assuming the P=0.95 ($\chi^2 \le \chi^2_{0.95}(6) = 12.59$) probability for the upper limit of the possible scatter range of the coordinate transformation and P=0.05 ($K^2 \ge F^2_{0.05}(6) = 1.63$) for the lower limit from the (2) dependence, the equal probability hyperellipsoids limiting the (uncertainty) space were established. The computations and graphical illustrations (Fig. 11, Fig. 12) of the results were performed in the Matlab software. The asterisks represent the transformation vector deviations scatter. It can be observed that the deviations of the transformation vector from their expected value, obtained in the experiments, are in the space within the uncertainty contours.

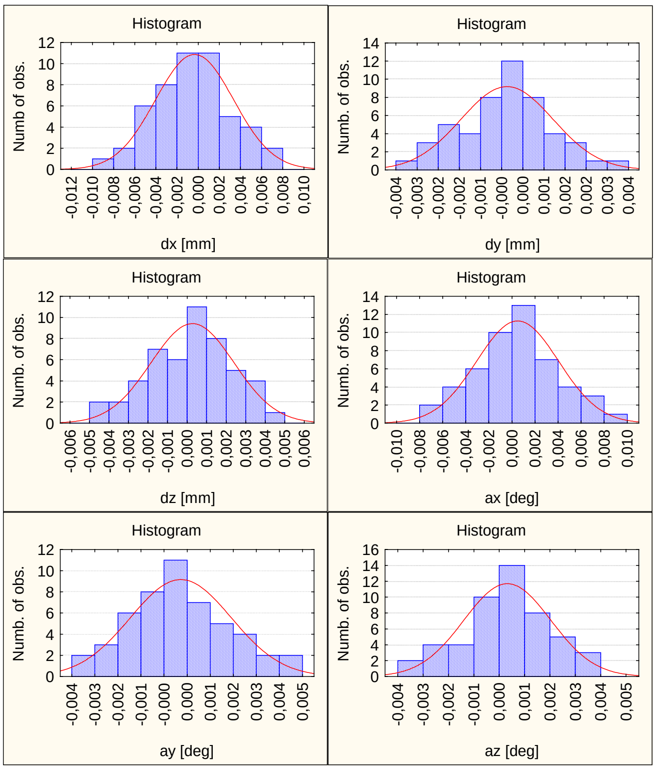

Fig. 10. Probability distributions of transformation parameters deviations from the expected values (without radius compensation)

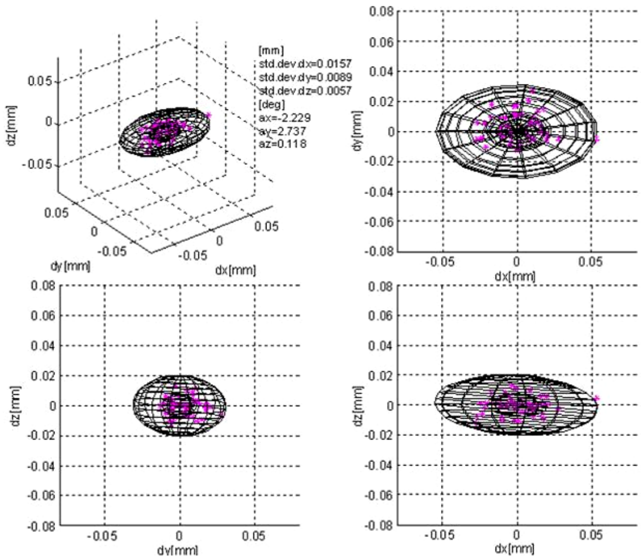

Fig. 11. Uncertainty contours and their projections on the coordinate system main planes, with applying radius compensation

Comparing the contours of the uncertainty of determining the coordinate system through fitting the measurement data to the CAD model in measurements with and without the radius compensation, it can be observed that, as in the case of the scatter of the determined surface geometric deviations, the uncertainty proportions are 3-4 times bigger for the measurements in which compensation was applied.

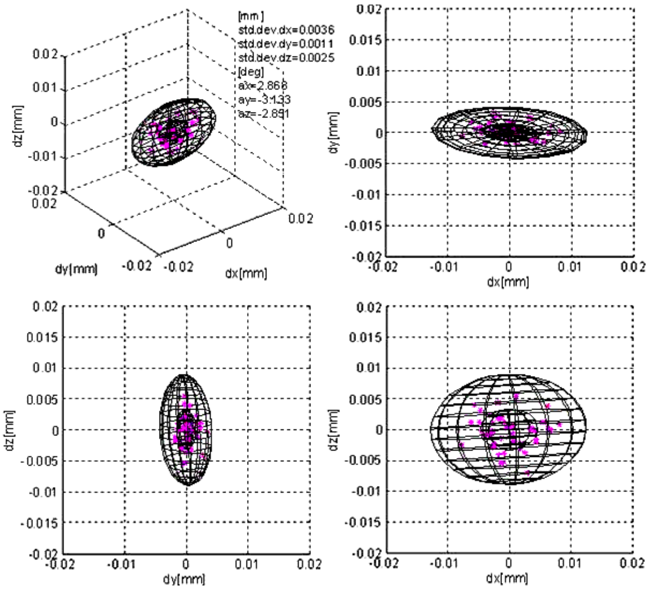

Fig. 12. Uncertainty contours and their projections on the coordinate system main planes, without compensation

4. CONCLUSIONS

This paper concerns the accuracy inspection of producing free surfaces, performed with the use of coordinate measuring machines equipped with touch trigger probes and software capable of programming the measuring track on the basis of CAD models. The authors concentrated on the major problems connected with measurement accuracy, i.e. the probe radius compensation error and the uncertainty of fitting data to the CAD model. The idea and results of research on the influence of probe radius compensation on accuracy inspection results were presented. In the paper, the authors describe and apply the method of determining the uncertainty area characteristic to measurement data, taking into account the correction vector (corrected scan points) and also characteristic to data consisting of indicated measured points. The tests were performed on a milled surface. In the described case, the mean value of geometric deviations obtained in measurements performed without applying radius compensation was smaller by 0.004 mm when compared to the value in measurements with compensation, and the uncertainty resulting from fitting the data to the CAD model approx. four times smaller for one of the transformation directions, respectively.

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