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CALCULATION OF REFERENCE SURFACE PARAMETERS FOR ELEMENTS WHOSE GENERATRIX IS A FRAGMENT OF A CIRCLE

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Abstract – Accurate measurement of irregularities of the geometrical surface structure requires mathematical definition of relevant reference surface, which is called according to the newest terminology, an associated surface. Such surface is the reference feature in relation to which appropriate surface parameters are calculated. When measuring cylindricity deviations, the least squares or minimum zone cylinders have been mostly used as associated features. However, an ideal cylinder cannot be employed as an associated feature when measured elements are not nominally cylindrical but for example barrel- or saddle-shaped. The paper presents mathematical model of calculation of the associated feature for elements whose generatrix is the fragment of a circle and results of the experimental verification of developed concept.

Keywords: associated feature, barrel, saddle

1. INTRODUCTION

Accurate measurement of irregularities of the geometrical surface structure requires mathematical definition of relevant reference surface, which is called according to the newest terminology, an associated surface [1]. Such surface is the reference feature in relation to which appropriate surface parameters are calculated.

Usually, it is assumed that associated surface refers to well-known nominal surface. However, sometimes consideration of wider class of surfaces instead of nominal surface is useful (particularly for purpose of evaluation of usability properties of elements). For example, in the case of conical surfaces the associated surface can belong to the group of cones with clearly defined nominal cone angle. However, the associated surface for conical elements can belong also to the whole group of cones with unrestricted cone angle. In the second case, besides the parameter defining the deviations from the conical surface, we can define also an additional parameter – the angle of the best fitted associated cone. We can define this parameter also as a deviation of the real (extracted) angle from the nominal one.

In cylindricity measurements, the least squares or minimum zone cylinders have been mostly used as associated features [2]. However, an ideal cylinder cannot be employed as an associated feature when measuring elements are not nominally cylindrical but for example barrel- or saddle-shaped.

2. MATHEMATICAL DESCRIPTION OF ASSOCIATED SURFACES

Let us consider two Cartesian coordinate systems denoted as *XYZ* and *X'Y'Z'*: *XYZ* system is associated to the measuring table and the *Z* axis coincides the rotation axis of the spindle. *X'Y'Z'* system is associated to the measured element and its *Z'* axis coincides the axis of the nominal rotational surface. Taking to account the method that macrogeometry of rotational elements is measured (scanning of the workpiece surface is the result of rotations of the spindle and vertical displacement of the measuring sensor) application of cylindrical coordinates is more comfortable. Coordinates of the point in the cylindrical system associated to the table *XYZ* are represented by the set of three values (φ, r, z) and in the system associated to the workpiece *X'Y'Z'* by the set of values (φ', r', z'). Assuming that deviation of *Z* axis from *Z'* axis is small in relation to the dimensions of the workpiece, one can write relationships between the coordinates in both systems as follows:

$$\varphi' = \varphi \tag{1}$$

$$r' = r - e_x \cos\varphi - e_y \sin\varphi - d_x z \cos\varphi - d_y z \sin\varphi \tag{2}$$

$$z' = z\ . \tag{3}$$

Parameters e_x, e_y, d_x, d_y define mutual location of axes *Z'* and *Z* [5].

Rotary surface is obtained by the rotation of so-called surface generatrix around *Z'* axis. Surface generatrix can be described as a function of the surface radius in relation to the *z'* variable.

$$r' = R_n(z') \tag{4}$$

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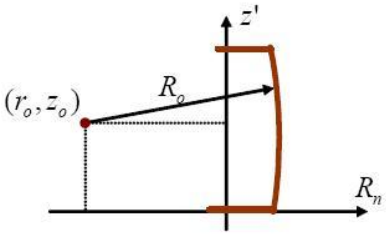
This paper concerns surfaces, whose generatrices constitute a fragment of a circle. Two types of such surfaces can be distinguished: barrel-shaped and saddle-shaped surfaces (Fig. 1).

a)



b)

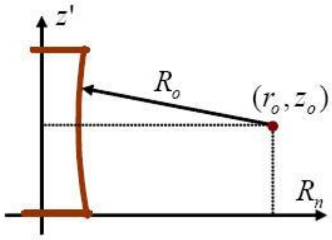


Fig. 1. Mathematical description of the associated surface:

a) a barrel shaped surface, b) a saddle shaped surface

Equation of the generatrix of a barrel can be formulated as follows:

$$R_n(z') = r_0 + \sqrt{R_0^{\,2} - (z' - z_0)^2}\ , \tag{5}$$

and equation of the generatrix of a saddle

$$R_n(z') = r_0 - \sqrt{R_0^{\,2} - (z' - z_0)^2}\ , \tag{6}$$

In the definitions given above parameter R_0 is the radius of the circle fragment that defines the generatrix and r_0, z_0 are the coordinates of the centre of the circle (see Fig. 1). One can notice that for $z' = z_0$ the radius of barrel-shaped surface reaches its maximum ($r_0 + R_0$) and the radius of the saddle-shaped surface reaches its minimum ($r_0 - R_0$).

