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ON-LINE ESTIMATION OF PARAMETERS OF A TIME SERIES

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Abstract − In the paper the paradigm of on-line (real-time) computation of parameters of a time series is developed. As an illustration the computation of the time deviation and Maximum Time Interval Error of the time error signal is shown.

Keywords: time error, paradigm, on-line.

1. INTRODUCTION

The usual practice of characterizing the time series obtained from the measurement is off-line estimation: the computation of some set of parameters of interest after the measurement is completed. Sometimes, however, it is reasonable to build up a parameters' value during the course of measurement. This on-line approach obviously saves the total time of measurement and parameters' calculation making it practically equal to the time of measurement. Sometimes tracking the current value of a parameter of some process during its course helps us to identify an influence of an environment on the process represented by a time series. In telecommunication there are good reasons (resulting from the Recommendation of International Telecommunication Union [1]) to stop collecting samples of time error on some network interface if a current value of Maximum Time Interval Error exceeds the limits set by ITU Recommendation; so after identifying the bad quality of the timing signal, we quit usually long-lasting measurements and save corresponding resources. We can imagine such a case not only in telecommunication. But in the following considerations we concentrate on a time series being a train of samples of time error. We will consider on-line (real-time) estimation of two parameters usually used to characterize time error signal (timing signal), namely time deviation (TDEV) and Maximum Time Interval Error (MTIE).

2. OFF-LINE ANALYSIS OF TIMING SIGNALS

Time deviation TDEV and Maximum Time Interval Error MTIE are commonly used for describing the quality of synchronization signal in the telecommunication network [1, 2, 3]. The parameters allow assessing the variations of time interval provided by the synchronization signal. Time deviation allows recognition of the type of phase noise affecting the signal. Maximum Time Interval Error is used for dimensioning of the buffers (elastic stores) at the border between time scales in the network. The estimates of the parameters are computed for a series of observation intervals, starting from some τ_{min} until some τ_{max}, using the sequence of time error samples previously measured at some network interface. The evaluation of the synchronization signal is commonly a two-stage process. First, the sequence of samples of time error between the analyzed signal and some reference has to be measured. Then, when the measurement is completed, the calculation of the parameter's estimate is performed.

Time deviation and Maximum Time Interval Error are computed usually for the observation intervals from the range between 0.1 s and 100 000 s, because the international standard institutions (ITU, ETSI) have defined recommendations (limit values of TDEV and MTIE) for this range of observation intervals. Some conditions for the measurement of time error which is used for reliable timing parameters' estimation are also specified. Time error must be sampled uniformly with a sampling interval τ_0. The minimum observation interval τ_{min} must be three times greater than the sampling interval τ_0. The minimum measurement time, necessary for reliable estimation of the parameters, should be twelve times longer than the maximum observation interval τ_{max}. According to these conditions, if we want to compute the parameters for the range of observation intervals 0.1 s – 1000 s, time error must be measured with the sampling interval τ_0=1/30 s during the time of 12 000 s (3 hours 20 minutes). Computation for the observation interval of 100 000 s requires the time error measurement during almost two weeks.

The formula of the parameter's estimator (especially for MTIE) and the large number of data (resulting from long lasting measurement) may cause rather long time of the estimate's computation. The computation of TDEV may last several minutes, but the MTIE computation for the same range of observation intervals may last several hours. E.g. time of MTIE computation performed by means of direct method (plain computation) for 21 observation intervals from the range 0.1 s – 1000 s for the time error sequence having 120 001 samples taken with sampling interval τ_0=1/30 s exceed nine hours. Long time of computation process together with long lasting time error measurement make the analysis of timing signal very time consuming. Additionally, according to off-line estimation rule, we have not any knowledge about the quality of the signal analyzed during the measurement until we start the computation.

3. PARADIGM OF ON-LINE COMPUTATION

A general paradigm of on-line computation is: *compute all what you can compute till a given moment and recognize and store at this moment all current outcomes you can profitably use in the forthcoming computation, but remember that all necessary job has to be done before the next sample (next item of a time series) is gained*. The form of the current outcomes depends on the expression for a parameter's estimator. The current outcomes might be some sums (e.g. TDEV) or some series of extremes (e.g. MTIE).