Relationships given above show that analyzed surfaces can be described by three parameters: the radius of the circle coinciding the generatrix R_0 and the coordinates of the centre of this circle r_0, z_0. Because measurements of the radius value are relative, so parameter r_0 does not really represent any geometrical quantity and it depends on mean value of extracted radius.

Assuming that deviation of the workpiece axis from the rotation axis is small in relation to nominal dimensions of the workpiece, the relationship between coordinates of given point in the coordinate system associated to the table and in the coordinate system associated to the workpiece can be described by equations (1) - (3).

Let us denote the series of values of the workpiece radius obtained through measurements of the profile in points whose coordinates are $(\varphi_i, z_i), i = 1,2,..., M$ by Δr_i. The indicator that defines the quality of the fitting the associated surface to coordinates of measuring points can be described by the following formula

$$J(e_x, e_y, d_x, d_y, r_0, R_0, z_0) = \frac{1}{2}\sum_{i=1}^{M} h(\varphi_i, \Delta r_i, z_i, e_x, e_y, d_x, d_y, r_0, R_0, z_0)^2 \tag{7}$$

where

$$h(\varphi, \Delta r, z, e_x, e_y, d_x, d_y, r_0, R_0, z_0) = \Delta r - e_x \cos\varphi - e_y \sin\varphi - d_x z \cos\varphi - d_y z \sin\varphi - R_n(z), \tag{8}$$

and the function $R_n(z)$ is described by (5) or (6).

The formula h is the distance of the point on the measured surface from the projection of this point on the associated surface in the direction perpendicular to Z axis.

In order to minimize the indicator given by (7), for example, so-called Gauss-Newton algorithm can be applied. This algorithm requires knowledge of good evaluations of initial parameters values. However, such evaluations are often not available, for example if the values of the nominal radiii are very large. For instance, in bearing industry there are barrel-shaped elements, whose surface convex is relatively small. In such cases development of the method that allows good preliminary evaluation of parameters R_0, z_0 is very important. Below the procedure allowing determination of associated surface parameters is described. The procedure consists of three steps: the aim of first two steps is determination of initial evaluations of parameters. The third step is the Gauss-Newton procedure.

Step 1. Calculation of approximate axis parameters of the associated surface and approximation of the function $R_n(z)$ using the polynomial of appropriate order

$$p(z) = a_0 + a_1 z + a_2 z^2 + ... + a_n z_n\ . \tag{9}$$

In order to assure good conditioning of this task, the series $1, z, z^2, ..., z^n$ can be replaced series of appropriately scaled Legendre or Tchebyshev polynomials. Associated surface is described by

$$R_s(\varphi, z) = p(z) - e_x \cos\varphi - e_y \sin\varphi - d_x z \cos\varphi - d_y z \sin\varphi \tag{10}$$

Optimum parameters values can be obtained from the formula

$$\theta = -\left(\sum_{i=1}^{M} w_i w_i^T\right)^{-1}\left(\sum_{i=1}^{M} w_i v_i\right), \tag{11}$$

where

$$v_i = \Delta r_i,$$

$$w_i = [1 \quad z_i \quad ... \quad z_i^{\,2} \quad -\cos\varphi_i \quad -\sin\varphi_i \quad -z_i \cos\varphi_i \quad -z \sin\varphi_i]^T$$

$$\hat{\theta} = [a_0 \quad a_1 \quad ... \quad a_n \quad \hat{e}_x \quad \hat{e}_y \quad \hat{d}_x \quad \hat{d}_y]^T$$

(12)-(14)