Very often the computed parameter of a time series is a function of some measurement's time dependent parameter. The set of the values of that parameter grows in number along the course of measurement of time series items. Growing number of observation intervals (windows) on which TDEV and MTIE depend is a good example. In such situation we are interested in some collection of the time series parameter's estimates conditioned on different values of the measurement's time dependent parameter.

Obviously, on-line computation becomes more complicated in such case as we have to perform parallel (quasi-parallel) computing. The reduction of a primary data (raw data), whenever it is possible, may ease the computation.

The aspects of real-time parallel computing will be illustrated in the examples of TDEV and MTIE.

4. REAL-TIME TDEV COMPUTATION

In order to gain some knowledge about the quality of a timing signal, we usually compare it with some reference, thus obtaining time error signal $x(t)$ [1, 2, 3]. Computation of time deviation involves the averaging of second differences of time error signal $x(t)$, assuming a negligible influence of frequency drift [4].

The formula for the estimator of the time deviation TDEV [1, 2, 3] takes the form:

$$T\hat{D}EV(\tau)=\sqrt{\frac{1}{6n^2(N-3n+1)}\sum_{j=1}^{N-3n+1}\left[\sum_{i=j}^{j+n-1}(x_{i+2n}-2x_{i+n}+x_i)\right]^2} \quad (1)$$

where $\{x_i\}$ is a sequence of N samples of time error function $x(t)$ taken with interval τ_0; $\tau=n\tau_0$ is an observation interval. To simplify the calculation of sums in (1) TDEV estimator's formula is rearranged to the form [4]:

$$T\hat{D}EV(n\tau_0)=\sqrt{\frac{1}{6}\cdot\frac{1}{N-3n+1}\cdot\frac{1}{n^2}\sum_{j=1}^{N-3n+1}S_j^2(n)} \quad (2)$$

where

$$S_j(n)=S_{j-1}(n)-x_{j-1}+3x_{j+n-1}-3x_{j+2n-1}+x_{j+3n-1} \quad (3)$$

$$S_1(n)=\sum_{i=1}^{n}(x_{i+2n}-2x_{i+n}+x_i) \quad (4)$$

The indexes in the formula for TDEV estimator must be changed in the case of the real-time calculation. We have no access to the time error samples indexed by $i+n$ or $i+2n$ for a time instant described by index i (currently measured sample). This rearrangement of indexes was performed in [5]. After changing the indexes of the simplified formulae (2-4), we have obtained:

$$T\hat{D}EV_i(n\tau_0)=\sqrt{\frac{1}{6}\cdot\frac{1}{i-3n+1}\cdot\frac{1}{n^2}S_{ov,i}(n)} \quad (5)$$

where $S_{ov,i}(n)$ is the overall sum updated for each sample i, given in the form:

$$S_{ov,i}(n)=S_{ov,i-1}(n)+S_i^2(n) \quad (6)$$

where

$$S_i(n)=S_{i-1}(n)-x_{i-3n}+3x_{i+2n}-3x_{i+n}+x_i,\ i>3n \quad (7)$$

$$S_{3n}(n)=\sum_{j=2n+1}^{3n}(x_j-2x_{j-n}+x_{j-2n}),\ j>2n \quad (8)$$

Finally, the operations of TDEV computation for the i-th sampling interval are performed using the formula (9) [5]:

$$T\hat{D}EV_i(n\tau_0)=\sqrt{\frac{1}{6n^2(i-3n+1)}\left[S_{ov,i-1}(n)+(S_{i-1}(n)+\Delta_i(n))^2\right]} \quad (9)$$

where

$$\Delta_i(n)=x_i-3x_{i+n}+3x_{i+2n}-x_{i-3n} \quad (10)$$

As a result of the rearrangement of the parameter's formula, in order to compute TDEV, for a current sampling instant i and given observation interval $\tau=n\tau_0$, we need the values of appropriate sum $S_{ov,i-1}(n)$ and $S_{i-1}(n)$, currently measured sample x_i and the samples x_{i-n}, x_{i-2n}, and x_{i-3n} previously measured and stored in memory.