Step 2. One should notice that coordinates of the point (φ, r', z) lying on the nominal surface fulfil the equation

$$R_0^2 - (z - z_0)^2 - (r' - r_0)^2 = 0 \tag{15}$$

(one should remember that $z = z'$). Developing above equation one can rewrite it as follows

$$(R_0^2 - r_0^2 - z_0^2) + 2zz_0 + 2r' r_0 - (z^2 + r^2) = 0\,, \tag{16}$$

and then in the following form

$$v = -(z^2 + r'^2)\,, \tag{17}$$

$$w = [1 \quad 2z \quad 2r']^T\,, \tag{18}$$

$$\vartheta = [R_0^2 - r_0^2 - z_0^2 \quad z_0 \quad r_0]^T\,. \tag{19}$$

Let us assume that the interval $[0, H]$ refers to the range of the variable *z*. If $r' = a_0 + a_1 z + a_2 x^2 + \ldots + a_n z^n$, then respective approximate values of the vector ϑ can be calculated from the condition of the minimization of the function

$$J = \frac{1}{2}\int_0^H (v + w^T \vartheta)\,dz\,, \tag{20}$$

which gives

$$\vartheta = \left(\int_0^H ww^T dz\right)^{-1}\left(\int_0^H wv\,dz\right). \tag{21}$$

After calculation of relevant integrals analytically, one can obtain following relationship for $n = 2$

$$\vartheta = \begin{pmatrix} -\frac{a_0(1+a_1^2)}{a_2} + \frac{3}{35}a_2^2H^4 - a_0(a + 3a_1H) + \frac{1}{70}a_2H^2(-120a_0 + 7a_1H) \\ -\frac{1}{70a_2}\left(35a_1^3 + 105a_1^2a_2H + 32a_2^3H^3 + a_1(35 + 102a_2^2H^2)\right) \\ a_0 + \frac{1+a_1^2}{2a_2} + \frac{3a_1H}{2} + \frac{6a_2H^2}{7} \end{pmatrix} \tag{22}$$

It can be shown that in most cases approximation order $n = 2$ is sufficient. Equations allowing calculation of the elements of the vector ϑ and approximation coefficients for higher values of *n* can be easily calculated by software for symbolic calculations. From obtained value of the vector ϑ approximate values of the surface generatrix are calculated

$$\hat{z}_0 = \vartheta_3\,,\ \hat{r}_0 = \vartheta_2\,,\ \hat{R}_0 = \sqrt{\vartheta_1 + \hat{r}_0^2 + \hat{z}_0^2}\,. \tag{23}$$

Finally, one should check if an analyzed surface is a barrel ($\hat{r} < 0$) or a saddle ($\hat{r} > 0$).

Step 3. In the third step of the procedure the Gauss-Newton algorithm is applied. In order to do this a vector of parameters $\theta = [e_x \quad e_y \quad d_x \quad d_y \quad r_0 \quad R_0 \quad z_0]^T$, a vector of initial parameters values calculated in the first two steps of the procedure $\hat{\theta} = [\hat{e}_x \quad \hat{e}_y \quad \hat{d}_x \quad \hat{d}_y \quad \hat{r}_0 \quad \hat{R}_0 \quad \hat{z}_0]^T$ and a vector of spatial variables $\xi = [\varphi \quad \Delta r \quad z]^T$ are defined.

Relevant partial derivates of the function $h(\xi, \theta) \overset{def}{=} h(\varphi, \Delta r, z, e_x, e_y, d_x, d_y, r_0, R_0, z_0)$ are equal to

$$\frac{\partial h(\xi,\theta)}{\partial \theta} = -\begin{bmatrix} \cos\varphi \\ \sin\varphi \\ z\cos\varphi \\ z\sin\varphi \\ 1 \\ \frac{-R_0}{\sqrt{R_0^2 - (z - z_0)^2}} \\ \frac{-(z - z_0)}{\sqrt{R_0^2 - (z - z_0)^2}} \end{bmatrix} \tag{24}$$

for barrel-shaped surface and

$$\frac{\partial h(\xi,\theta)}{\partial \theta} = -\begin{bmatrix} \cos\varphi \\ \sin\varphi \\ z\cos\varphi \\ z\sin\varphi \\ 1 \\ \frac{R_0}{\sqrt{R_0^2 - (z - z_0)^2}} \\ \frac{(z - z_0)}{\sqrt{R_0^2 - (z - z_0)^2}} \end{bmatrix} \tag{25}$$

for saddle-shaped surface.