The formula of TDEV estimator (1), the TDEV estimator given by the simplified formulae (2-4), and the formula in the form given by (9) for the sampling instant $i=N$, are identical. The formula (9) allows us to perform the calculation in the real time, during the measurement of time error samples. A general procedure of the real-time quasi-parallel TDEV computation for a series of observation intervals is as follows [5]:

1. Measure a new time error sample and store it in a data file.
2. Compute the difference for a given n (observation interval $\tau=n\tau_0$) using a current sample, and the samples measured n, $2n$ or $3n$ sampling intervals earlier.
3. Update the sum and compute the square.
4. Compute the current average and its square roots.
5. Execute steps 2 to 4 for successive greater observation intervals (greater n).
6. Execute step 1 (measure a new sample).
7. When the measurement is finished, the values of the parameter's estimate for the observation intervals considered are known.

Steps 2 to 5 can be executed, when a sufficient number of time error samples were measured, i.e. $2n+1$ samples for a given n. For this instant the first item of internal sum $S_{3n}(n)$ can be computed. The sum $S_{3n}(n)$ is updated until the sample number $3n$ is measured. Starting from this instant, the sum $S_i(n)$ is updated using the samples number $i-3n$, $i-2n$, $i-n$, and i, according to (7), and the overall sum $S_{ov,i}(n)$ is updated according to (6). When the updating for a given n is finished, the conditions for the successive (greater) observation intervals are checked, and necessary operations for the intervals are performed.

An example of the real-time TDEV computation process for the two observation intervals – $3\tau_0$ and $5\tau_0$ – is presented in Fig. 1 [5]. The stage of the process after the measurement of the sample number 16 is presented. The overall sum $S_{ov,i}(3)$ is updated using the samples number 10, 13, and 16. The internal sum $S_1(3)$ was computed at the early stages of the process and its operator (second difference operator) is not active now. The internal sum $S_1(5)$ was computed and the overal sum $S_{ov,i}(5)$ is updated using the samples number 1, 6, 11, and 16.

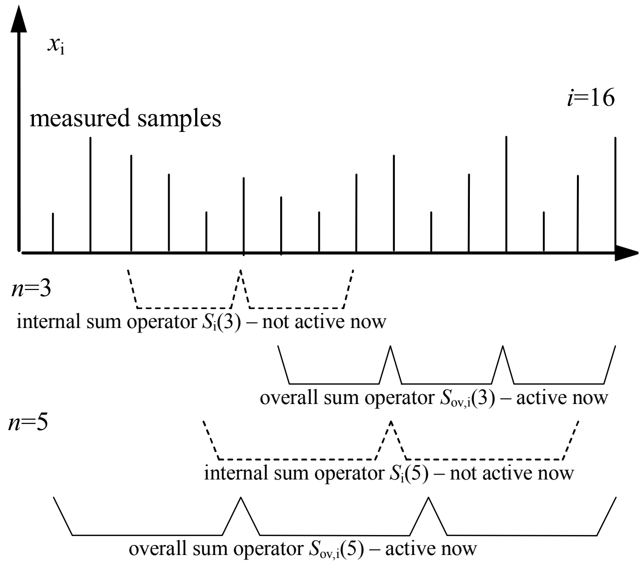

Fig. 1. Real-time TDEV calculation for the observation intervals $3\tau_0$ and $5\tau_0$, sample number 16 has been measured.

The method of real-time TDEV computation described above was tested in a calculation experiment [5]. The calculations were performed off-line but the on-line work was imitated. Data sequence containing time error samples taken with the sampling interval τ_0=1/30 s was considered. This sequence, representing white phase noise (WPM) of timing signal, is presented in Fig. 2. The observed quantity was the maximum time spent for calculation within one sampling interval. The goal of the experiment was to test, whether or not, the time of operations performed within one sampling interval (between two successive samples) exceed the length of the sampling interval. We have assumed that this time cannot exceed the length of 1/30 s = 33.3… ms.

The computational tests were performed for changing numbers of simultaneously analyzed observation intervals (5, 10, and 20 intervals per decade) from the range between 0.1 s and 1000 s. The longest observation interval was changed from 1 s till 1000 s. The maximum time spent for computation within one sampling interval (maximum between-two-samples-computation time) using a personal computer with Pentium IV 3GHz microprocessor is presented in Table 1. The values of TDEV estimate computed in the experiment are presented in Fig. 3. The results of tests have proved the ability of real-time TDEV computation performed simultaneously for numerous series of observation intervals (up to 81 simultaneously analyzed observation intervals from the range 0.1 s – 1000 s were considered).

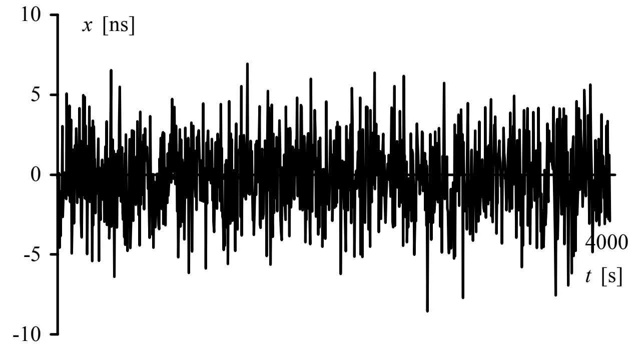

Fig. 2. WPM time error sequence.

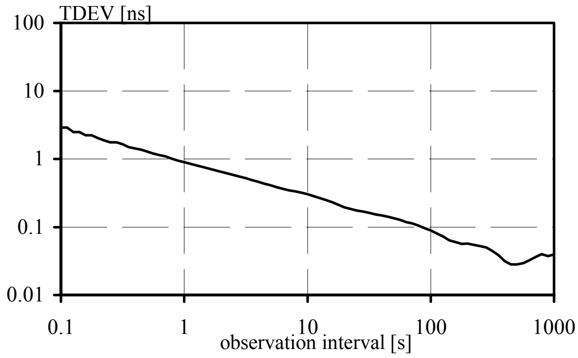

Fig. 3. Time deviation for the WPM time error sequence.

Table 1. The maximum between-two-samples-computation time of real-time TDEV assessment.

Range of observation intervals [s]	Number of intervals per decade		
	5	10	20
	t-max [ms]	t-max [ms]	t-max [ms]
0.1-1	0.18	0.3	0.6
0.1-10	0.3	0.6	1.2
0.1-100	0.5	0.9	1.8
0.1-1000	0.7	1.3	2.6

5. REAL-TIME MTIE COMPUTATION

The point estimate of the Maximum Time Interval Error is defined in the international standards as the maximum peak-to-peak variation of a given time error signal within some observation interval [1, 2, 3]. MTIE can be estimated from the formula:

$$M\hat{T}IE(n\tau_0) = \max_{1\le k\le N-n}\left(\max_{k\le i\le k+n} x_i - \min_{k\le i\le k+n} x_i\right) \qquad (11)$$

where $\{x_i\}$ is a sequence of N samples of time error signal $x(t)$ taken with sampling interval τ_0, $\tau=n\tau_0$ is an observation interval, and n can change from 1 till N–1 depending on the considered values of observation intervals.

Following directly the formula (11) in order to find the estimate of MTIE for the observation interval τ, all intervals having the width of τ, existing in the sequence of N time error samples, must be reviewed. The window having the width of $\tau=n\tau_0$ and including n+1 samples is set at the beginning of the data sequence $\{x_i\}$ and then it is shifted with the step of τ_0 to the end of the sequence. For each window's location the peak-to-peak value of time error in the window is found. The maximum peak-to-peak value found for all existing locations of the window is the value of MTIE(τ) estimate. The process of window reviewing does not depend on the data value. The complexity of calculation grows with n and therefore the direct method is really time-consuming. The idea of direct search (plain computation) of MTIE is presented in Fig. 4.

The formula of the MTIE estimator allows us to perform the calculation of the parameter in the real-time, during the time error measurement. Therefore we can observe the value of the parameter during the long lasting measurement process. Any possible wrong behavior of the analyzed signal (recognized, if MTIE exceeds the limit) enables applying proper activity of a maintenance team.

A general procedure of the real-time quasi-parallel MTIE calculation for a series of observation intervals is as follows [7, 8]:

1. Measure a new time error sample.
2. Compare the new sample with current maximum and minimum.
3. If the current window's location is filled out with samples, fix the extremes for this location.
4. Check if the current window's location is filled out with samples for the next longer observation interval.
5. If so, find the extremes for this location and check the conditions for the next longer observation interval.
   If no, measure a new sample.
6. When the measurement is finished, continue the computation for the remaining locations of each longer observation interval.