In the last step we regard also to the tip radius of the measuring sensor. If analyzed surface has shape of a barrel then the generatrix radius is equal to the difference of R_0 and the tip radius. And if analyzed surface is a saddle, then the generatrix radius is the sum of R_0 and the tip radius

3. EXPERIMENTAL VERIFICATION OF DEVELOPED CONCEPT

In order to evaluate accuracy of proposed methodology and procedures measurements of nominally barrel- and saddle-shaped elements were performed. The measurements were carried out with a special-purpose measuring device. The device has been designed and built on the basis of Talyrond 3 measuring instrument and equipped with an original amplifier that has been integrated with a microprocessor system. The driver of the device is controlled by a PC and original software. The original software contains procedures allowing minimization of the influence of nonlinearities between the changes of the workpiece radius and indications of the measuring sensor (including the most significant ones: nonlinearity related to the rotation of the sensor and nonlinear characteristics of the measuring sensor). The software contains also original procedures allowing filtration of rotary surfaces, whose radius is not constant. The procedures enable separation of individual types of irregularities (mainly waviness and roughness).

In order to carry out comparing research, the workpieces were measured also by the coordinate measuring machine Eclipse 550 by Zeiss.

The research was performed using the typical in mechanical engineering rotary workpieces distinguished by large dimensional variation. In hitherto practice, such workpieces were evaluated only as a special case of cylindrical elements.

Taking to account that such verification is necessary, following workpieces have been designed:
- three barrel-shaped elements with generatrix radii equal to: R = 63[mm], R = 250[mm], R = 1000[mm],
- three saddle-shaped elements with generatrix radii equal to: R = 63 [mm], R = 250 [mm], R = 1000 [mm].

Obtained measurements results are given in following tables.

Table 1. Comparison of measurements results of the barrels' radii obtained by proposed method and by coordinate measuring machine Eclipse 550

Barrel radius [mm]	**Result from CMM [mm]**	**Results from the tested measuring device [mm]**	**Difference (absolute value) [mm]**	**Relative error [%]**
R=1000	1000,1595	999,301	0,859	0,09
R=250	250,1814	250,022	0,159	0,06
R=63	62,9736	62,9695	0,004	0,01

Table 2. Comparison of measurements results of the saddles' radii obtained by proposed method and by coordinate measuring machine Eclipse 550

Saddle radius [mm]	**Result from CMM [mm]**	**Results from the tested measuring device [mm]**	**Difference (absolute value) [mm]**	**Relative error [%]**
R=1000	1000,330	998,771	1,559	0,16
R=250	250,344	250,107	0,237	0,09
R=63	62,868	63,008	0,140	0,22

Measurements results given in the tables 1 and 2 show that relative difference of measurements results of radii lies within the range from 0,01% to 0,09% for barrels and from 0,09% do 0,22% for saddles. The errors are comparable for different radii, so one can assume that the new concept can be used for evaluation of elements, whose radii differ a lot.

3. SUMMARY AND CONCLUSIONS

Elements, whose generatrix is nominally a fragment of a circle constitute significant group of machine elements. They occur in a large number, for example, in the bearing industry (barrel bearings).

In hitherto practice, evaluation of such elements was carried out with use of methodology and procedures relating to cylindrical surfaces. However, such approach does not allow accurate measurement and evaluation of profiles, whose nominal generatrix constitutes a fragment of a circle. In order to evaluate a profile precisely, right definition of reference surface is necessary (reference surfaces according to the new standards are called associated surfaces). Therefore, research efforts aiming at development of the procedure allowing definition of relevant associated features for elements, whose generatrix constitutes a fragment of a circle were taken. The procedure consists of three steps. The first two steps enable obtaining good approximations of values of associated surface parameters. The third step of the Gauss-Newton procedure. Developed methodology has been tested by computer simulations and verified in practice through experiments. During the experiments a special purpose measuring device was used. The basis of the device is Talyrond 3 measuring instrument that has been equipped with original amplifier integrated with a microprocessor system. The device has been equipped with an original software allowing accurate measurement of elements, whose generatrix constitutes a fragment of a circle. The software includes original procedures that enable correction of different types of nonlinearities (for example nonlinearities related to the angular displacement of the sensor and nonlinear characteristics of the sensor). Developed software procedures allow also filtration of rotary surfaces, whose radius is not constant (including barrel and saddle shaped ones). The filtration procedures enable separation of individual types of surface irregularities (mainly waviness and roughness). The results of comparing research presented in the table 1 and 2 show that developed concept proves correct in practice and it allows accurate measurement and evaluation of profiles of elements, whose generatrix constitutes a fragment of a circle.

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