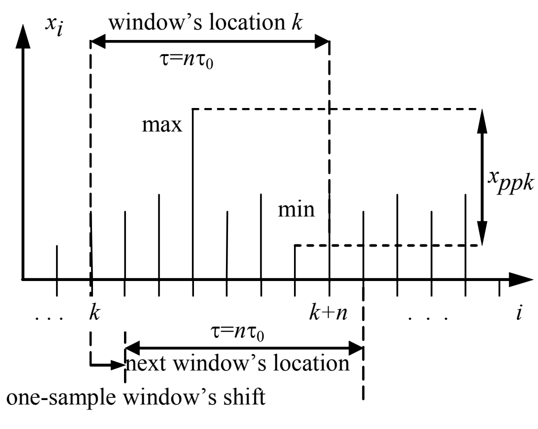

Fig. 4. The idea of direct search for MTIE.

The choice of the algorithm suited for the real-time parallel calculation is very important. Because all necessary operations have to be performed in the time period between two successive sampling instants (during the sampling interval τ_0), the calculation algorithm should be time effective – the time of operations performed within one sampling interval must be shorter than the length of the sampling interval. The authors of the paper have adopted and tested three time effective methods for real-time calculations [7, 8]. The principles of one of these methods will be presented below.

The real-time computation using direct search with sequential data reducing (DSDR) method [7] for the first (shortest) observation interval τ_{min} begins with the first measured time error sample. Each new sample measured is compared with current maximum and minimum values, until the first window's location is filled out with the samples. Then the extreme values for this location are fixed. Each successive measured sample creates a new window's location. The extreme samples found for each window's location (indicated by white and black stars for current window's location) create new data sequences with reduced number of items. The items of reduced data sequences are used for the MTIE estimate calculation for the observation intervals longer than τ_{min}. The first location of the next longer window is not analyzed until all samples situated in this location are reviewed by the preceding window. The example of computation using the DSDR method is presented in Fig. 5 [7]. Fourteen samples have been measured and the first location of the 8-sample window can be analyzed, because all samples situated in this location have been analyzed by the preceding 6-sample window. The 10-sample window has not been activated yet.

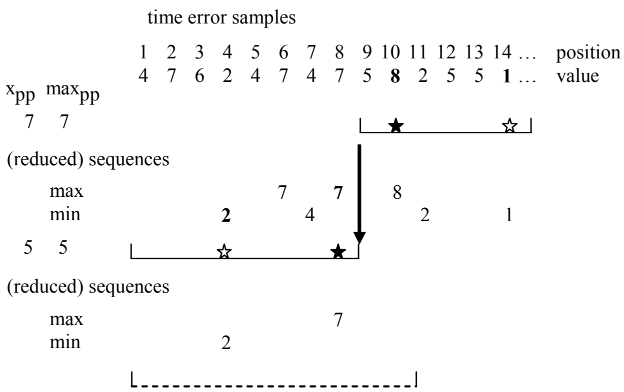

Fig. 5. Real-time MTIE computation using DSDR method for observation intervals having 6, 8 and 10 samples.

The method presented above was tested similarly as the method of real-time TDEV computation [7, 8]. The calculations were performed for the time error samples taken with the sampling interval τ_0=1/30 s. Because the data reduction process depends on the data behaviour, different types of time error data were considered. The first data sequence, being the result of the comparison of two oscillators disciplined by the signals from the Global Positioning System (GPS), is presented in Fig. 6. The second data sequence (Fig. 7) was the result of comparison of two independent internal oscillators of a timing signal measurement system (MSG). The calculations were performed for 5 observation intervals per decade. The maximum time spent for computation within one sampling interval (maximum between-two-samples-computation time) using two computers – the first with Pentium IV 3 GHz microprocessor and the second with Core 2 Quad 2.83 GHz microprocessor – is presented in Table 2.

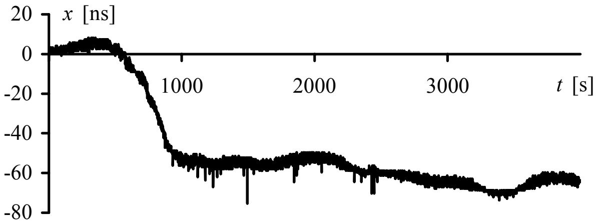

Fig. 6. GPS time error sequence.

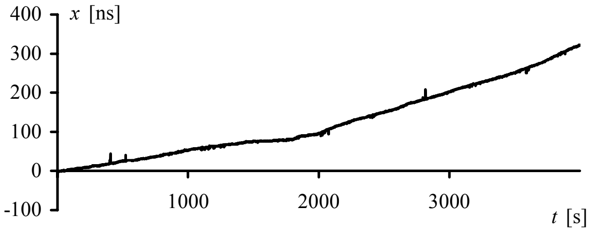

Fig. 7. MSG time error sequence.

Table 2. The maximum between-two-samples-computation time of real-time MTIE assessment.

Range of observation intervals [s]	Pentium IV 3.0 GHz		Core 2 Quad 2.83 GHz	
	GPS	MSG	GPS	MSG
	t-max [ms]	t-max [ms]	t-max [ms]	t-max [ms]
0.1-1	2.2	2.7	-	-
0.1-10	4.4	7.1	-	-
0.1-100	7.7	24.7	4.4	10.4
0.1-1000	11.0	54.9	5.5	21.4

The results of tests have showed very good performance of the method for narrow ranges of the observation intervals (0.1 s – 1 s and 0.1 s – 10 s). If the data sequence is affected by monotonic changes (being result of frequency difference, as for the MSG sequence), the between-two-samples-computation-time for the computer with older microprocessor may exceed the sampling interval, especially for wider range of observation intervals considered simultaneously. Application of the computer with modern microprocessor (computations were performed for wider ranges of observation intervals only) brought very good results for this range of observation intervals – the observed time was shorter than the sampling interval considered. The results of the MTIE estimate computation for both data sequences considered are presented in Fig. 8 and 9.

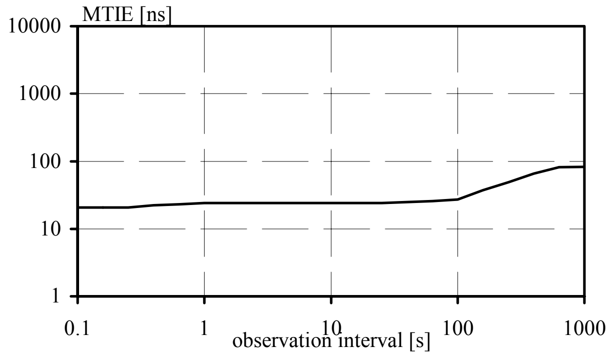

Fig. 8. MTIE for the GPS time error sequence.

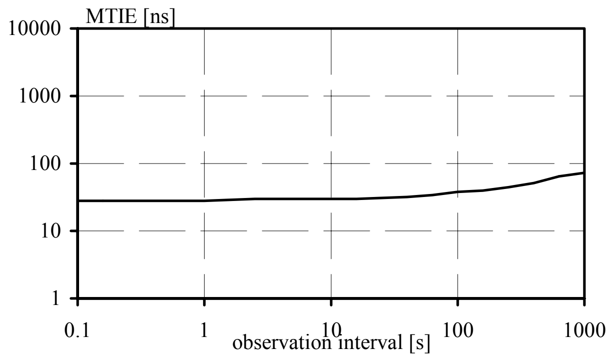

Fig. 9. MTIE for the MSG time error sequence.

6. CONCLUSIONS

The real-time calculation of some number of parameters appears to be challenging task. The ability of real-time computation depends on the several conditions: number and length of the observation intervals considered, computational power of the measurement equipment, computation method applied, and time error data behavior. Therefore we should make the algorithm of computation as effective as possible.

The results of the experimental tests presented in the paper have proved the ability of the real-time computation of the time deviation and the Maximum Time Interval Error. The computation complexity of the time deviation does not depend on the length of observation interval; the number of observation intervals considered is the only limiting factor. The behavior of the time error data may influence on the time of MTIE computation within one sampling interval, especially for long observation intervals. Application of fast computation equipment or limitation of considered range of observation intervals simultaneously analyzed may be necessary in such a case. Both parameters considered can be computed in the real time, simultaneously for the series of observation intervals in the course of the time error measurement process performed with the sampling interval τ_0=1/30 s often used in the telecommunication applications.

The real benefit of on-line computation is a current knowledge about the process under observation. The value of the synchronization signal's parameter (TDEV or MTIE) can be observed during the long lasting time error measurement process. We have also knowledge about the variation in time of the considered parameter's value.

These remarks have rather strong support arising from our experience of assessment of timing signal in the telecommunication systems and networks.

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